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Angular Distribution of the Process $e^+e^-\to J/\psi\to\gamma+X(J^{PC})$, $X\to B\overline{B}$, and the Spin-Parity Analysis of the Resonance X

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The process $e^+e^-\to J/\psi\to\gamma+X(J^{PC})$, $X\to B\overline{B}$ (baryon and anti-baryon) is analyzed by using the helicity angular distribution method. The angular and the projection angular distribution formulas for different spin-parity of the resonance X are obtained. With these formulas, we can determine the spin of the resonance X through the analysis of the experimental data.

Key words: spin parity, resonance, angular distribution, helicity amplitude.

1. INTRODUCTION

Among the radiative decay modes of J/ψ to baryon and anti-baryon, $J/\psi\to\gamma p\overline{p}$ (p is proton) is very interesting. Through the study of this mode, we may probe the nature of mesons whose masses are around 2–3 GeV [1].

DM2 [2] and MARKIII [3] have studied this process, respectively, but they didn't see anything, especially no $\xi(2230)$ signal except a small signal of η_c. The upper limit given by MARKIII is: $BR(J/\psi\to\gamma\xi(2230))BR(\xi(2230)\to p\overline{p}) < 2\times10^{-5}$. However, an obvious resonant structure around $\sim$ 2.2GeV has appeared in the recent study of this process at BES [4].

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Table 1

l \ s	0	1
0	0^{-+}	
1		0^{++}, 1^{++}, 2^{++}
2	2^{-+}	

$\xi(2230)$ was firstly observed in 1983 by MARKIII in the process $J/\psi \to \gamma K^+K^-$, and soon it was verified again in $K_s^0 K_s^0$ final states. However, due to the limited events, whether it has spin 2 or 4 has not been determined so far. Reference 5 has come up with a new method: generalized momenta analysis, which hopefully will allow to determine the spin of X.

It is not long after the discovery of $\xi(2230)$, many hypotheses have been proposed to explain its existence, such as Higgs, Glue-ball (gg, ggg), Hybrid ($q\bar{q}g$), Four-quark state ($qq\bar{q}\bar{q}$), and High Spin $s\bar{s}$ state, $\Lambda\bar{\Lambda}$ bound state, etc. However, we cannot presently make a definite judgement as to which theory is more reasonable.

Is the resonance observed at BES $\xi(2230)$ or another new state? Is it a glueball, a hybrid state, or four-quark state? In order to answer these questions, the determination of its spin-parity is required.

In addition, the g_T [6] states found in the fixed-target experimental processes $\pi^- p \to n\varphi\varphi$, $nK_s^0 K_s^0$ are also around this mass region, and their ($\varphi\varphi$) resonant signal has not yet been observed in J/ψ decay process. It is also interesting to explore what kind of relation the resonance observed at BES has with these g_T states.

2. FORM OF HELICITY ANGULAR DISTRIBUTION

For a system of one baryon and one anti-baryon, the parity and charge parity are $P = (-1)^{l+1}$, $C = (-1)^{l+s}$, where l, s stands for the orbital angular momenta between the two baryons and their spin, respectively. Because the charge parity is conserved in the process $J/\psi \to \gamma + X$, so the charge parity of the resonance X is plus and therefore $l + s$ should be an even number. The possible spin and parity of the resonance X is listed in Table 1.

The sub-matrices for various sub-processes of the processes $e^+e^- \to J/\psi \to \gamma + X(J^{PC})$, $X \to B\bar{B}$ are follows:

$$
\begin{aligned}
&\langle \psi_{\lambda_J} | T_1 | e_r^+ e_{r'}^- \rangle \sim e_\mu^{\lambda_J} \bar{v}_r(p_+)\gamma^\mu u_{r'}(p_-), \\
&\langle \gamma_{\lambda_\gamma} X_{\lambda_X} | T_2 | \psi_{\lambda_J} \rangle \sim A_{\lambda_\gamma, \lambda_X} \delta_{\lambda_J, \lambda_\gamma - \lambda_X}, \qquad (1) \\
&\langle B_{\lambda_B} \bar{B}_{\lambda_{\bar{B}}} | T_3 | X_{\lambda_X} \rangle \sim B_{\lambda_B, \lambda_{\bar{B}}} D^{J^*}_{\lambda_X, \lambda_B - \lambda_{\bar{B}}}(\varphi, \theta, -\varphi),
\end{aligned}
$$

where λ is the helicity of the corresponding particle, r and r' are the polarized parameters of the positron and electron, respectively, $A_{\lambda_\gamma,\lambda_X}$, $B_{\lambda_B,\lambda_{\bar{B}}}$ are the helicity amplitudes of the above sub-processed. In writing these sub-matrices, we take the reference frames as: 1. the rest frame of e^+e^- system. In this frame we select the z axis along with the moving direction of γ, y axis the direction of $p_\gamma \times p_{e^-}$; 2. the rest frame of the resonance X, and the z' axis is along with the moving direction of X in the J/ψ rest frame. $\Omega = (\theta, \varphi)$ describes the polar angle and the azimuthal angle of B or $\bar{B}$ momenta direction in this frame.

