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Analytical Study of Two-Dimensional Self-Affine Fractal in Multiparticle Production of High Energy Collisions

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The self-affine cascading model of 2-dimensional phase space is discussed in some detail. The analytical expression for the scaled probability moments, C_N^2 and C_N^3, are deduced for the self-similar analysis of a 2-dimensional self-affine fractal object with Hurst exponent $H = 0.5$. The Levy stability indices of the model are calculated analytically. It is shown that the results of Levy indices from the Monte Carlo simulation are consistent with those from the analytical calculation. The recent experimental data from NA22 collaboration are explained qualitatively.

Key words: intermittency, Levy stability law, self-affine fractal, random cascading process.

1. INTRODUCTION

As a signal of fractal of multiparticle production spectrum in high energy collisions, the anomalous scaling behavior of the scaled factorial moments (SFM's) F_q with diminishing phase space cell δ [1]

$$F_q \sim \delta^{-\varphi_q}. \tag{1}$$

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has been investigated extensively [2]. It was shown by almost all the corresponding experimental collaborations in this field that the power-law (1) could not be satisfied exactly. When the phase space scale becomes very small, the 1-dimensional (1D) data of SFM's tend to saturation in the plot of $\ln F_q - \ln(1/\delta)$ and the higher dimensional data are bending upward, which urge people to proposes self-affine fractal in multi-particle production processes [3]. Meanwhile, in order to fit both the experimental data in different dimensions and the Monte Carlo results of some theoretical models, a new scaling formula has been proposed as a generalization of Eq.(1), i.e., [4]

$$F_q(\delta)=b_q[g(\delta)]^{\varphi_q}. \tag{2}$$

where $g(\delta)$ is an arbitrary function independent of the order q. Expressing $g(\delta)$ in terms of $F_2(\delta)$, one can find the linear relation

$$\ln F_q(\delta)=\ln b_q+\frac{\varphi_q}{\varphi_2}\ln F_2=\overline{b}_q+r_q\ln F_q, \tag{3}$$

where $r_q = \varphi_q/\varphi_2$ is directly proportional to the fractal dimensions d_q of the phase space. In terms of the Levy stability law of self-similar cascading process [5,6,7], r_q can be related to a parameter μ called the Levy stability index, i.e.,

$$\frac{d_q}{d_2}=\frac{r_q}{(q-1)}=\frac{q^\mu-q}{2^\mu-2}\,\frac{1}{q-1} \qquad (0<\mu\leqslant 2), \tag{4}$$

The Levy stability index μ is related to the strength of dynamical fluctuations of the cascading process. When the dynamical fluctuations are very small, the Gaussian law of probability distribution is satisfied approximately, $\mu = 2$; if the fractal is originated from a second-order phase transition, the dynamical fluctuations are the largest, and $\mu = 0$, which corresponds to a monofractal structure, i.e., $d_q = d_2$.

The Levy stability description of self-similar cascading process is investigated in Refs. 5, 6, and 8, which shows that the dependence of Levy stability index upon the dynamical fluctuations is consistent with the prediction of Levy stability law. However, since the dynamical fluctuations are most probably anisotropic [3], the Levy stability description of multiparticle production processes should be analyzed in self-affine way. The problem is that one does not know at the beginning what kind of self-affine fractal exists in multiparticle production processes, in particular the inherent Hurst exponent H which characterizes the anisotropy of the fractal system is unknown. Therefore, although the phase space should be shrunk anisotropically according to the value of Hurst exponent from the theoretical point of view, the SFM's had been calculated with the phase space being shrunk isotropically in present experiments. We call this the "self-similar analysis of self-affine fractal."

Until now, the investigation about the self-affine fractal of multiparticle production has remained at the Monte Carlo simulation by means of computer. In this paper, the 2-dimensional (2D) self-affine random cascading model with Hurst exponent $H = 0.5$ is studied analytically and a formula of SFM's in self-similar analysis of the model is deduced. The Levy stability indices μ are then calculated analytically from the obtained SFM's, and compared with the corresponding results from self-affine analysis. It turns out that the value of μ depends upon the dynamical fluctuation parameter of the cascading process but never beyond the limitation of $0 \leq \mu \leq 2$ of Levy stability law, in consistency with the results of Monte Carlo simulations. Meanwhile, the Levy stability indices of the model with some other Hurst exponents are also calculated by means of the Monte Carlo simulation.

