High Energy Physics and Nuclear Physics

Volume 20, Number 2

Multifragmentation and Fluctuation of Interaction Potential

Li Junqing, Yuan Xiuqing, Liu Jianye, Ge Linxiao, and Liu Hang

(Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, China)

The variation of equipotential lines of interaction potential with time intuitively gives the information about the fluctuation of the potential in intermediate energy heavy ion collisions. At the early stage of collisions, the potential value in the central region of the system could be positive. The pursued irregular deformation of equipotential lines in the region, especially the appearance of the negative curvature of the equipotential lines, causes the chaotic nucleonic motion. The phase difference of the neighboring nucleons increases with time exponentially, and the phase space of nucleons therefore separates and causes the multifragmentation.

Key words: multifragmentation, equipotential lines, chaotic motion.

1. INTRODUCTION

In intermediate-ener gy heavy-ion collisions, the multifragmentation is the dominant decay mode [1-3] for highly excited compound systems. The available theories indicate that the hot and compressed system formed at the beginning of the collision expands. At the subsaturation density, there may exist a liquid-gas region of instability and at even lower densities ($\rho/\rho_0 \approx 0.3$), the EOS predicts that there may be an anomaly region where the pressure decrease with the increasing density. Exponential growth of density fluctuations can lead to the separation of the phase space and cause the multifragmentation. However, this is only an assumption at the theoretical level: the true mechanism of the multifragmentation has not yet been unambiguously determined. In order to test this mechanism,

Received on February 22, 1995. Supported by the National Natural Science Foundation of China and Science Foundation of the Chinese Academy of Sciences.

we observe the time evolution of the nucleon–nucleon interaction potential in the framework of the quantum molecular dynamics approach (QMD) [4.5], which contains the many-body correlation and can describe the fluctuation and the formation of the fragments during the reaction [6].

2. EQUIPOTENTIAL LINES SHAPE AND THE DYNAMICAL MOTION PATTERN OF NUCLEONS

The interaction potential in the collision is

$$V = V^{\mathrm{loc}} + V^{\mathrm{Yuk}} + V^{\mathrm{Coul}}, \tag{1}$$

where

$$V^{\mathrm{loc}} = t_1 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) + t_2 \delta(\boldsymbol{r}_1 - \boldsymbol{r}_3)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{2}$$

$$V^{\mathrm{Yuk}} = t_3 \exp\{[-|\boldsymbol{r}_1 - \boldsymbol{r}_2| / m] / [|\boldsymbol{r}_1 - \boldsymbol{r}_2| / m]\}. \tag{3}$$

and V^{Coul} is an effective charge Coulomb interaction. The two- and three-body interactions are taken to be an equivalent density-dependent interaction of the form

$$V^{\mathrm{loc}} = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}, \tag{4}$$

with $\alpha = -124$ MeV, $\beta = 70.5$ MeV, and $\gamma = 2$. We observe the time evolution of the summation of the n–n interaction. The observation points are set in the x–z plane at $y = 0$. For a symmetric system $^{40}\mathrm{Ca}+^{40}\mathrm{Ca}$, we calculate all interaction potential lines from Eq.(1) by projecting it onto the x–z plane at $y = 0$, at incident energies: (1) 10 MeV/u; (2) 30 MeV/u; (3) 60 MeV/u; and (4) 100 MeV/u, respectively with impact parameter $b = 0$.

In Fig. 1, the time evolution of the equipotential lines are shown for mentioned cases. V_{min} indicated in the figure measures the minimum value of the potential. It is shown that although the potential is out of a simple summation of the N–N interaction over total nucleons, the mean field is indeed formed. For all the cases, we can observe two separated mean fields at the beginning, and later the formation of an unified field is seen, and then the equipotential lines irregularly deformed, separated... etc. The shape of equipotential lines can be divided roughly into three types: concentric circles; approximately quadrupole deformations; and other complicated shapes.

The equipotential lines consisting of concentric circles can be considered as the projection of a three-dimensional harmonic oscillator potential. The nucleon motion therefore has an analytical solution. The phase space of the system is limited in a ring, i.e., the phase deviation remains unchanged and there is no separation from the phase space. The system is stable.

