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Glauber Theory on Transverse Energy Distributions in Ultra-Relativistic Heavy Ion Collisions

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A geometrical model is proposed for the transverse energy distributions in ultra-relativistic heavy ion collisions based on the Glauber theory. The calculated distributions $d\sigma/dE_T$ for 60 and 200 AGeV $^{16}O+^{197}Au$ are in good agreement with the data of CERN NA35 Collaboration. The relation with other geometrical models is discussed briefly.

Key words: ultra-relativistic heavy ion collision, transverse energy distribution, Glauber theory.

The main purpose of experiments on ultra-relativistic heavy ion collisions is to search for the signals of quark-gluon plasma (QGP) within high energy density areas (>2 GeV/fm^3) and to find out the physical instruction of the vacuum and its stimulation under extreme conditions. A series of experiments have been carried out since the fall of 1986, when the alternative gradient simultaneous accelerator (AGS) at the Brookhaven National Laboratory (BNL) (USA) was set up and accelerated ^{16}O and ^{32}S to 14.5 GeV and CERN SPS accelerator accelerated ^{16}O and ^{32}S to 60 and 200 AGeV. During the last eight years, enormous data have been collected. Moreover, the relativistic heavy ion collider (RHIC) at BNL will be built by 1999 and colliding experiments between ^{238}U nuclei at 200 AGeV will come into reality, which means the collision energy will have been so greatly improved to produce more interesting physical phenomena.

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In the process of heavy ion collisions with high energies, a great many poins and other hadrons will be produced. Generally, the particle multiplicity distribution $d\sigma/dm$, transverse energy distribution $d\sigma/dE_T$, and zero-degree energy distribution $d\sigma/dE_{ZDC}$ are called the global observables of high-energy heavy ion collisions. They are such important experimental backgrounds that to understand their producing mechanism, quality and regularity should be the basis and premise to find new physical phenomena in this field. Since 1987, some phenomenal models and theories [1-10] have been proposed to discuss the transverse energy distribution $d\sigma/dE_T$ and their conclusions are rather concordant with the experimental data. However, those models and theories have used some phenomenal assumptions and experimental results as their basis. Our purpose of this work is to reduce this kind of assumptions as many as possible to get the transverse energy distribution from a more fundamental point of view.

From the level of the nucleon-nucleon (N–N) collisions, we define that the differential cross section for one N–N collision, which has the transverse energy within ε_T to $\varepsilon_T + d\varepsilon_T$, is $d\sigma_{NN}$, and its distribution on ε_T is $d\sigma_{NN}/d\varepsilon_T$.

$$d\sigma_{NN} = d\varepsilon_T \frac{d\sigma_{NN}}{d\varepsilon_T}, \tag{1}$$

then the N–N inelastic cross section can be indicated as

$$\sigma_{NN} = \int d\varepsilon_T \frac{d\sigma_{NN}}{d\varepsilon_T}. \tag{2}$$

In the process of the nucleus-nucleus collision (B+A), when the projectile nucleus B collides on the target nucleus A with impact parameter $\boldsymbol{b}$, the average number of collisions is

$$\overline{N} = \overline{N}(\boldsymbol{b}) = \sigma_{NN} D_{BA}(\boldsymbol{b}), \tag{3}$$

where $D_{BA}(\boldsymbol{b})$ is the overlap of the nuclear density profile distributions of the two nuclei A and B.

$$D_{BA}(\boldsymbol{b}) = \int d^2 s\, D_B(\boldsymbol{s})\, D_A(\boldsymbol{b}+\boldsymbol{s}), \tag{4}$$

$D_A(\boldsymbol{b})$ and $D_B(s)$ are the nuclear profile density distribution of A and B, respectively,

$$D_A(\boldsymbol{b}) = \int dz_A\, n_A(\boldsymbol{b}, z_A), \tag{5}$$

$$D_B(\boldsymbol{s}) = \int dz_B\, n_B(\boldsymbol{s}, z_B), \tag{6}$$

where $n_A(\boldsymbol{r}_A)$ and $n_B(\boldsymbol{r}_B)$ are the nucleon number density distributions of A and B, and $\boldsymbol{r}_A = \boldsymbol{r}_A(\boldsymbol{b}, z_A)$ and $\boldsymbol{r}_B = \boldsymbol{r}_B(s, z_B)$ are vectors whose origin points are fixed at the centers of A and B, respectively. The direction of the z-axis is defined in the direction of the projectile nucleus and vectors $\boldsymbol{b}$ and s are perpendicular to the z-axis. One has,

$$A = \int d^3 r_A\, n_A(\boldsymbol{r}_A) = \int d^2 \boldsymbol{b} D_A(\boldsymbol{b}), \tag{7}$$

$$B=\int d^3 r_B\, n_B(r_B)=\int d^2 s\, D_B(s), \tag{8}$$

$$BA=\int d^2 b\int d^2 s\, D_B(s)\, D_A(b+s)=\int d^2 b\, D_{BA}(b), \tag{9}$$

where B and A are, respectively, the nucleon numbers of the projectile nucleus B and the target nucleus A.