Table 2

J^{PC}	independent amplitude	
0^{-+}	$A_{1,0}=-A_{-1,0}$	$B_{\frac{1}{2},\frac{1}{2}}=-B_{-\frac{1}{2},-\frac{1}{2}}$
0^{++}	$A_{1,0}=A_{-1,0}$	$B_{\frac{1}{2},\frac{1}{2}}=B_{-\frac{1}{2},-\frac{1}{2}}$
1^{++}	$A_{1,1}=-A_{-1,-1}$, $A_{1,0}=-A_{-1,0}$	$B_{\frac{1}{2},\frac{1}{2}}=-B_{-\frac{1}{2},-\frac{1}{2}}$, $B_{\frac{1}{2},-\frac{1}{2}}=-B_{-\frac{1}{2},\frac{1}{2}}$
2^{-+}	$A_{1,1}=-A_{-1,-1}$ $A_{1,0}=-A_{-1,0}$, $A_{1,2}=-\dot{A}_{-1,-2}$	$B_{\frac{1}{2},\frac{1}{2}}=-B_{-\frac{1}{2},-\frac{1}{2}}$ $B_{\frac{1}{2},-\frac{1}{2}}=-B_{-\frac{1}{2},\frac{1}{2}}$
2^{++}	$A_{1,1}=A_{-1,-1}$ $A_{1,0}=A_{-1,0}$, $A_{1,2}=A_{-1,-2}$	$B_{\frac{1}{2},\frac{1}{2}}=B_{-\frac{1}{2},-\frac{1}{2}}$ $B_{\frac{1}{2},-\frac{1}{2}}=B_{-\frac{1}{2},\frac{1}{2}}$

Considering the parity conservation, we have the following relations between these helicity amplitudes:

$$
\begin{aligned}
A_{\lambda_\gamma,\lambda_X} &= P(-1)^J A_{-\lambda_\gamma,-\lambda_X}, \\
B_{\lambda_B,\lambda_{\bar{B}}} &= P(-1)^J B_{-\lambda_B,-\lambda_{\bar{B}}},
\end{aligned}
\tag{2}
$$

where η is the parity, J is the spin of X. For the various cases listed in Table 1, we can get the independent helicity amplitudes listed in Table 2: The angular distribution of the process in question is:

$$
\begin{aligned}
W_J(\theta_\gamma, \Omega) \sim & \sum_{\substack{\lambda_J, \lambda_J', \lambda_X, \lambda_X' \\ \lambda, \lambda_B, \lambda_{\bar{B}}}} I_{\lambda_J,\lambda_J'}(\theta_\gamma) A_{\lambda_\gamma,\lambda_X} A^{*}_{\lambda_\gamma,\lambda_X'} |B_{\lambda_B,\lambda_{\bar{B}}}|^2 \\
& D^{J*}_{\lambda_X,\lambda_B-\lambda_{\bar{B}}}(\varphi, \theta, -\varphi) D^{J}_{\lambda_X',\lambda_B-\lambda_{\bar{B}}}(\varphi, \theta, -\varphi) \delta_{\lambda_J,\lambda_\gamma-\lambda_X} \delta_{\lambda_J',\lambda_\gamma-\lambda_X'}
\end{aligned}
\tag{3}
$$

The value of $I_{\lambda_J,\lambda'_J}(\theta_\gamma)$ can be found in Ref. 5.