For the quadrupole deformation of equipotential lines, the potential can be considered as a harmonic oscillator plus a quadrupole Legendre potential by projecting it onto the x–z plane at $y = 0$ as follows:

$$V = \frac{1}{2} m\omega^2 r^2 + C_2 P_2\left(\frac{z}{r}\right) + V_0, \tag{5}$$

with ω the frequency of the harmonic oscillator, C_2 the strength of the quadrupole Legendre potential,

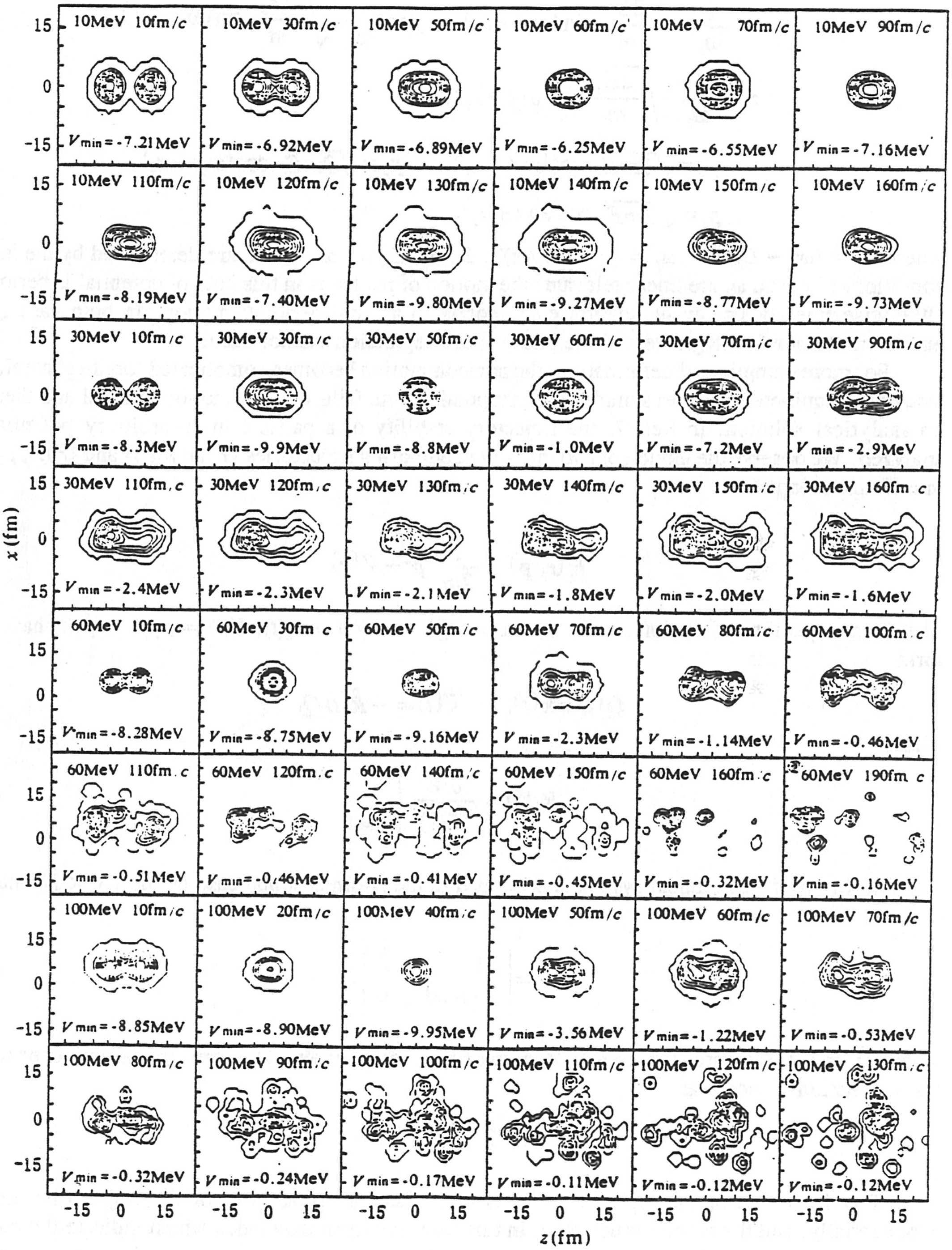

Fig. 1

The time evolution of equipotential lines for $^{40}Ca+^{40}Ca$ system at four different energies with impact parameter $b = 0$.

and V_0 is a constant. In this case, there exists an analytical solution [7]:

$$x=\frac{1}{\omega_1}\sqrt{\frac{2E_x}{m}}\sin(\omega_1 t+\alpha_x),\quad y=\frac{1}{\omega_1}\sqrt{\frac{2E_y}{m}}\sin(\omega_1 t+\alpha_y),\qquad (6)$$
$$z=\frac{1}{\omega_2}\sqrt{\frac{2E_z}{m}}\sin(\omega_2 t+\alpha_z).$$

$$p_x=\sqrt{2mE_x}\cos(\omega_1 t+\alpha_x),\quad p_y=\sqrt{2mE_y}\cos(\omega_1 t+\alpha_y),\qquad (7)$$
$$p_z=\sqrt{2mE_z}\cos(\omega_2 t+\alpha_z).$$

where $\omega_1 = (\omega^2 - C_2/m)^{1/2}$, $\omega_2 = (\omega^2 + C_2/m)^{1/2}$. E_x, E_y, E_z, α_x, α_y, and α_z are determined by the initial condition. If ω_1 and ω_2 are linear relevant, the motion of nucleons in this kind of potential is periodic. Otherwise linearly irrelevant relation corresponds to a quasi-periodic motion. In both cases, the nucleonic motion is integrable and the phase space separation cannot occur.