Furthermore, the Levy stability in the 2D experimental data of $\pi^+/K - p$ interaction at 250 GeV/c from NA22 collaboration is analyzed too. It is found that the Levy indices μ are different for different combination of variables, which means that the fractal property is different in different directions.

2. 2-DIMENSIONAL SELF-AFFINE RANDOM CASCADING MODEL

In the experimental analysis, the variables of 2D phase space are often chosen as $(y, \ln p_\perp)$ or (y, φ), where y, $\ln p_\perp$, and φ indicate the rapidity, logarithm of transverse momentum, and the azimuthal angle, respectively. In the following we will denote the variables used in longitudinal and transverse directions simply as $x_\parallel$ and $x_\perp$, respectively.

In self-affine fractal model, the partition numbers in both transverse $(\lambda_\perp)$ and longitudinal $(\lambda_\parallel)$ directions may be different in different steps of cascading, but the Hurst exponent $H = \ln\lambda_\parallel / \ln\lambda_\perp$ must keep constant in the whole cascading process. In has been shown in Ref. 10 through analyzing the 1-dimensional experimental data of hadron-hadron interaction at 250 GeV/c [9] that the Hurst exponent $H = 0.516 \pm 0.012$ for $x_\parallel = y$, $x_\perp = \ln p_\perp$. As an approximation to this experimental result we choose $H = 0.5$ in our discussion, and use the simple partition numbers $\lambda_\parallel = 2$, $\lambda_\perp = 4$.

Consider a square with unit area. The probability for a particle to appear in this square is supposed to be unity, cf., Fig. 1. With the chosen partition numbers $\lambda_\parallel$ and $\lambda_\perp$, the unit length in $x_\parallel$ direction is divided into 2 portions and that of $x_\perp$ into 4 portions, so 8 smaller rectangles are obtained. We call such a phase space partition the 2—4 splitting and the corresponding fractal the 2—4 fractal. To ensure probability conservation in each step, the probability for the particle to appear in the i'-th rectangle is chosen as [3]

$$w_i^{(1)} = \frac{1+\alpha r_i}{8+\alpha\sum_{j=1}^{8} r_j}, \quad (i = 1, 2, \cdots, 8), \tag{5}$$

where $0 \le \alpha \le 1$ indicates the strength of dynamical fluctuation of the cascading process; r_i $(i = 1, 2, \ldots, 8)$ are the random numbers distributed uniformly in $[0,1]$; the superscript of $w_i^{(1)}$ indicates the number of steps in cascading.

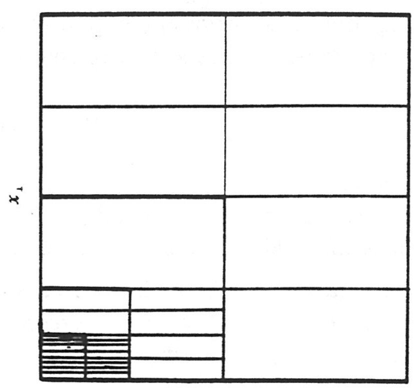

Fig. 1

The sketch map of 2D self-affine random cascading model with $\lambda_\parallel = 2$, $\lambda_\perp = 4$, where the thin lines indicate the cascading process of model and the thick ones indicate the division of self-similar analysis.

Table 1

The ratios r_q of intermittency indices for different values of the fluctuation parameter α and elementary partition number $\lambda_{\|} - \lambda_{\perp}$.

	$\alpha=0.1$	0.4	0.6	0.8
$r_3^{(2-3)}$	2.978	2.885	2.712	2.550
$r_3^{(2-4)}$	3.096	2.867	2.722	2.565
$r_3^{(2-5)}$	2.838	2.775	2.660	2.540
$r_3^{(2-6)}$	3.073	2.852	2.713	2.567

Table 2

The Levy stability index μ of the 2D self-affine fractal for different α and $\lambda_{\|} - \lambda_{\perp}$.