In fact, we have adopted the collision-independent assumption while we write out Eq.(3): namely, every N–N collision is independent. When the colliding energy is high enough, the de Broglie wavelengths of nucleons can be much shorter than the lengths of distances between nucleons in the colliding nuclei, the above assumption can be rational and practical. In addition, when the colliding nuclei are large enough, we can reasonably employ the sharp radius model for the nuclei for further simplification. Then the differential cross section of the N–N collision $d\sigma_{NN}/d\varepsilon_T$ will be independent of the impact parameter b, and this is equivalent to neglecting the EMC effect.

The N–N inelastic collision probability $P(b)$ at impact parameter b in the B+A collision can be defined as:

$$P=P(b)=\frac{\overline{N}(b)}{BA}\ . \tag{10}$$

We suppose a binomial distribution form for the n-th N–N inelastic collisions probability $P(n,b)$,

$$P(n,\ b)=\binom{BA}{n}p^n(1-p)^{BA-n}\ . \tag{11}$$

when $BA \gg n$ and $p \ll 1$, the above distribution function will turn into a Poisson one:

$$P(n,\ b)=\frac{(\overline{N})^n}{n!}\ e^{-\overline{N}}\ . \tag{12}$$

Substituting Eq.(2) into Eq.(3) and then into Eq.(12), we find the contribution to the total inelastic BA cross section from the n-th N–N inelastic collisions

$$\sigma_n=\int d^2 b\, P(n,\ b)=\int d^2 b\, e^{-\int d\varepsilon_T \frac{d\sigma_{NN}}{d\varepsilon_T} D_{BA}(b)}\ \frac{1}{n!}\prod_{i=1}^{n}\int d\varepsilon_{T^i}\, \frac{d\sigma_{NN}}{d\varepsilon_{T^i}}\, D_{BA}(b). \tag{13}$$

On the other hand, if at impact parameter b the probability density having N–N inelastic collisions and producing transverse energies ε_{T_1}, ε_{T_2},...ε_{T_n} on each collision, separately, is $P(n,\ b,\ \varepsilon_{T_1},...\varepsilon_{T_n})$, then, one finds

$$\sigma_n=\int d^2 b\int d\varepsilon_{T_1}\cdots\int d\varepsilon_{T_n} P(n,\ b,\ \varepsilon_{T_1},\ \cdots,\ \varepsilon_{T_n}). \tag{14}$$

With the comparison between Eqs.(13) and (14), we have

$$P(n, \boldsymbol{b}, \varepsilon_{\mathrm{T}_1}, \cdots, \varepsilon_{\mathrm{T}_n}) = \frac{1}{n!} e^{-\int d\varepsilon_{\mathrm{T}} \frac{d\sigma_{\mathrm{NN}}}{d\varepsilon_{\mathrm{T}}} D_{\mathrm{BA}}(\boldsymbol{b})} \prod_{i=1}^{n} \frac{d\sigma_{\mathrm{NN}}}{d\varepsilon_{\mathrm{T}_i}} D_{\mathrm{BA}}(\boldsymbol{b}). \tag{15}$$

the total transverse energy produced in n N–N inelastic collisions is

$$E_{\mathrm{T}} = \varepsilon_{\mathrm{T}_1} + \varepsilon_{\mathrm{T}_2} + \cdots + \varepsilon_{\mathrm{T}_n} \tag{16}$$

The differential cross section can be written as

$$\frac{d\sigma_n}{dE_{\mathrm{T}}} = \int d^2\boldsymbol{b} \int d\varepsilon_{\mathrm{T}_1} \cdots \int d\varepsilon_{\mathrm{T}_n} P(n, \boldsymbol{b}, \varepsilon_{\mathrm{T}_1}, \cdots, \varepsilon_{\mathrm{T}_n})\, \delta(E_{\mathrm{T}} - \varepsilon_{\mathrm{T}_1} - \cdots - \varepsilon_{\mathrm{T}_n}), \tag{17}$$

and the total transverse energy distribution is

$$\frac{d\sigma}{dE_{\mathrm{T}}} = \sum_{n=1}^{n_{\max}} \frac{d\sigma_n}{dE_{\mathrm{T}}}, \tag{18}$$

