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Relativistic Mean-field Calculation on Λ Hypernuclei

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The ground-state properties of Λ hypernuclei are studied theoretically using the relativistic mean-field theory with *TM*1 and *NL-SH* force parameters. It is found that these new parameters can very satisfactorily reproduce the ground-state properties of Λ hypernuclei. The parameter sets *TM*1 and *NL-SH* give very similar descriptions on hypernuclei. The properties of hypernuclei in baryon-hyperon systems are insensitive to the force parameters of effective nucleon-nucleon interactions. The influence of the pairing forces is also taken into account in the study of the Λ hyperon and it is found that it leads to an even-odd effect.

Key words: relativistic mean-field theory, Λ hypernuclei, even-odd effect.

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1. INTRODUCTION

The relativistic mean-field (RMF) theory has been proved very successful for describing many nuclear properties during the last decade. It has been applied in many aspects, such as properties of nuclear matter and hot neutron matter, description of finite nuclei, nuclear equation-of-state, and even dynamical calculations [1–3]. It has been shown that the relativistic mean-field theory is a good theory to understand the nuclear properties from the numerical results. On the other hand, baryonic systems contain not only nucleons but also hyperons such as Λ, Σ hyperons, etc.

Hypernuclei, as a bound system of different types of baryons, link closely the underlying hyperon -nucleon and hyperon-hyperon interactions. The study of hypernuclei has attracted much theoretical and experimental attention since the first observation of hypernuclei in 1950s. The success of the RMF

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theory in the nuclear system has inspired quite natural attempts to describe the more general baryon -system with strangeness within the same formalism [4–15].

Recently, Sharma *et al.* [16, 17] have proposed a new set of RMF force parameters named *NL-SH* that remedies the defect in *NL*1 parameters which have a too-large neutron skin thickness and symmetry energy. Sugahara *et al.* [18] have introduced the ω^4 term into the RMF theory and obtained the new parameter sets *TM*1 by the least-squares fitting on 21 nuclei. Both new RMF parameter sets can satisfactorily reproduce ground-state properties of nuclei far from the β stable line.

In this paper, the ground-state properties of Λ hypernuclei will be investigated with the new RMF parameter set *TM*1 and *NL-SH*. The difference between the two RMF parameter sets on hypernuclei will also be studied.

2. RELATIVISTIC MEAN-FIELD THEORY

The Lagrangian density in RMF theory can be written as the following form [4–13]

$$\mathscr{L}_{\mathrm{Total}} = \mathscr{L}_{\mathrm{N}} + \mathscr{L}_{\Lambda} , \tag{1}$$

where $\mathscr{L}_{\mathrm{N}}$ is the nucleon Lagrangian density [1–3, 16–18],

$$\begin{aligned}\mathscr{L}_{\mathrm{N}} = {} & \overline{\Psi}_{\mathrm{N}}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - M - g_{\sigma\mathrm{N}}\sigma - g_{\omega\mathrm{N}}\gamma^{\mu}\omega_{\mu} - g_{\rho\mathrm{N}}\gamma^{\mu}\rho^{a}_{\mu}\tau^{a})\Psi_{\mathrm{N}} \\ & + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} + \frac{1}{4}c_{3}(\omega_{\mu}\omega^{\mu})^{2} \\ & - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu} - \frac{1}{4}R^{a\mu\nu}\cdot R^{a}_{\mu\nu} + \frac{1}{2}m_{\rho}^{2}\rho^{a\mu}\cdot\rho^{a}_{\mu} \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\Psi}_{\mathrm{N}}\gamma^{\mu}A^{\mu}\frac{1}{2}(1-\tau^{3})\Psi_{\mathrm{N}} ,\end{aligned} \tag{2}$$

with

$$\Omega^{\mu\nu} = \partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu} , \tag{3}$$

$$R^{a\mu\nu} = \partial^{\mu}\rho^{a\nu} - \partial^{\nu}\rho^{a\mu} + g_{\rho}\varepsilon^{abc}\rho^{b\mu}\rho^{c\nu} , \tag{4}$$

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} , \tag{5}$$

where σ, ω_{μ}, and ρ^{a}_{μ} denote scalar, vector, and vector-isovector mesons, respectively. It involves nucleon Ψ_{N} and the photon A^{μ} for the Coulomb field. The inputs in the calculation are the *NL-SH* and *TM*1 parameters which are listed in Table 1. $g_{\sigma\mathrm{N}}$, $g_{\omega\mathrm{N}}$, and $g_{\rho\mathrm{N}}$ are the coupling constants of the σ, ω, and ρ mesons with nucleon. g_2 and g_3 are the constants of nonlinear self-interactions of the σ meson and c_3 is the constant of nonlinear self-interactions of the ω meson.