We discuss here several cases of J^{PC}:

A. $J^{PC} = 0^{\pm +}$

From formula (3), we have:

$$
W_0(\theta_\gamma, \Omega) \sim 1+\cos^2\theta_\gamma, \tag{4}
$$

B. $J^{PC} = 1^{++}$

The ratio of helicity amplitude is defined as:

$$
\begin{aligned}
xe^{i\varphi_x} &= \frac{A_{1,1}}{A_{1,0}}, \\
ze^{i\varphi_z} &= \frac{B_{\frac{1}{2},-\frac{1}{2}}}{B_{\frac{1}{2},\frac{1}{2}}},
\end{aligned}
\tag{5}
$$

So we have

$$
\begin{aligned}
W_1(\theta_\gamma,\ \Omega) \sim\ & 2(1+\cos^2\theta_\gamma)(2\cos^2\theta+z^2\sin^2\theta) \\
& +2x^2\sin^2\theta_\gamma[2\sin^2\theta+(1+\cos^2\theta)z^2] \\
& +x\cos\varphi\cos\varphi_x\sin2\theta_\gamma\sin2\theta(z^2-2).
\end{aligned}
\tag{6}
$$

C. $J^{PC} = 2^{++}$

We define here:

$$
\begin{aligned}
xe^{i\varphi_x} &= \frac{A_{1,1}}{A_{1,0}}, \\
ye^{i\varphi_y} &= \frac{A_{1,2}}{A_{1,0}}, \\
ze^{i\varphi_z} &= \frac{B_{\frac{1}{2},\,-\frac{1}{2}}}{B_{\frac{1}{2},\,\frac{1}{2}}},
\end{aligned}
\tag{7}
$$

therefore,

$$
\begin{aligned}
W_2(\theta_\gamma,\ \Omega) \sim\ & (1+\cos^2\theta_\gamma)[(3\cos^2\theta-1)^2+\frac{3}{2}z^2\sin^22\theta \\
& +\frac{3}{2}y^2\sin^4\theta+z^2y^2\sin^2\theta(1+\cos^2\theta)]+3x^2\sin^2\theta_\gamma\sin^22\theta \\
& +2x^2z^2\sin^2\theta_\gamma(4\cos^4\theta-3\cos^2\theta+1) \\
& -\sqrt{3}\ x\sin2\theta_\gamma\sin2\theta\cos\varphi\cos\varphi_x(3\cos^2\theta-1) \\
& +\sqrt{6}\ y\sin^2\theta_\gamma\sin^2\theta\cos2\varphi\cos\varphi_y(3\cos^2\theta-1) \\
& +3\sqrt{2}\ xy\sin2\theta_\gamma\sin^3\theta\cos\theta\cos\varphi\cos(\varphi_x-\varphi_y) \\
& -\sqrt{3}\ xz^2\sin2\theta_\gamma\sin2\theta\cos\varphi\cos\varphi_x(1-2\cos^2\theta) \\
& -\frac{\sqrt{6}}{2}\ yz^2\sin^2\theta_\gamma\sin^22\theta\cos2\varphi\cos\varphi_y \\
& +\sqrt{2}\ xyz^2\sin2\theta_\gamma\cos\varphi\sin2\theta\cos^2\theta\cos(\varphi_x-\varphi_y)
\end{aligned}
\tag{8}
$$

For the case of $J > 2$, we have the following general formula:

$$
\begin{aligned}
W_J(\theta_\gamma,\ \Omega) \sim\ & 2I_{1,1}(\theta_\gamma)\{2d^J_{00}(\theta)^2+2z^2d^J_{01}(\theta)^2 \\
& +y^2[2d^J_{20}(\theta)^2+z^2d^J_{21}(\theta)^2+z^2d^J_{2-1}(\theta)^2]\} \\
& +2I_{0,0}(\theta_\gamma)x^2[(d^J_{11}(\theta)^2+d^J_{1-1}(\theta)^2)z^2+2d^J_{10}(\theta)^2] \\
& +4I_{1,0}(\theta_\gamma)x\cos\varphi\cos\varphi_x[2d^J_{00}(\theta)d^J_{10}(\theta) \\
& +z^2d^J_{01}(\theta)(d^J_{11}(\theta)-d^J_{1-1}(\theta))] \\
& -4I_{1,0}(\theta_\gamma)xy\cos\varphi\cos(\varphi_x-\varphi_y)[2d^J_{20}(\theta)d^J_{10}(\theta) \\
& +z^2(d^J_{11}(\theta)d^J_{21}(\theta)+d^J_{1-1}(\theta)d^J_{2-1}(\theta))] \\
& +4I_{1,-1}(\theta_\gamma)y\cos2\varphi\cos\varphi_y[2d^J_{00}(\theta)d^J_{20}(\theta) \\
& +z^2d^J_{01}(\theta)(d^J_{21}(\theta)-d^J_{2-1}(\theta))]
\end{aligned}
\tag{9}
$$