	$\alpha=0.1$	0.4	0.6	0.8
$\mu^{(2-3)}$	1.972	1.736	1.495	1.246
$\mu^{(2-4)}$	2.018	1.746	1.503	1.252
$\mu^{(2-5)}$	1.846	1.701	1.472	1.228
$\mu^{(2-6)}$	2.007	1.743	1.502	1.254

In the second step of cascade, each rectangle is divided again in the way of 2—4 partition, and 8^2 smaller rectangles are produced. The probability for a particle to appear in one of these rectangles reads

$$p_n^{(2)} = w_i^{(1)} w_j^{(2)}, \qquad (i, j = 1, 2, \cdots, 8 \qquad n = 1, 2, \cdots, 8^2), \tag{6}$$

where $w_j^{(2)}$ indicates the splitting probability from first step to second step, which has the same form as Eq.(5). Consequently, there are 8^ν cells in the ν'-th step and the probability in one cell is

$$p_n^{(\nu)} = w_i^{(1)} w_j^{(2)} \cdots w_k^{(\nu)}, \qquad (n = 1, 2, \cdots, 8^\nu, \qquad i, j, k = 1, 2, \cdots, 8), \tag{7}$$

Figure 1 shows this cascading process.

In Table 1 are listed the ratios r_q of intermittency indices for different fluctuation parameters α in self-affine analysis of the above 2D model. The Levy indices μ calculated from r_q and Eq.(4) are shown in Table 2. In the tables, $r_3^{(a)}$ (a = 2—3, 2—4, 2—5, and 2—6) indicate the ratio $r_3^{(a)} = \varphi_3^{(a)} / \varphi_2^{(a)}$ of the third to the second order intermittency indices with corresponding partition numbers $\lambda_{\|} - \lambda_{\perp}$, and $\mu^{(a)}$ denote the corresponding Levy stability indices.

3. SELF-SIMILAR ANALYSIS OF 2D SELF-AFFINE CASCADING MODEL

3.1. The recurrence formula of probability moments in self-similar analysis

Because the logarithmic ratio of transverse and longitudinal partition numbers is unknown at the beginning, the phase space was shrunk isotopically, i.e., self-similarly, in experimental analysis. In order to compare with the present experimental data, we also analyze the sample of the 2D self-affine fractal model in the self-similar way, i.e., we calculate the probability moments of a square window with length of sides equal to $1/2^{\nu}$ in the ν'-th step of cascading, i.e., the 2—2 self-similar analysis.

It is obvious from Fig. 1 that if a function X_m is defined by the recurrence formula

$$X_m = w^{(m)}\left(X_{m-1}(1) + X_{m-1}(2) + X_{m-1}(3) + X_{m-1}(4)\right), \tag{8}$$

the probability moment z_ν in the square window of the 2—2 self-similar analysis after ν steps of cascading can be expresses as

$$z_\nu = \begin{cases} w^{(1)}w^{(2)}\cdots w^{(m-1)}X_m, & \nu = 2m \\ w^{(1)}w^{(2)}\cdots w^{(m)}[X_m(1) + X_m(2)], & \nu = 2m+1 \end{cases}, \tag{9}$$

where $w^{(\nu)}$ indicates the elementary partition probability in the ν'-th step of cascade. Since the phase space is divided with equal weight the average probability in each sub-window is the same. Therefore, the subscript of $w_i^{(\nu)}$ has been omitted in the above equation.

3.2. The analytic expression of the 2nd and 3rd order probability moments

When $\nu = 2m$, it can be seen from Eq. (9) that

$$\langle z_\nu^q \rangle = \langle w^q \rangle^{m-1} \langle X_m^q \rangle, \tag{10}$$

$$\langle z_\nu \rangle^q = \langle w \rangle^{q(m-1)} \langle X_m \rangle^q = \frac{\langle X_m \rangle^q}{8^{q(m-1)}}, \tag{11}$$

Since $\langle z_\nu \rangle = 1/4^{2m}$, we have

$$\langle X_m \rangle = \frac{1}{2^{m+3}}. \tag{12}$$

Therefore, the q-th order probability moment reads

$$C_\nu^q = \frac{\langle z_\nu^q \rangle}{\langle z_\nu \rangle^q} = 8^{q(m-1)} \langle w^q \rangle^{(m-1)} \frac{\langle X_m^q \rangle}{\langle X_m \rangle^q}, \tag{13}$$

In the following let us deduce the analytic expression of C_ν^q for $q = 2, 3$.