where $n_{\max}$ is the maximum number of the N–N inelastic collisions in the B+A collision. Substituting Eq.(15) into Eq.(17), and then Eq.(17) into Eq.(18), with the restriction of the energy conservation,

$$\delta(E_{\mathrm{T}} - \varepsilon_{\mathrm{T}_1} - \cdots - \varepsilon_{\mathrm{T}_n}) = \frac{1}{2\pi} \int d\tau\, e^{i(E_{\mathrm{T}} - \varepsilon_{\mathrm{T}_1} - \cdots - \varepsilon_{\mathrm{T}_n})\tau}, \tag{19}$$

and then taking the limit of $n_{\max} \to \infty$, we can obtain:

$$\frac{d\sigma}{dE_{\mathrm{T}}} = \int d^2\boldsymbol{b} \frac{1}{2\pi} \int d\tau\, e^{iE_{\mathrm{T}}\tau + \int d\varepsilon_{\mathrm{T}} \frac{d\sigma_{\mathrm{NN}}}{d\varepsilon_{\mathrm{T}}} D_{\mathrm{BA}}(b)(e^{-i\varepsilon_{\mathrm{T}}\tau} - 1)}. \tag{20}$$

Because the transverse energy produced by one N–N collision is very small, we take the approximation:

$$e^{-i\varepsilon_{\mathrm{T}}\tau} \approx 1 - i\varepsilon_{\mathrm{T}}\tau - \frac{1}{2} \varepsilon_{\mathrm{T}}^2 \tau^2, \tag{21}$$

Finally, we get the total transverse energy distribution as the following:

$$\frac{d\sigma}{dE_{\mathrm{T}}} \approx \int d\overline{E}_{\mathrm{T}} \frac{d^2\boldsymbol{b}}{d\overline{E}_{\mathrm{T}}} \frac{1}{\sqrt{2\pi\,\Delta(\boldsymbol{b})}} e^{-[E_{\mathrm{T}} - \overline{E}_{\mathrm{T}}(\boldsymbol{b})]^2 / 2\Delta(\boldsymbol{b})}, \tag{22}$$

where $\overline{E}_{\mathrm{T}} = \overline{E}_{\mathrm{T}}(\boldsymbol{b})$ is the average transverse energy produced in the B+A collision:

$$\overline{E}_{\mathrm{T}}(\boldsymbol{b}) = \int d\varepsilon_{\mathrm{T}} \cdot \varepsilon_{\mathrm{T}} \frac{d\sigma_{\mathrm{NN}}}{d\varepsilon_{\mathrm{T}}} D_{\mathrm{BA}}(\boldsymbol{b}) = \bar{\varepsilon}_{\mathrm{T}} \overline{N}(\boldsymbol{b}), \tag{23}$$

and $\Delta(b)$ is the width of the Gaussian distribution

$$\Delta^2(b) = \int d\varepsilon_T \cdot \varepsilon_T^2 \frac{d\sigma_{NN}}{d\varepsilon_T} D_{BA}(b) = \overline{\varepsilon_T^2}\, \overline{N}(b), \tag{24}$$

where $\bar{\varepsilon}_T$ and $\overline{\varepsilon_T^2}$ are, respectively, the first-order and second-order moments of the distribution function $d\sigma_{NN}/d\varepsilon_T$:

$$\bar{\varepsilon}_T = \frac{1}{\sigma_{NN}} \int d\varepsilon_T \cdot \varepsilon_T \frac{d\sigma_{NN}}{d\varepsilon_T}, \tag{25}$$

$$\overline{\varepsilon_T^2} = \frac{1}{\sigma_{NN}} \int d\varepsilon_T \cdot \varepsilon_T^2 \frac{d\sigma_{NN}}{d\varepsilon_T}. \tag{26}$$

Equations (22)-(26) are the relations describing the transverse energy distribution. We can understand explicitly the physical meaning of the results: the observed energy distribution $d\sigma/dE_T$ is a Gaussian diffusion of the average transverse energy distribution d^2b/dE_T. This is concordant with out previous phenomenal assumption [9]. The only difference is that the Gaussian distribution width $\Delta(b)$ here is dependent on the impact parameter b. Based on this point, using Eqs.(22)-(26) to do the calculations similar to Ref. 9, our results are in good agreement with the observations by the CERN NA35 collaboration for the 60 and 200 AGeV $^{16}O+^{197}Au$ experiments. Especially in the lower energy region of the transverse energy distribution obvious improvements in our results over that in Ref. 9 can be seen as shown in Figs. 1 and 2, respectively. We have chosen $\sigma_{NN} = 32$ mb for both processes