In our experiment, generally, we could measure three projection angular distributions. They are:

$$
\begin{aligned}
W_J^1(\theta_\gamma) &\sim \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi\, W_J(\theta_\gamma,\ \Omega) \\
W_J^2(\theta) &\sim \int_{-1}^{1} d\cos\theta_\gamma \int_0^{2\pi} d\varphi\, W_J(\theta_\gamma,\ \Omega) \\
W_J^3(\varphi) &\sim \int_{-1}^{1} d\cos\theta_\gamma \int_{-1}^{1} d\cos\theta\, W_J(\theta_\gamma,\ \Omega)
\end{aligned}
\tag{10}
$$

Therefore, for the above A, B, and C cases, we have:

A. $J^{PC} = 0^{\pm +}$

$$
\begin{aligned}
W_0^1(\theta_\gamma) &= \frac{3}{8}\,(1+\cos^2\theta_\gamma) \\
W_0^2(\theta) &= \frac{1}{2} \\
W_0^3(\varphi) &= \frac{1}{2\pi}
\end{aligned}
\tag{11}
$$

B. $J^{PC} = 1^{++}$

$$
\begin{aligned}
W_1^1(\theta_\gamma) &= \frac{3}{8}\,\frac{1}{1+x^2}\,(1+\cos^2\theta_\gamma+2x^2\sin^2\theta_\gamma) \\
W_1^2(\theta) &= \frac{3}{8}\,\frac{1}{(1+x^2)(1+z^2)}\,\{2(2\cos^2\theta+z^2\sin^2\theta)+x^2[2\sin^2\theta+z^2(1+\cos^2\theta)]\} \\
W_1^3(\varphi) &= \frac{1}{2\pi}.
\end{aligned}
\tag{12}
$$

C. $J^{PC} = 2^{\pm +}$

$$
\begin{aligned}
W_2^1(\theta_\gamma) &= \frac{3}{8}\,\frac{1}{1+x^2+y^2}\,[(1+\cos^2\theta_\gamma)(1+y^2)+2x^2\sin^2\theta_\gamma] \\
W_2^2(\theta) &= \frac{5}{8}\,\frac{1}{(1+z^2)(1+x^2+y^2)}\,\{(3\cos^2\theta-1)^2 \\
&\quad +\frac{3}{2}\,z^2\sin^2 2\theta+\frac{3}{2}\,y^2\sin^4\theta \\
&\quad +z^2y^2\sin^2\theta(1+\cos^2\theta)+\frac{3}{2}\,x^2\sin^2 2\theta \\
&\quad +x^2z^2(4\cos^4\theta-3\cos^2\theta+1)\} \\
W_2^3(\varphi) &= \frac{1}{2\pi}\left[1-\frac{\sqrt{6}}{6}\,\frac{y\cos\varphi_y\cos 2\varphi}{1+x^2+y^2}\right]
\end{aligned}
\tag{13}
$$

3. DISCUSSION

From the above theoretical formulas (4), (6) (8), (11), (12), and (13), we could determine the spin of the resonance X through the analysis of the real data.

The formulas (11), (12), and (13) tell us that for the $J^{PC} = 0^{\pm +}$ = case, the distribution of the particles depends only on θ_γ, but not on θ and ϕ. However, for the $J^{PC} = 1^+$ case, the distribution depends on θ_γ, and θ, but not on φ. Only for the $J^{PC} = 2^\pm$ case, the distribution depends on all the angles,θ_γ,θ,andφ.Thisindicatesinthenormalizedprojectionangulardistributionthatthevalueof $W_0^2(\theta)$, $W_0^3(\varphi)$ and $W_1^3(\varphi)$ are a constant, but the value of $W_1^2(\theta)$, $W_2^2(\theta)$ and $W_2^3(\varphi)$ are not. It is not much difficult to distinguish if the resonance X has spin 0, 1, or 2. For the case of $J^{PC} = 4$, we can obtain the angular distribution from Eq.(9). However, to distinguish if $J = 2$ or 4, we will discuss it using generalized momenta analysis method in our next paper.

Yet, it is important to notice that there is a little difference between the radiative decay under consideration and the hadronic decay process discussed in Ref. 7. While we cannot determine the parity of the resonance X, we can determine it for the process in Ref. 7. This is because for different parity of resonance X, the hadronic amplitude A_{00} will be zero or nonzero, and this will cause a difference in the angular distribution. However, this does not happen in the radiative situation, because in this case, the photon is only transversely polarized. In other words, the helicity amplitude A_{00} does not exist. Therefore, for same spin value but different parity, the angular distribution is completely the same.

In conclusion, for different spin states we obtained the corresponding angular distribution, and the angular distribution has its own nature. We can therefore determine the spin of the resonance X according to the nature of the measured angular distribution and the projection angular distribution.

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