When $q = 2$, a recurrence relation of moments can be obtained from Eq.(8),

$$\langle X_m^2 \rangle = 4\langle w^2 \rangle \left(\langle X_{m-1}^2 \rangle + \frac{3\langle w_1 w_2 \rangle}{4(m-1)} \right). \tag{14}$$

Let

$$a = 4\langle w^2 \rangle, \qquad b = 3\langle w_1 w_2 \rangle \tag{15}$$

where $\langle w^2 \rangle$ and $\langle w_1 w_2 \rangle$ denote, respectively, the second order moment and correlation moment of elementary partition probability. The recurrent result of Eq.(14) is

$$\langle X_m^2 \rangle = \frac{a^{m+1}}{4} + a^m b + \frac{a^m b - \dfrac{ab}{4^{m-1}}}{4a-1}. \tag{16}$$

Substituting into Eq.(13), we obtain the second order scaled probability moment

$$\begin{aligned} C_v^2 &= 8^{2m}\left(a^{2m} + 4a^{2m-1}b + \frac{4a^{2m-1}b - \dfrac{4a^m b}{4^{m-1}}}{4a-1}\right) \\ &= 8^v\left(a^v + 4a^{v-1}b + \frac{4a^{v-1}b - \dfrac{a^{v/2}b}{2^{v-4}}}{4a-1}\right) \\ &\qquad v = 0,\ 2,\ 4,\ \cdots, \end{aligned} \tag{17}$$

where ν is the number of cascading steps.

When $q = 3$, it can be deduced from Eq.(8) that

$$\langle X_m^3 \rangle = 4\langle w^3\rangle(\langle X_{m-1}^3\rangle + 6 \cdot 4^3\langle w_1 w_2 w_3\rangle\langle X_{m-2}\rangle^3 + 9\langle X_{m-1}^2(1) X_{m-1}(2)\rangle). \tag{18}$$

where $\langle X_{m-2}\rangle$ can be calculated from Eq.(12). $\langle X_{m-2}^2(1)\, X_{m-1}(2)\rangle$ is obtained from Eqs.(8) and (14)

$$\langle X_{m-1}^2(1) X_{m-1}(2)\rangle = \langle w_1^2 w_2\rangle \frac{A_m}{2^{(m-1)}}, \tag{19}$$

and

$$A_m = a^{m-1} + 4a^{m-2}b + \frac{4a^{m-2}b - \dfrac{b}{4^{m-3}}}{4a-1}. \tag{20}$$

Let

$$t = 4\langle w^3\rangle \quad u = 6\langle w_1 w_2 w_3\rangle, \quad s = 9\langle w_1^2 w_2\rangle, \tag{21}$$

where $\langle w^3\rangle$, $\langle w_1 w_2 w_3\rangle$, and $\langle w_1^2 w_2\rangle$ indicate, respectively, the third order moment and correlation moments of elementary partition probability. The recurrence relation of $\langle X_m^3\rangle$ can be obtained by substituting Eqs.(19)—(21) into Eq.(18),

$$\langle X_m^3\rangle = t\left(\langle X_{m-1}^3\rangle + \frac{s \cdot A_m}{2^{m-1}} + \frac{u}{8^{m-1}}\right). \tag{22}$$

The recurrent result reads

$$\langle X_m^3 \rangle = \frac{t^{m+1}}{4} + \left(\frac{u}{8^m} - \frac{bs}{4a-1} \frac{16}{8^m} \right) \left\{ \frac{8t-(8t)^{m+1}}{1-8t} \right\}$$

$$+ s \left(\frac{16b}{4a-1} + 1 \right) \frac{a^m}{2^m} \left\{ \frac{\frac{2t}{a} - \left(\frac{2t}{a} \right)^{m+1}}{1 - \frac{2t}{a}} \right\}. \tag{23}$$

From Eq.(13), the third order of scaled probability moment is therefore

$$C_\nu^3 = 8^3 (128t)^{\frac{\nu}{2}-1} \left\{ 8^{\nu/2} \frac{t^{\frac{\nu}{2}+1}}{4} + \left(u - \frac{16bs}{4a-1} \right) \left(\frac{8t-(8t)^{\frac{\nu}{2}+1}}{1-8t} \right) \right.$$

$$\left. + s(4a)^{\nu/2} \left(\frac{16b}{4a-1} + 1 \right) \left(\frac{\frac{2t}{a} - \left(\frac{2t}{a} \right)^{(\frac{\nu}{2}+1)}}{1 - \frac{2t}{a}} \right) \right\}, \tag{24}$$

$$\nu = 0, 2, 4, \cdots.$$

where ν is the number of steps of splitting.

