High Energy Physics and Nuclear Physics

Volume 19, Number 3

Further Studies on Halo Nuclei ^{11}Li, ^{14}Be and ^{17}B

Ren Zhongzhou[1,2] and F. Carstoiu[2,3]

[1](Department of Physics, Nanjing University, Nanjing, China)
[2](GANIL, BP 5027, 14021, Caen CEDEX, France)
[3](Institute of Atomic Physics, Dept. of Theor. Phys., Bucharest, Romania)

Based on the three-body model with Yukawa interactions, halo nuclei ^{11}Li, ^{14}Be and ^{17}B are further studied by the variational calculation. An analytical expression for the density distribution of the halo neutron is given. The theoretical results agree well with recent experimental data.

Key words: halo nuclei, halo neutron density, three-body model.

1. INTRODUCTION

The study of the properties of halo nuclei ^{11}Li, ^{14}Be and ^{17}B is a hot point nuclear physics. It is clearly shown by experiments that the last two neutrons in ^{11}Li, ^{14}Be and ^{17}B are weakly bound and have formed the neutron halo. It has lead to the abnormally large radii of these nuclei. It is also observed [2] in the fragmentation process of ^{11}Li and ^{14}Be that the momentum distribution of nucleons has a two-peak structure, a narrow one superposing on a wide one.

On the theoretical research side, the results of the nuclear shell model [3] and Hartree-Fock shell model [4] do not agree with the experimental data of ^{11}Li and ^{14}Be. The three-body model [1] can almost reproduce the experimental data of the ground state energies and extraordinarily large radii of the halo nuclei. But, in the previous variational three-body calculation [1], the trial wave function is too simple so that the theoretical result does not agree well with the newest experimental data [5-7]. Therefore, we shall carry out further variational calculations in the three-body model to study the ground state properties of these nuclei, in this paper.

Received on April 4, 1994

2. VARIATIONAL CALCULATION OF THE THREE-BODY PROBLEM

Assuming [1] that ^{11}Li, ^{14}Be and ^{17}B are three-body systems composing of a core and two neutrons. The core-neutron and neutron-neutron interactions are taken as attractive exponential potentials. The trial wave function $\psi = e^{-\alpha(r_1+r_2)-\beta r_3}$ was used to calculate the ground state energy and the root-mean-square radius of the halo nuclei in the previous paper. Here two improvements will be made in this paper: one is on the interaction and the other is on the trial wave function.

In the frame of the core-neutron-neutron three-body model, the relative distances of the core-neutron (C-N), core-neutron and neutron (N-N) are, respectively, denoted by r_1, r_2 and r_3. The interactions of the core-neutron and neutron-neutron are chosen as attractive Yukwawa potentials.

The N-N potential in the singlet S-state is determined by fitting the low energy data of two-nucleon system [9]

$$V(r_3) = -s \cdot (147.585) b^{-2} \left(\frac{b}{r}\right) \exp(-2.1196 r_3 / b), \tag{1}$$

where $s = 0.949$ and $b = 2.06$fm are, respectively, the well depth parameter and the force range parameter.

The C-N potential is chosen as

$$U(r_i) = -s_c \cdot (147.585) b_c^{-2} \left(\frac{b_c}{r}\right) \cdot \left(\frac{N_c + 1}{2N_c}\right) \exp(-2.1196 r_i / b_c), \tag{2}$$

where $i = 1, 2$ and s_c is the well depth parameter. There is no bound state for the C-N system while $s_c \leq 1$. The force range b_c is chosen as 5.0fm, which is approximately equal to the sum of the core radius and the range of the N-N potential. N_c is the nucleon number of the core, i.e. $N_c = 9$ for ^{11}Li.

We shall solve the three-body problem by the variational method in the triangular coordinate system. After the center-mass motion is eliminated in the coordinate system, the wave function of the ground state will depend only on three radial variables r_1, r_2 and r_3 which are subject to the usual triangular inequalities, $r_1 + r_2 \geq r_3$, $r_2 + r_3 \geq r_1$, $r_3 + r_1 \geq r_2$. The trial wave function of the core-neutron-neutron three-body system is taken to be

$$\Psi = (e^{-a_1 r_1} + x e^{-b_1 r_1})(e^{-a_1 r_2} + x e^{-b_1 r_2}) \cdot (e^{-a_3 r_3} + y e^{-b_3 r_3}) \cdot G, \tag{3}$$

where a_1, b_1, x and a_3, b_3, y are variational parameters. G is the normalization factor of the wave function.