For more complicated deformation, the nucleon motion becomes complicated too. In general, the lose of the equipotential lines symmetry corresponds to the failure of the motion integral and there is no analytical solution. In Ref. 7, the trajectory stability of a particle in an arbitrary potential is analyzed. We observe the motion of two initially neighboring trajectories $\{r_1(t), p_1(t)\}$ and $\{r_2(t), p_2(t)\}$ in a Hamiltonian

$$H(r, p)=\frac{1}{2m}p^2+V(r)\qquad (8)$$

The linear equations of motion for the deviations $Q(t) = r_1(t) - r_2(t)$; $K(t) = p_1(t) - p_2(t)$ have the form

$$\dot{Q}(t)=\vec{\kappa}(t),\qquad \dot{\vec{\kappa}}(t)=-\hat{R}(t)Q,\qquad (9)$$

where

$$R_{ij}(t)=\left.\frac{\partial^2 V}{\partial r_i \partial r_j}\right|_{r=r_1(t)}.\qquad (10)$$

The stability of the dynamical system is described in the N-dimensional case by the $2N \times 2N$ matrix

$$\Gamma=\begin{vmatrix} 0 & \hat{I} \\ -\hat{R}(t) & \hat{0} \end{vmatrix}.\qquad (11)$$

where $\hat{0}$ and $\hat{I}$ are zero and unit $N \times N$ matrices, respectively. One can find a time-dependent transformation T such that

$$(\hat{T}\Gamma(t)\hat{T}^{-1})_{ij}=\lambda_i(t)\delta_{ij},\qquad (12)$$

If at least one of the eigenvalues λ_i is real, then the separation of the trajectories grows exponentially, and the motion is unstable. In this case, the Lyapunov index which indicate the rate of the deviation for neighboring trajectories with time is not zero. Imaginary eigenvalues correspond to a stable motion, i.e., the phases only vibrate with time and they cannot reach to infinite.

The diagonalization of the matrix $\hat{\Gamma}(t)$ is equivalent to solving the original equations of motion. The time dependence of $\hat{R}(t)$ can be eliminated by the replacement of the time-dependent point $r(t)$ in the phase space by a time-independent coordinate r.

This reduces Eq.(9) to

$$\dot{Q}=\vec{\kappa}, \qquad \dot{\vec{\kappa}}=-R(t)\,Q, \tag{13}$$

For a system with 2 degrees of freedom, the equation for the eigenvalues of the matrix Γ takes the form

$$\det\begin{vmatrix} -\lambda & 0 & 1 & 0 \\ 0 & -\lambda & 0 & 1 \\ -\dfrac{\partial^2 V}{\partial r_1^2} & -\dfrac{\partial^2 V}{\partial r_1 \partial r_2} & -\lambda & 0 \\ -\dfrac{\partial^2 V}{\partial r_1 \partial r_2} & -\dfrac{\partial^2 V}{\partial r_2^2} & 0 & -\lambda \end{vmatrix}=0. \tag{14}$$

and its solution is

$$\lambda_{1,2,3,4}=\pm[-b\pm\sqrt{b^2-4c}\,]^{1/2}, \tag{15}$$

where

$$\left.\begin{aligned} b&=S_p\,\hat{R}(r)=\frac{\partial^2 V}{\partial r_1^2}+\frac{\partial^2 V}{\partial r_2^2}, \\ c&=\det\,\hat{R}(r)=\frac{\partial^2 V}{\partial r_1^2}\frac{\partial^2 V}{\partial r_2^2}-\left(\frac{\partial^2 V}{\partial r_1 \partial r_2}\right)^2. \end{aligned}\right\} \tag{16}$$

If $b > 0$, then under the condition $c > 0$, the solution λ is purely imaginary and the motion is stable. For $c < 0$, there is a pair of real solution, and this leads to an exponential separation of the neighboring trajectories, the motion is unstable and chaotic.