When $\nu = 2m + 1$, it can be deduced from Eq.(9) that

$$C_\nu^2 = 4a^m 8^{2m+1} \left(\langle X_m^2 \rangle + \frac{b}{3 \cdot 4^m} \right), \tag{25}$$

and

$$C_\nu^3 = \frac{2t^m 8^{4m+2}}{4^m} \left(\langle X_m^3 \rangle + \frac{s}{3a} \frac{1}{2^{(m-2)}} \langle X_m^2 \rangle \right), \tag{26}$$

where a, b, s, and t are defined by Eqs.(15) and (21), and $\langle X_m^2 \rangle$ and $\langle X_m^3 \rangle$ are referred to Eqs.(16) and (23), respectively.

4. LEVY STABILITY INDEX OF SELF-SIMILAR ANALYSIS IN 2D PHASE SPACE

4.1. NA22 experimental data

In NA22 experimental data [9], the variables of 2D phase space were chosen as (y, φ) and $(y, \ln p_\perp)$, which have been transformed at first by

$$X(x_a) = \frac{\int_{x_{a\,\min}}^{x_a} \rho(x_a)\,\mathrm{d}x_a}{\int_{x_{a\,\min}}^{x_{a\,\max}} \rho(x_a)\,\mathrm{d}x_a}, \tag{27}$$

Table 3

The NA22 data of r_3, r_4, and r_5 for different 2D variables.

	r_3	r_4	r_5
$y-\varphi$	3.4 ± 0.2	$6.1+0.6$	9 ± 2
$y-\ln p_\perp$	2.75 ± 0.07	5.2 ± 0.2	8.6 ± 0.4

The ratios r_3, r_4, r_5 of intermittency indices can then be obtained by fitting 2-5th order SFM's to Eq.(3). The results are listed in Table 3.

Through fitting the data of Table 3 to Eq.(4) the Levy stability indices μ for different 2D variables can be obtained as

$$\mu=\begin{cases} 1.973, & (y-\varphi) \\ 1.701, & (y-\ln p_\perp) \end{cases}. \tag{28}$$

The fitting curve is shown in Fig. 2.

Obviously, Levy stability indices are different for different 2D variables, which implies that the dynamical fluctuations are different in different directions. However, all the resulting μ stay within the limit of Levy stability law, i.e., $0 \leq \mu \leq 2$.

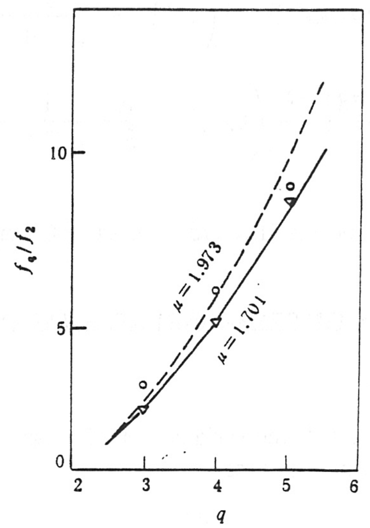

Fig. 2

The fitting curve of Eq.(4) to NA22 data, where ○ and ∇ indicate respectively $y-\varphi$ and $y-\ln p_\perp$.

Table 4
The numerical results of M_1, M_2, M_3, M_4, and M_5 versus different values of the fluctuation parameter α.

	M_1	M_2	M_3	M_4	M_5
0.05	1.563639	1.562337	1.957398	1.954142	1.952514
0.1	1.567062	1.561848	1.970239	1.957197	1.950681
0.2	1.580807	1.559884	2.021880	1.969455	1.943322
0.4	1.636667	1.551901	2.233070	2.019119	1.913460
0.6	1.733347	1.538107	2.602820	2.104240	1.862100
0.8	1.876503	1.517642	3.163619	2.228780	1.786480
1	2.065690	1.490616	3.929909	2.389570	1.687840

4.2. The analytical calculation for the self-similar analysis of 2D self-affine fractal and its Monte Carlo simulation

It can be seen from Eqs.(17), (24—26) that the second and third order SFM C_v^2 and C_v^3 depend on a, b, t, u, and s. In order to calculate the SFM's and the Levy indices analytically, one must obtain at first the 5 moments $\langle w^2\rangle$, $\langle w_1w_2\rangle$, $\langle w^3\rangle$, $\langle w_1^2w_2\rangle$, and $\langle w_1w_2w_3\rangle$ of the elementary partition probability. Appendix A shows how to calculate them. The resulting analytical expressions depend only upon the fluctuation parameter α. In Table 4 are listed the numerical results of these moments for different values of the fluctuation parameter α, where $M_1 = \langle w^2\rangle \times 10^2$, $M_2 = \langle w_1w_2\rangle \times 10^2$, $M_3 = \langle w^3\rangle \times 10^3$, $M_4 = \langle w_1^2w_2\rangle \times 10^3$, and $M_5 = \langle w_1w_2w_3\rangle \times 10^3$.