The expectation value of the Hamiltonian in the triangular coordinate system is as follows:

$$\begin{aligned} E &= \langle \Psi | H | \Psi \rangle \\ &= 8\pi^2 \int r_1 dr_1 r_2 dr_2 r_3 dr_3 \\ &\quad \cdot \left\{ \frac{\hbar^2(m_c + m)}{2mm_c} \left[\sum_{i=1}^{3} \left(\frac{\partial \Psi}{\partial r_i}\right)^2 + t(1,2,3) + t(2,3,1) + t(3,1,2) \right] \right. \\ &\quad + \frac{\hbar^2(m_c - m)}{2mm_c} \left[\left(\frac{\partial \Psi}{\partial r_3}\right)^2 - t(1,2,3) + t(2,3,1) + t(3,1,2) \right] \\ &\quad \left. + [U(r_1) + U(r_2) + V(r_3)] \Psi^2 \right\}, \end{aligned} \tag{4}$$

Table 1.
The numerical results of ^{11}Li, ^{14}Be and ^{17}B.

	s_c	b_c (fm)	E_0 (MeV)	$\overline{r_1^2}$ (fm^2)	$\overline{r_3^2}$ (fm^2)	r_c (exp.)	R_m (fm)	E_0 (exp.)	R_m (exp.)
^{11}Li	0.82	5.0	−0.35	33.60	42.47	2.31	3.12	−0.35 (±0.05)	3.10 (±0.17)
^{14}Be	0.90	5.0	−0.72	22.36	29.94	2.57	2.92	−1.34 (±0.11)	3.10 (±0.30)
^{17}B	0.91	5.0	−0.78	21.10	28.69	2.50	2.82	−1.39 (±0.14)	3.00 (±0.40)

where

$$t(i,j,k) = \frac{r_i^2 + r_j^2 - r_k^2}{2r_i r_j} \frac{\partial \Psi}{\partial r_i} \frac{\partial \Psi}{\partial r_j}, \tag{5}$$

with $i \neq j \neq k = 1, 2, 3$; m the mass of the neutron, $m_c = N_c \cdot m$ the mass of the core.

Letting the variational parameters a_1, b_1, x, and a_3, b_3, y vary, we can obtain the minimum of the binding energy. Then we shall further obtain the ground state energy and wave function. The mean-square radii of C-N and N-N will be calculated from the wave function. They are defined as fellows:

$$\overline{r_1^2} = \langle \Psi | r_1^2 | \Psi \rangle, \tag{6}$$
$$\overline{r_3^2} = \langle \Psi | r_3^2 | \Psi \rangle. \tag{7}$$

The matter root-mean-square (RMS) radius of the C-N-N system is calculated by the following equation [1]

$$R_m = \overline{R_m^2}^{\frac{1}{2}} = \left\{ \frac{N_c}{N_c + 2} \left[\overline{r_c^2} + \frac{2}{N_c + 2} \overline{r^2} \right] + \frac{\overline{r_3^2}}{2(N_c + 2)} \right\}^{\frac{1}{2}}, \tag{8}$$

where

$$\overline{r^2} = \overline{r_1^2} - \overline{r_3^2}/4 \tag{9}$$

is the mean-square distance between the center of mass of the two neutrons and the core. r_c is the RMS radius of the core.

The numerical results are listed in Table 1. In numerical calculations, b_c is chosen to be 5.00fm, we adjust the well depth parameter s_c to give the binding energies and radii of nuclei ^{11}Li, ^{14}Be and ^{17}B.

In Table 1, E_0 and R_m are theoretical values of binding energies and radii; E_0(exp) and R_m(exp) are the newest data of the ground state energies and RMS radii [5-7]. For the RMS radii, theoretical results agree with experimental data. The theoretical binding energy of ^{11}Li also agree with experimental datum but those of ^{14}Be and ^{17}B are slightly higher than experimental data.

If we compare present results with Refs. [1] and [8], we find the results of this paper are closer to the experimental data.