$\mathscr{L}_{\Lambda}$ denotes the Lagrangian density of Λ hyperon,

$$\mathscr{L}_{\Lambda} = \overline{\Psi}_{\Lambda}\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - M_{\Lambda} - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^{\mu}\omega_{\mu} + \frac{f_{\omega\Lambda}}{2M_{\Lambda}}\sigma^{\mu\nu}\partial_{\mu}\omega_{\nu}\right)\Psi_{\Lambda} , \tag{6}$$

The last term in the above equation is an ω-Λ tensor coupling term. $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$, and $f_{\omega\Lambda}$ are the Λ-σ, Λ-ω, and Λ-ω tensor coupling constants, respectively.

From the Lagrangian density, we obtain by standard techniques of field variation the equations with static solutions [1–3]

$$\varepsilon_{N\alpha}\phi_{N\alpha} = \gamma_0\Big(-i\boldsymbol{\gamma}\cdot\nabla + m_N + g_{\sigma N}\sigma + g_{\omega N}\omega_0\gamma_0 + \frac{1}{2}g_{\rho N}R_{0,0}\tau_0 + e^2A_0\frac{1+\tau_0}{2}\gamma_0\Big)\phi_{N\alpha}\,, \tag{7}$$

$$\varepsilon_{\Lambda\alpha}\phi_{\Lambda\alpha} = \gamma_0\Big(-i\boldsymbol{\gamma}\cdot\nabla + M_\Lambda + g_{\sigma\Lambda}\sigma + g_{\omega\Lambda}\,\omega_0\gamma_0 - \frac{f_{\omega\Lambda}}{2M_\Lambda}i\boldsymbol{\gamma}\cdot\nabla\omega_0\Big)\phi_{\Lambda\alpha}\,, \tag{8}$$

$$(-\Delta + m_\sigma)\sigma = -g_{\sigma N}\rho_{SN} - g_{\sigma\Lambda}\rho_{S\Lambda} - g_2\sigma^2 - g_3\sigma^3, \tag{9}$$

$$(-\Delta + m_\omega)\omega_0 = g_{\omega N}\rho_{0N} + g_{\omega\Lambda}\,\rho_{0\Lambda} + c_3\omega_0^3 - \frac{f_{\omega\Lambda}}{2M_\Lambda}\,\rho_{T\Lambda}, \tag{10}$$

$$(-\Delta + m_\rho)R_{0,0} = \frac{1}{2}g_{\rho N}\,\rho_{0,0}\,, \tag{11}$$

$$-\Delta A_0 = e^2\rho_{Pr,0}\;. \tag{12}$$

where the subscribes N and Λ denote nucleon and hyperon, respectively, and the density ρ_{SN}, ρ_{0N}, $\rho_{0,0}$, and $\rho_{pr,\,0}$ are nucleon densities defined as

$$\rho_{SN} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\phi_{N\alpha}\,, \tag{13}$$

$$\rho_{0N} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\gamma_0\,\phi_{N\alpha}\,, \tag{14}$$

$$\rho_{0,0} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\gamma_0\tau_0\,\phi_{N\alpha}\,, \tag{15}$$

$$\rho_{pr,0} = \sum_{\alpha=1}^{\Omega} w_{N\alpha}\bar{\phi}_{N\alpha}\frac{1+\tau_0}{2}\gamma_0\,\phi_{N\alpha}\;. \tag{16}$$

$\rho_{S\Lambda}$, $\rho_{0\Lambda}$, and $\rho_{T\Lambda}$ are the hyperon densities,

$$\rho_{S\Lambda} = \sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}\phi_{\Lambda\alpha}\,, \tag{17}$$

$$\rho_{0\Lambda} = \sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}\gamma_0\,\phi_{\Lambda\alpha}\,, \tag{18}$$

$$\rho_{T\Lambda} = \nabla\cdot\sum_{\alpha=1}^{\Omega} w_{\Lambda\alpha}\bar{\phi}_{\Lambda\alpha}\,i\boldsymbol{\alpha}\,\phi_{\Lambda\alpha}\;. \tag{19}$$

where the subscribes T and S denote tensor and scalar, respectively.