By using the data of Table 4 and the Eqs.(17), (24—26), the C_v^3 and C_v^2 can be calculated analytically. It is found that the Ochs-Wosiek relation Eq.(3) is satisfied quite well for all the values of α up to 20 steps of the cascading process. By fitting the data of $\ln C^3 - \ln C^2$ to Eq.(3), one can get the value of $r_3 = \varphi_3/\varphi_2$. The Levy stability index μ can therefore be obtained by minimizing the error function

$$f(\alpha;\mu)=\left(\frac{3^\mu-3}{2^\mu-2}-r_3(\alpha)\right)^2, \tag{29}$$

In Table 5 is shown the numerical results of r_3 and μ for different values of the fluctuation parameter α.

It can obviously be seen from Table 5 that the Levy stability index increases with the decreasing dynamic fluctuation parameter, and vice versa. However, it never goes beyond the scope of [0,2] as it is limited by the Levy stability law (as the NA22 data confirm).

In the above analysis of the 2D self-affine model the Hurst exponent has been chosen as $H = 0.5$ according to the experimental results from NA22. Therefore, the results are approximately applicable to the case of $x_\| = y$ and $x_\perp = \ln p_\perp$ [10]. Comparing the data for (y, $\ln p_\perp$) in Table 3 and Eq.(28)

Table 5

The analytical results of r_3 and μ for different α.

	α=0.05	0.1	0.2	0.4	0.6	0.8	1
r_3	2.998	2.992	2.970	2.888	2.768	2.635	2.508
μ	1.997	1.990	1.961	1.853	1.681	1.475	1.255

to the results shown in Table 5, it can be seen that the results of model with α = 0.6 fit the experimental data approximately.

On the other hand, we have made a 20000-event MC simulation for this 2—4 self-affine fractal model. The Levy stability indices are calculated through self-similar analysis of the MC sample. The results are listed in the second row of Table 6 for α = 0.01, 0.1, 0.4, 0.6, and 0.8, and agree with the analytic results of Table 5 quite well.

In order to investigate the dependance of Levy stability index upon the Hurst exponent, we have also made the Monte Carlo simulation of the 2D self-affine cascading model with some other values of the Hurst exponent. The results are listed in Table 6. Just like the case of the 2—4 self-affine fractal, Levy stability index μ decreases with the increasing fluctuation parameter α, but never go beyond the limitation of Levy stability law. It can also been seen from Table 6 that when the dynamic fluctuation is large (e.g., $\alpha \geq 0.5$), the Hurst exponent influences the Levy index obviously. With the decreasing α, all the Levy indices approach to 2 and the dependence upon Hurst exponent reduces gradually.

In Fig. 3 is shown the μ-α curve of the 2D self-affine model in a self-affine analysis (full line) and in a self-similar analysis (dashed line).

Comparing Table 3 and Eq.(28) with Table 6 it can be seen that the Hurst exponent H and dynamic fluctuation strength in NA22 data may be different for different 2D variables (y,ϕ) and $(y, \ln p_{\perp})$, which means that the fractal behavior are different in different directions.

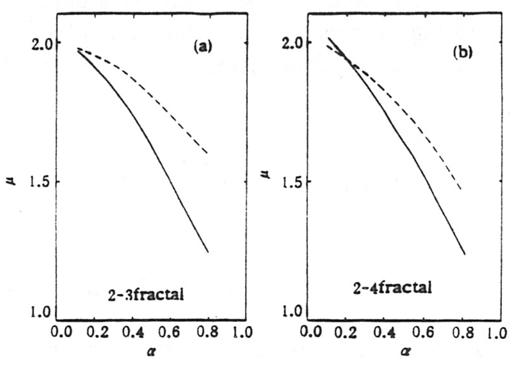

Fig. 3

The Levy stability index μ of 2D self-affine fractal model versus the dynamic fluctuation parameter α: (a), $\lambda_{\|} - \lambda_{\perp}$ ~2—4 fractal; (b), $\lambda_{\|} - \lambda_{\perp}$ ~2—3 fractal. The full curves indicate the self-affine analysis and the dashed ones the self-similar analysis.