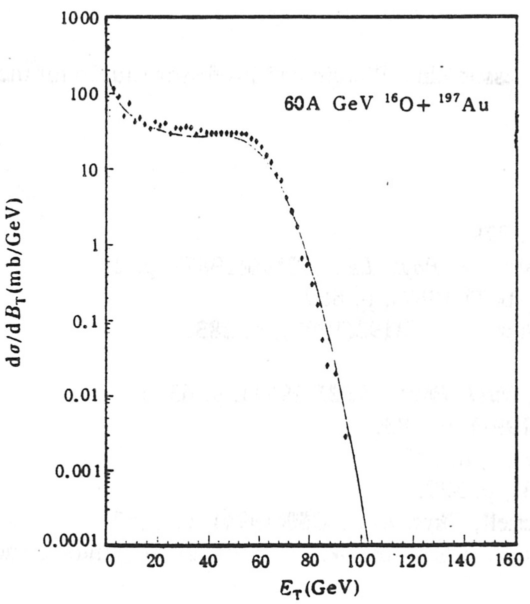

Fig. 1

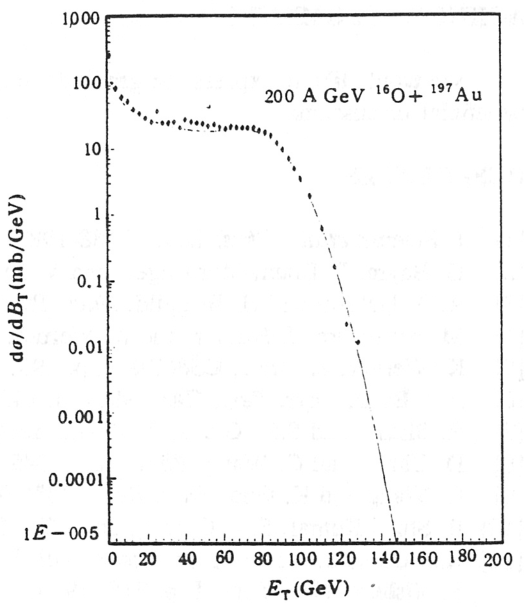

Fig. 2

and are $(\overline{\varepsilon_T}, \overline{\varepsilon_T^2})$ (0.0578 GeV, 0.0906 GeV^2) and (0.0863 GeV, 0.155 GeV^2) for 60 and 200A GeV collisions, respectively.

We can expect reasonably that by substituting the transverse energy ε_T and E_T with the corresponding particle multiplicity m and M, we will be able to get the equations directly for the total multiplicity distribution $d\sigma/dM$.

We should point out that our final conclusion is independent from the behavior of the transverse energy distribution of N–N inelastic collisions, $d\sigma_{NN}/d\varepsilon_T$, but it is related to the first and second order moments integrals $\bar{\varepsilon}_T$ and $\overline{\varepsilon_T^2}$.

Noticing that Eq.(13) is just the conclusion of the Glauber theory [11,12]. Therefore our work is actually a Glauber theory for the transverse energy distributions of high-energy heavy ion collisions. Extending the energy to very high limit, the classical description can be employed and the overlapping profile density distribution function $D_{BA}(b)$ of the two colliding nuclei plays an important role in the theory. This kind of theory actually becomes a phenomenal geometrical model theory as those in Refs. 1-10. Most of other geometrical models have adopted the following experimental simulating curve for the transverse energy distribution of N–N collisions $d\sigma_{NN}/d\varepsilon_T$:

$$\frac{d\sigma_{NN}}{d\varepsilon_T} = \frac{1}{\bar{\varepsilon}_T} e^{-\varepsilon_T / \bar{\varepsilon}_T}, \tag{27}$$

However, in our theory, $d\sigma_{NN}/d\varepsilon_T$ appears in the integrand of $E_T(b)$ and $\Delta(b)$. This indicates that $d\sigma_{NN}/d\varepsilon_T$ has only the integral effect through $\bar{\varepsilon}_T$ and $\overline{\varepsilon_T^2}$ on the total transverse energy distribution $d\sigma/dE_T$ and the results are not sensitive to $d\sigma_{NN}/d\varepsilon_T$.

The key points for the simplification of our model are the use of the Poisson limit $BA \gg n$ and $p \ll 1$, and to take the limit $n_{max} \to \infty$ in Eq.(21). This is equivalent to the use of the limit $BA \to \infty$ and for cases of very high energies. Numerical calculations indicated that in the CERN SPS very energy region, these conditions are fulfilled.

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