It is seen from Table 1 that although the force range of C-N and N-N are about 1-2fm, the RMS radii of C-N and N-N are about 4-7fm and this indicates that C-N and N-N are mainly outside of the force range and neutron halos appear.

In Table 2, we have given corresponding values of variational parameters a_1, b_1, x and a_3, b_3 and y.

Using the above expression of the ground state wave function in the triangular coordinate system and completing the length integral on r_2 and r_3, we have obtained an analytical expression of the

Table 2.
The values of variational parameters.

	a_1	a_3	b_1	b_3	x	y
^{11}Li	0.1477	0.1077	0.3561	0.8500	1.9085	2.8180
^{14}Be	0.1865	0.1115	0.3751	0.8796	2.2802	2.6401
^{17}B	0.1905	0.1093	0.3721	0.8808	2.6335	2.6447

probability distribution of the halo neutron relative to the core:

$$\begin{aligned}\rho_{c-n} = C\cdot(e^{-a_1r_1} + xe^{-b_1r_1})^2\cdot\Big\{&\phi(a_1+a_3, a_1-a_3)\\
&+2x\phi\left(\frac{a_1+b_1+2a_3}{2},\frac{a_1+b_1-2a_3}{2}\right)\\
&+2y\phi\left(\frac{2a_1+b_3+a_3}{2},\frac{2a_1-a_3-b_3}{2}\right)+x^2\phi(b_1+a_3,b_1-a_3)\\
&+y^2\phi(a_1+b_3,a_1-b_3)+4xy\phi\left(\frac{a_1+b_1+a_3+b_3}{2},\frac{a_1+b_1-a_3-b_3}{2}\right)\\
&+2x^2y\phi\left(\frac{2b_1+a_3+b_3}{2},\frac{2b_1-a_3-b_3}{2}\right)\\
&+2xy^2\phi\left(\frac{a_1+b_1+2b_3}{2},\frac{a_1+b_1-2b_3}{2}\right)+x^2y^2\phi(b_1+b_3,b_1-b_3)\Big\},\end{aligned}\tag{10}$$

where C is the normalization factor, $\int_0^\infty 4\pi r_1^2\rho_{c-n}dr_1 = 1$. $\phi(\alpha,\beta)$ is a function of r_1,

$$\begin{aligned}\phi(\alpha,\beta) = r_1^3\frac{e^{-\alpha r_1}}{\alpha r_1}\Big\{&\frac{\sinh\beta r_1}{\beta r_1}\left[\frac{1}{\alpha r_1}+\frac{1}{(\alpha r_1)^2}\right]\\
&+\frac{1}{(\beta r_1)^2}\left[\cosh\beta r_1-\frac{\sinh\beta r_1}{\beta r_1}\right]\Big\}.\end{aligned}\tag{11}$$

The sum of all terms in Eq. (10) is a monotonous decreasing function of r_1. The analytical expression of the probability distribution of the neutron-neutron is:

$$\begin{aligned}\rho_{n-n} = C\cdot(e^{-a_3r_3}+ye^{-b_3r_3})^2&\\
\cdot\Big\{\phi(2a_1, 0)+4x\phi\left(\frac{3a_1+b_1}{2}, a_1-b_1\right)&+2x^2[2\phi(a_1+b_1,0)\\
+\phi(a_1+b_1,a_1-b_1)]+4x^3\phi&\left(\frac{a_1+3b_1}{2},\frac{a_1-b_1}{2}\right)+x^4\phi(2b_1, 0)\Big\},\end{aligned}\tag{12}$$

where C is the normalization factor, $\int_0^\infty 4\pi r_3^2 \rho_{n-n} dr_3 = 1$. $\phi(\alpha,\beta)$ is a function of r_3 and its expression is exactly the same as Eq. (11) except that r_1 is replaced by r_3. As the second variable in $\phi(\alpha,\beta)$ is zero, it should be replaced by the following expression:

$$\phi(\alpha,0) = r_3^3 \frac{e^{-\alpha r_3}}{\alpha r_3}\left[\frac{1}{2} - \frac{1}{6} + \frac{1}{(\alpha r_3)} + \frac{1}{(\alpha r_3)^2}\right]. \tag{13}$$

In order to check the correctness of the probability distribution Eq. (10) and Eq. (12), we have used $\overline{r_1^2} = \int \rho(r_1) r_1^4 dr_1 \cdot 4\pi$ and $\overline{r_3^2} = \int \rho(r_3) r_3^4 dr_3 \cdot 4\pi$ to calculate the mean-square radii of C-N and N-N and found that they agree with results by Eq. (6) and Eq. (7).