Table 6
The Monte Carlo results of Levy stability index μ for self-similar analysis of 2D self-affine fractal.

$\lambda_{\parallel}-\lambda_{\perp}$	$\alpha=0.01$	0.1	0.4	0.6	0.8
2−3	1.996	1.981	1.865	1.735	1.595
2−4	2.000	1.986	1.824	1.662	1.491
2−5	2.008	1.992	1.871	1.744	1.600
2−6	1.999	1.990	1.867	1.734	1.588
3−4	2.055	2.032	1.841	1.673	1.493

5. CONCLUSION

The 2D self-affine random cascading model with Hurst exponent $H = 0.5$ is investigated analytically and the analytical expressions for the scaled probability moments C_N^2 and C_N^3 are obtained. They are functions of 5 kinds of moments of elementary partition probability w.

The Levy stability indices of the model are obtained both from analytic calculation and from the Monte Carlo simulation. The results indicate the following conclusions.

(1) The Levy stability indices of a 2D phase space satisfy the stability law quit well, i.e., $0 \leq \mu \leq 2$, for both self-affine and self-similar analysis.

(2) The Levy index μ depends upon the dynamic fluctuation parameter α. The stronger the dynamic fluctuation, the smaller the Levy index μ, and vice versa. When α is large, the Levy stability index noticeably depends upon the Hurst exponent.

(3) The NA22 data of the Levy stability index are different for different 2D variables; this fact implies that dynamic fluctuations are different in different directions.

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APPENDIX A

The calculations of the 2nd and 3rd order moments and the correlation moments of elementary partition probability in self-affine cascading model.

The five kinds of moments of elementary partition probability in 2D self-affine random cascading model, i.e., $\langle w^2\rangle$, $\langle w_1w_2\rangle$, $\langle w^3\rangle$, $\langle w_1^2w_2\rangle$, and $\langle w_1w_2w_3\rangle$, will be calculated analytically in this appendix.

From the expression Eq.(5) for the elementary partition probability of 2D self-affine model we have

$$\langle w^2\rangle=\frac{1}{2^8}\underbrace{\int_{-1}^{1}\cdots\int_{-1}^{1}}_{8}\frac{(1+\alpha r_1)^2}{[8+\alpha(r_1+r_2+\cdots+r_8)]^2}\,dr_1dr_2\cdots dr_8. \tag{A.1}$$

By calculating the integration of the above equation step by step from r_8 to r_2, one can obtain

$$\langle w^2\rangle=-\frac{1}{120\cdot 2^8\alpha^8}\sum_{n=0}^{7}(-1)^nC_7^nf_1(\alpha,n). \tag{A.2}$$

where

$$f_1(\alpha,n)=\alpha\int_{-1}^{1}dr_1(1+\alpha r_1)^2[8+\alpha r_1+\alpha(7-2n)]^5\ln[8+\alpha r_1+\alpha(7-2n)]. \tag{A.3}$$

The result of the integration of $f_1(\alpha,n)$ reads

$$f_1(\alpha,n)=Y_1(\alpha,n)-P_1(\alpha,n). \tag{A.4}$$

Let

$$x_1(m)=8+\alpha(m-2n),\quad x_2(i)=7+\alpha(i-2n), \tag{A.5}$$

$$y_1(m,i,\alpha,n)=\left(\frac{x_1^8(m)}{8}-\frac{2x_2(i)x_1^7(m)}{7}+\frac{x_2^2(i)x_1^6(m)}{6}\right)\ln x_1(m), \tag{A.6}$$

$$p_1(m,i,\alpha,n)=\frac{x_1^8(m)}{8^2}-\frac{2x_2(i)x_1^7(m)}{7^2}+\frac{x_2^2(i)x_1^6(m)}{6^2}. \tag{A.7}$$

then $Y_1(\alpha,n)$, $P_1(\alpha,n)$ can be written as

$$\begin{aligned}Y_1(\alpha,n)&=y_1(8,7,\alpha,n)-y_1(6,7,\alpha,n),\\P_1(\alpha,n)&=p_1(8,7,\alpha,n)-p_1(6,7,\alpha,n),\end{aligned} \tag{A.8}$$