Since c has the same sign with the Guassian curvature of the potential energy surface

$$K(r_1, r_2)=\frac{\dfrac{\partial^2 V}{\partial r_1^2}\dfrac{\partial^2 V}{\partial r_2^2}-\left(\dfrac{\partial^2 V}{\partial r_1 \partial r_2}\right)^2}{\left[1+\left(\dfrac{\partial V}{\partial r_1}\right)^2+\left(\dfrac{\partial V}{\partial r_2}\right)^2\right]^2} \tag{17}$$

from Eq.(15), one finds that if the potential has negative curvature the motion becomes unstable. When the neighboring nucleons come to the negative curvature region, the trajectories would be separated exponentially with time. The nucleons are scattered. Due to many scatterings, the motion of particles be comes chaotic and the separation of the whole phase space becomes inevitable.

3. EQUIPOTENTIAL LINES SHAPE AND MULTIFRAGMENTATION MECHANISM

From the four incident energy cases shown in Fig. 1, one can see that when the incident particle is bumping into a target, a unified field is formed and then the equipotential lines will deviate from a spherical shape in two ways. One is the quadrupole deformation of the whole shape by stretching in

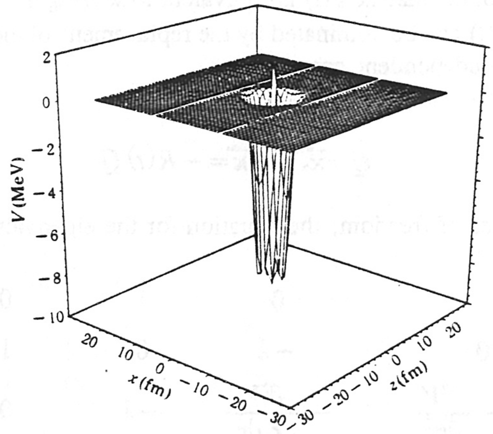

Fig. 2

The interaction potential for $^{40}Ca+^{40}Ca$ system at 100 MeV/u, 30 fm/c in x–z plane at $y = 0$.

the z-direction. The other is the deformation in both the z and x directions. Therefore the deformation is irregular. In cases (1) and (2), the potentials are stretched. Since the incident nucleus is in the z-direction, the z-component is dominant before equilibrium. Before 90 fm/c, a mode of breath oscillation can be seen. This is due to the unbalanced attractive interactions among nucleons during the density expansion of the high-pressure system. The spherical and quadrupole lines indicate that the nucleon motion is regular and the phase difference of the two-nucleon motion keeps constant; therefore, a compound nuclear system is formed in this case. However, in case (2), the incident energy is higher, at 90 fm/c two centers of the colliding nuclei start to separate, but they are still confined in a common mean field. After 110 fm/c, the system continues to expand due to the higher energy and pressure and the density may reach anomaly in the outer region. The anomaly relation between density and pressure may lead to the fluctuation of the potential field. Therefore more striking deformation of the equipotential lines can be seen in the outer region. This deformation develops towards the center region when the density diffuses outward. At 160 fm/c, there are three circle distributed peak zones but they are well enveloped by equipotential lines. Up to 200 fm/c, there still exist enveloped lines, though fewer distributed peak zones are outside.

For the case (4) at 30 fm/c, in the center region, a positive interaction potential about 0.8 MeV is found (as shown in Fig. 2). In this region, the system is greatly unstable. The nucleons moving in this center area are not only repelled by a positive potential but also attracted by a negative potential in the surrounding. With the time increase, the positive potential disappears. It is unfortunate that we cannot mark the process of its vanishing. At about 80 fm/c for case (3) and 60 fm/c for case (4), there are fluctuations both in the x and z directions in the peak zone. The latter is a fluctuation. The equipotential lines in the peak zone deform irregularly, and there even exist negative curvatures. The phases of neighboring nucleons will separate exponentially. Because the potential is determined by the position of nucleons, this fluctuation will be amplified and ultimately leads to multifragmentations. After $t \geq 100$ fm/c for case (3) and $t \geq 80$ fm/c for case (4), many local sources can be observed. They may be stable or unstable, looking very much like some stable islands embedded in some hyperbolas in the phase space of a chaotic system. Finally, these enveloped lines disappear. There are no potential gradient among the sources. The system separates into a few fragments and reaches a status where the statistic theory can be applied.

4. CONCLUSIONS

From the variation of equipotential lines with time, it seems that the chaotic nucleonic motion which will later feed back to interaction potential is originated by the irregular fluctuation in the peak zone. Therefore, the fluctuation is amplified and the whole system becomes unstable. The fluctuation at the outer region of the potential would release some particles or clusters, but it cannot influence the stability of the whole system. One should notice that in the spherical or quadrupole deformation potential, the behavior of nucleons is regular and the phases of neighboring nucleons always keeps constant. Even the system oscillation in breath type does not cause fragmentations.

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