As the density distribution of halo neutrons relative to the core is needed in calculations on reactions with halo nuclei, Eq. (10) will be very useful.

If we assume that the density distribution of the core ^{9}Li in ^{11}Li is a Gasussian function, we can calculate the density distribution of ^{11}Li by Eq. (10) and the result is plotted in Fig.1. The detailed calculation in Fig. 1 is as follows,

$$\rho(^{11}\mathrm{Li}) = \rho(^{9}\mathrm{Li}) + 2 \cdot \rho_{c-n}, \tag{14}$$

$$\rho(^{9}\mathrm{Li}) = \frac{9}{\pi^{3/2} r_{0c}^3} e^{\frac{-r^2}{r_{0c}^2}}, \tag{15}$$

where $\rho(^{11}\mathrm{Li})$, $\rho(^{9}\mathrm{Li})$ and $\rho_{c\text{-}n}$ are, respectively, the density distributions of ^{11}Li, ^{9}Li and the halo neutron. The relationship between the parameter r_{0c} and the RMS radius of ^{9}Li is

$$\frac{3}{2} r_{0c}^2 = r_c^2. \tag{16}$$

In Fig.1, we have also given the density distribution of the core ^{9}Li and halo neutrons. It is seen from Fig.1 that the density distribution of ^{11}Li has a long tail due to the role of the halo neutrons.

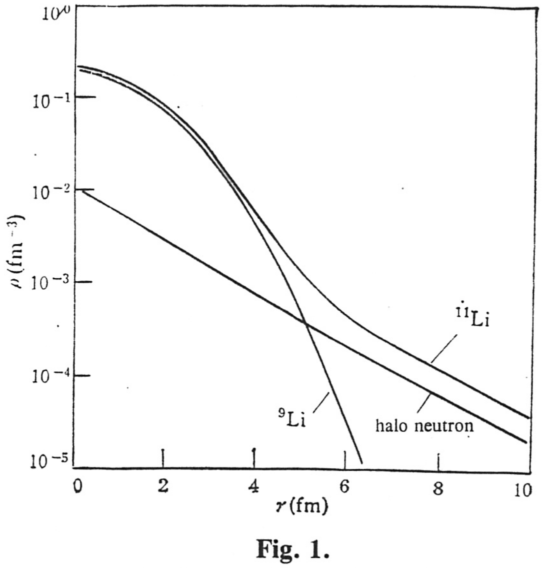

Fig. 1.
The density distribution of ^{11}Li.

3. CONCLUSIONS

Based on the three-body model and assuming that the C-N and N-N interactions in ^{11}Li, ^{14}Be and ^{17}B are Yukawa potentials, we have calculated the ground state energies, RMS radii, density distributions of the halo neutrons and ^{11}Li. The theoretical results agree better with the experimental data compared with other theoretical results.

REFERENCES

[1] Ren Zhongzhou, Xu Gongou, *High Energy Phys. and Nucl. Phys. (Chinese Edition)*, **15**(1991): p. 737.

[2] E. Roeckl, *Rep. Prog. Phys.*, **55**(1992): p. 1661.

[3] T. Hoshino *et al.*, *Nucl. Phys.*, **A506**(1990): p. 271.

[4] J. M. G. Gomez *et al.*, *Phys. Lett.*, **B295**(1992): p. 1.

[5] T. Kobayashi *et al.*, Preprint RIKEN-AF-NP-106 (1991).

[6] J. M. Wouters *et al*, *Z. Phys.*, **A331**(1988): p. 229.

[7] G. Audi, A. H. Wapstra, *Nucl. Phys.*, **A565**(1993): p. 1.

[8] Li Culiang, Duan Yiwu, and Huang Xizhi, *Acta. Physica Sinica (Chinese Edition)*, **43**(1994): p. 14.

[9] J. M. Blatt, J. D. Jackson, *Phys. Rev.*, **76**(1949): p. 18.