The mean field localizes the nucleus and thus breaks the transnational invariance, which results in the center-of-mass of the whole nucleus oscillating in the mean-field. The spurious excitation of the

Table 1
Parameter sets *TM*1 and *NL-SH* in the RMF theory.

	*TM*1	*NL-SH*
M(MeV)	938.0	939.0
$m\sigma$(MeV)	511.198	526.059
$m\omega$(MeV)	783.0	783.0
$m\rho$(MeV)	770.0	763.0
g_σ	10.0289	10.4436
g_ω	12.6139	12.9451
g_ρ	4.6322	4.3828
$g_2(\text{fm})^{-1}$	−7.2325	−6.9099
g_3	0.6183	−15.8837
c_3	71.3075	0.000

center-of-mass has must be eliminated. A simple and reliable treatment on the center-of-mass correction is in Ref. [19],

$$E_{cm} = \frac{\langle P_{cm}^2 \rangle}{2Am} ,$$

$$\langle P_{cm}^2 \rangle = \sum_\beta \omega_\beta \langle \phi_\beta | \hat{p}^2 | \phi_\beta \rangle - \sum_{\alpha\beta} (\omega_\alpha \omega_\beta + \sqrt{\omega_\alpha (1-\omega_\alpha)\omega_\beta(1-\omega_\beta)}\,) |\langle \phi_\alpha | \hat{p} | \phi_\beta \rangle|^2, \tag{20}$$

where $P_{cm} = \sum_i \hat{p}_i$.

3. NUMERICAL RESULTS

Hypernuclei are the system in which the Λ hyperons, with 1/2 spin, 0 isospin, and 1 strangeness, are bound in the nuclear systems in addition to protons and neutrons. Beside the nucleon-nucleon interaction parameter sets (*TM*1 or *NL-SH*), there are now three additional Λ-nucleon parameters $g_{\sigma\Lambda}$, $g_{\omega\Lambda}$, and $f_{\omega\Lambda}$ in the RMF theory. The coupling of the σ and ω to the Λ hyperon $\alpha_{\sigma\Lambda} = g_{\sigma\Lambda}/g_{\sigma N} = 0.621$, $\alpha_{\omega\Lambda} = g_{\omega\Lambda}/g_{\omega N} = 0.667$ [4], and $f_{\omega\Lambda}/g_{\omega\Lambda} = -1$ [13] are used in the calculation. In the following, we use the above three parameters with relativistic mean-field parameter sets *TM*1 and *NL-SH* to describe the properties of ground state in hypernuclei.

In Fig. 1, we plot the binding energies of the single particle states of hypernuclei as a function of the mean number $A^{-2/3}$ with $A = 9$–209. The experimental data from (π^+, K^+) production [7, 14, 15] including the heavy nuclei results are also given for comparison. The figure shows good agreement of theoretical results with experimental data both for sets *TM*1 and *NL-SH*. The difference between *TM*1 and *NL-SH* is very small and the maximum error is only 1 MeV. Until now, there are not enough experimental data on double- and multi-Λ hypernuclei, and we will discuss this lack in the paper.