Substituting A.3—A.8 into A.2 we have

$$\langle w^2\rangle=-\frac{1}{120\cdot 2^8\alpha^8}\left(\sum_{n=0}^{7}(-1)^nC_7^nY_1(\alpha,n)-7680\alpha^8\right). \tag{A.9}$$

In the same way, the other four kinds of moments can be expressed as

$$\langle w_1w_2\rangle=-\frac{1}{24\cdot 2^8\alpha^8}\left(\sum_{n=0}^{6}(-1)^nC_6^nY_2(\alpha,n)+\frac{768}{7}\alpha^8\right), \tag{A.10}$$

$$\langle w^3 \rangle = \frac{1}{24 \cdot 2^9 \alpha^8} \left(\sum_{n=0}^{7} (-1)^n C_7^n Y_3(\alpha, n) + 4608\alpha^8 \right), \tag{A.11}$$

$$\langle w_1^2 w_2 \rangle = \frac{1}{6 \cdot 2^9 \alpha^8} \left(\sum_{n=0}^{6} (-1)^n C_6^n Y_4(\alpha, n) - \frac{384}{7} \alpha^8 \right), \tag{A.12}$$

$$\langle w_1 w_2 w_3 \rangle = \frac{1}{2^{10} \alpha^8} \left(\sum_{n=0}^{5} (-1)^n C_5^n Y_5(\alpha, n) + \frac{64}{21} \alpha^8 \right). \tag{A.13}$$

where

$$Y_2(\alpha, n) = y_2(8, 7, 0, \alpha, n) - y_2(6, 7, 0, \alpha, n) - y_2(6, 5, 1, \alpha, n) + y_2(4, 5, 1, \alpha, n), \tag{A.14}$$

$$y_2(m, i, j, \alpha, n) = \frac{1}{30} \left(-\frac{x_1^8(m)}{8} + \frac{(6z(j) + x_2(i)) x_1^7(m)}{7} - x_2(i) z(j) x_1^6(m) \right) \ln x_1(m), \tag{A.15}$$

$$z(j) = 1 + (-1)^j \alpha, \tag{A.16}$$

$$Y_3(\alpha, n) = y_3(8, 7, \alpha, n) - y_3(6, 7, \alpha, n), \tag{A.17}$$

$$y_3(m, i, \alpha, n) = \left(\frac{x_1^8(m)}{8} - \frac{3x_2(i) x_1^7(m)}{7} + \frac{3x_2^2(i) x_1^6(m)}{6} - \frac{x_2^3(i) x_1^5(m)}{5} \right) \ln x_1(m), \tag{A.18}$$

$$Y_4(\alpha, n) = y_4(8, 7, 0, \alpha, n) - y_4(6, 7, 0, \alpha, n) - y_4(6, 5, 1, \alpha, n) + y_4(4, 5, 1, \alpha, n), \tag{A.19}$$

$$y_4(m, i, j, \alpha, n) = \left[-\frac{x_1^8(m)}{8 \cdot 20} + \left(\frac{z(j)}{4} + \frac{x_2(i)}{10} \right) \frac{x_1^7(m)}{7} - \left(\frac{z(j) x_2(i)}{2} + \frac{x_2^2(i)}{20} \right) \frac{x_1^6(m)}{6} + \frac{z(j) x_2^2(i) x_1^5(m)}{20} \right] \ln x_1(m), \tag{A.20}$$

$$Y_5(\alpha, n) = y_5(8, 7, 0, 0, \alpha, n) - y_5(6, 7, 0, 0, \alpha, n) - 2[y_5(6, 5, 0, 1, \alpha, n) - y_5(4, 5, 0, 1, \alpha, n)] + y_5(4, 3, 1, 1, \alpha, n) - y_5(2, 3, 1, 1, \alpha, n), \tag{A.21}$$

$$Y_5(m, i, j, l, \alpha, n) = \left\{ \frac{2}{8 \cdot 6!} x_1^8(m) - \left[\frac{2(z(j) + z(l))}{7 \cdot 5!} + \frac{2x_2(i)}{7!} \right] x_1^7(m) + \left[\frac{2z(j) z(l)}{6 \cdot 4!} + \frac{2(z(j) + z(l)) x_2(i)}{6!} \right] x_1^6(m) - \frac{2z(j) z(l) x_2(i) x_1^5(m)}{5!} \right\} \ln x_1(m). \tag{A.22}$$