Figure 2 shows the ground-state properties of multi-Λ hypernuclei of Ca which is built out of the double closed-shell nucleus ^{40}Ca. ^{40}Ca is a stable nucleus and the single-particle levels of nucleon are much deeper than that of the hyperon. The pairing correlations have been taken into account first for hyperon in multi-Λ hypernuclei using the BCS formalism with a similar treatment as that in nucleon with $\Delta_\Lambda = 11.2/\sqrt{A}$. For odd Λ hypernuclei, there exists a pairing block in Λ hyperons and an approximate method is used in calculation. It is obvious that when the pairing correlations are taken

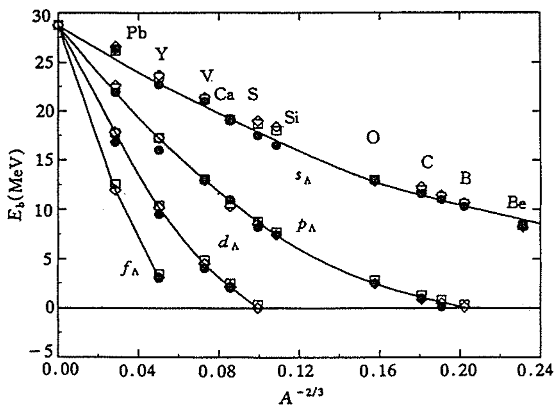

Fig. 1
s_Λ, p_Λ, d_Λ, and f_Λ single-particle energies are plotted versus the atomic mass of the nuclei.
• denotes experimental data [7, 14, 15]; □ and ◇ denote the results of *TM*1 and *NL-SH*, respectively.

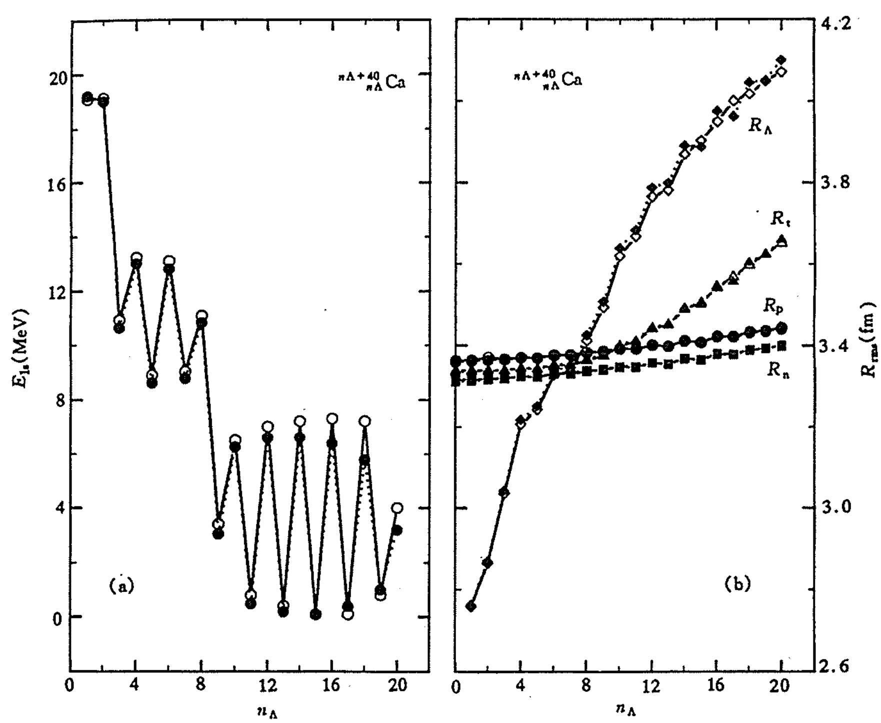

Fig. 2
(a) Lambda separation energy as a function of the number of Λ particles; (b) various radii of hypernuclei, Δ total (R_t), ○ proton (R_p), □ neutron (R_n), and ◇ Λ(R_Λ) rms radii. The full line denotes the results of *TM*1, the dashed line denotes the results of *NL-SH*.

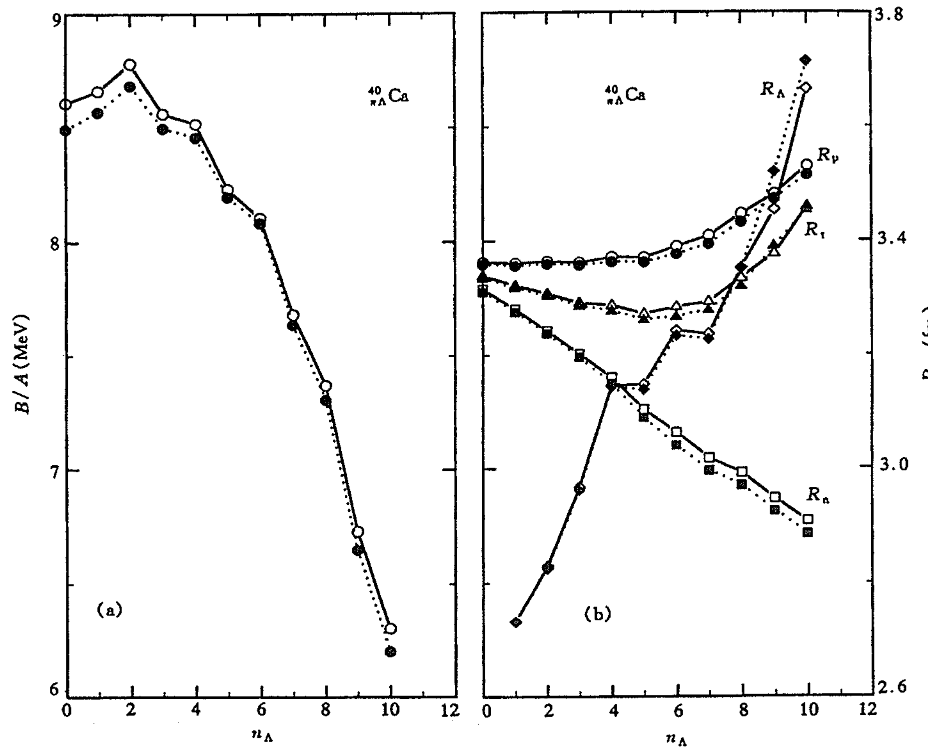

Fig. 3

(a) The binding energy per nucleon of $^{40}_{n\Lambda}$Ca; (b) various radii of hypernuclei and the caption is same as Fig. 2.

into account the odd-even effect appears also in the lambda separation energy. There occurs a dramatic decrease of the lambda separation energy for the number of lambda particles from 2 to 3 and from 8 to 9. This can be explained as a shell effect in $n_\Lambda = 2$, 8 and the 3th Λ has to go into the $1p_{3/2}$ level which is gives a very weak binding to the Λ particle. It is also the reason why the multi-Λ hypernuclei with $n_\Lambda > 2$ are not observed up to now. It also shows that the proton radius and the neutron radius only have a small change with the increase of lambda particle number. In the mean time, the Λ radius increases rapidly with the increase of lambda particle number and the Λ radius is greater than the neutron radius when $n_\Lambda > 8$. The total radius of hypernuclei is also effected by the lambda number obviously.

In Fig. 3, we show another case of multi-Λ hypernuclei, which keeps the constant value for $A = 40$ and $Z = 20$ and substitutes lambda hyperon to the neutron. This may go on until the outmost level is not bound theoretically. Half of the neutrons can be replaced by lambda particles for $^{40}_{n\Lambda}$Ca and this kind of hypernuclei may exist theoretically. It also shows in Fig. 3 that the binding energy per nucleon has a maximum value at $n_\Lambda = 2$ and has a rapid decrease at $n_\Lambda = 8$. This means that $^{40}_{n\Lambda}$Ca is the most stable hypernuclei and $n_\Lambda = 8$ is a closed-shell for hypernuclei. Figure 3(b) shows that the neutron radius decreases and the proton radius increases with the increase of the neutron number replaced by lambda hyperons and the lambda radius increases more rapidly than any other radius and it ultimately exceeds the proton radius.

It is indicated from the above discussion that the RMF theory provides a good description on the binding energy of single-Λ hypernuclei and explains why the multi-Λ hypernuclei have not been observed in experiments until now. The theoretical calculation shows that there occurs the even-odd effect when the pairing correlations are taken into account in multi-Λ hypernuclei and there is a strong shell effect when the Λ hyperon number is 2 and 8. Finally, the difference between two RMF parameter sets *TM*1 and *NL-SH* is studied and it is found that the difference is very small within the same Λ-σ, Λ-ω coupling parameters and Λ-ω tensor coupling parameter. This indicates that the ground-state properties of hypernuclei are insensitive to the parameters of effective nucleon-nucleon interactions.

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