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$osp(1|4)$Toda 模型解的构造

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摘要 将 Leznov - Saveliev 代数分析和 Drinfeld - Sokolov 构造这种方法推广到超对称情形,并运用这种方法给出 $osp(1|4)$Toda 模型的解,从而将这种方法推广到二秩情况.

关键词 Toda 超代数 最高权 手征

1 引言

二维可积场论在过去 20 年里取得了丰硕的成果,而 Toda 场论因为与现代理论物理领域中许多重大课题有着密切关系而得到人们广泛研究.例如,与扩展共形代数(W 代数)和 W 引力紧密联系的共形 Toda 场,与量子群对称紧密相关的仿射 Toda 场.过去几年中,对带有费米子的超对称 Toda 场的研究重新引起了物理学家的兴趣.Toppan 证明了带有玻色素根的仿射李超代数哈密顿约化确实可以产生超对称 Toda 模型[1],Leznov 等通过 $sl(n|n-1)$超群的基本表示以矩阵元形式给出了有固定端点 f - Toda 链的通解[2].Prata 在仿射超代数 $B^{(1)}(0,1)$基础上计算了超 Toda 模型可积边界的相互作用[3]……过去数年中,为解决 Toda 形二维可积场论而提出了许多种好的方法.例如,Hirota 双线性方程,Leznov - Saveliev 分析,Dressing - Backlund 变换,Drinfeld - Sokolov 构造……但是,这些方法并不是都被推广到超对称情形.有关 Leznov - Saveliev 分析和玻色(超)共形 Toda 模型解的 Drinfeld - Sokolov 构造的研究已经进行了多年[4—8],但将这种方法运用到超可积模型却并不多见,本文作者曾经在基本李超代数 $osp(1|2)$基础上,用 Leznov - Saveliev 代数分析和 Drinfeld - Sokolov 构造给出了超 Liouville 模型的精确解[9].从可积场观点看,每个基本李超代数都可建立相应的超 Toda 场论[10].而并不是所有超代数都有对应的超对称 Toda 模型,$osp(1|4)$Toda 模型就是一个显著例子.

2 $osp(1|4)$Toda 模型

李超代数 $osp(1|4)$是超代数中具有玻色素根的最简单例子.它是秩为二的 14 维超

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代数，具有10个玻色生成元和4个费米生成元. 两个玻色生成元 H_1, H_2 构成 Cartan 子代数. $osp(1|4)$的根系是由一个正(负)费米素根和一个正(负)玻色素根组成. 这里采用 Serre－Chevalley 基底，用$\{e_1, f_1\}$，$\{e_2, f_2\}$分别表示素根所对应玻色和费米生成元，其中$\{e_1, f_1\}$是具有 Grassmann 偶宇称(玻色)的，$\{e_2, f_2\}$是具有 Grassmann 奇宇称(费米)的，其生成元的对易关系式如下：

$$[H_i, H_j] = 0, \quad [H_i, e_j] = a_{ij}e_j, \quad [H_i, f_j] = -a_{ij}f_j, \quad [e_i, f_j] = \delta_{ij}H_j.$$

这里采用的 Cartan 矩阵已由文献[11]给出

$$A = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}$$

为了构造出 $osp(1|4)$Toda 场运动方程解，这里引入最高权表示. $osp(1|4)$拥有两个基本主权$|\lambda^{(1)}\rangle$，$|\lambda^{(2)}\rangle$而且它们都是 Grassmann 奇的权矢量$|\lambda^{(i)}\rangle$及其对偶矢量$\langle\lambda^{(i)}|$，其特点如下：

$$H_i|\lambda^{(j)}\rangle = \delta_{ij}|\lambda^{(j)}\rangle, \quad e_i|\lambda^{(j)}\rangle = 0,$$
$$f_i|\lambda^{(j)}\rangle = \delta_{ij}f_j|\lambda^{(j)}\rangle, \quad \langle\lambda^j|H_i = \langle\lambda^{(j)}|\delta_{ij},$$
$$\langle\lambda^{(j)}|e_i = \delta_{ij}\langle\lambda^{(j)}|e_j, \quad \langle\lambda^{(j)}|f_i = 0.$$

下面来介绍 $osp(1|4)$Toda 理论，由于这个模型具有良好的可积性，首先给出这个模型的 Lax pair. 这里，采用光锥坐标 $x_\pm = t \pm x$，$\partial_\pm = \partial_{x\pm}$. $osp(1|4)$Toda 模型的 Lax pair 被定义如下：

$$\partial_+ T = \left[\frac{1}{2}\partial_+\Phi + \exp\left(-\frac{1}{2}ad\Phi\right)(\bar{\Psi}_+ + \varepsilon_+)\right]T,$$
$$\partial_- T = -\left[\frac{1}{2}\partial_-\Phi + \exp\left(\frac{1}{2}ad\Phi\right)(\bar{\Psi}_- + \varepsilon_-)\right]T.$$

其中 $\Phi = \phi_1 H_1 + \phi_2 H_2$，$\Psi_- = \psi_-^{(1)}e_1 + \psi_-^{(2)}e_2$，$\Psi_+ = \psi_+^{(1)}f_1 + \psi_+^{(2)}f_2$，

$\varepsilon_+ = \theta[e_1, e_2]$，$\varepsilon_- = \bar{\theta}[f_1, f_2]$，$\bar{\Psi}_\pm = \pm[\varepsilon_\pm, \Psi_\pm]$.

$\phi_1, \phi_2, \psi_\pm^{(1)}$是玻色场，$\psi_\pm^{(2)}$是费米场，$\theta, \bar{\theta}$是 Grassmann 数. 其相容性条件给出场方程

$$\begin{aligned}\partial_+\Psi_- - \exp(-ad\Phi)\bar{\Psi}_+ &= 0,\\ \partial_-\Psi_+ - \exp(ad\Phi)\bar{\Psi}_- &= 0,\\ \partial_+\partial_-\Phi - [\bar{\Psi}_+, \exp(ad\Phi)\bar{\Psi}_-] - [\varepsilon_+, \varepsilon_-] &= 0.\end{aligned}$$

用分量场 $\phi_1, \phi_2, \psi_\pm^{(1)}, \psi_\pm^{(2)}$来表示上面的场运动方程，可写成如下形式：

$$\partial_+\psi_-^{(1)} - \theta\psi_+^{(2)}a_{21}e^{-a_{11}\phi_1}e^{-a_{21}\phi_2} = 0, \quad \partial_+\psi_-^{(2)} - \theta\psi_+^{(1)}a_{12}e^{-a_{12}\phi_1}e^{-a_{22}\phi_2} = 0,$$
$$\partial_-\psi_+^{(1)} - \bar{\theta}\psi_-^{(2)}a_{21}e^{-a_{11}\phi_1}e^{-a_{21}\phi_2} = 0, \quad \partial_-\psi_+^{(2)} + \bar{\theta}\psi_-^{(1)}a_{12}e^{-a_{12}\phi_1}e^{-a_{22}\phi_2} = 0,$$
$$\partial_+\partial_-\phi_1 + \theta\bar{\theta}\psi_+^{(2)}\psi_-^{(2)}a_{21}^2e^{-a_{11}\phi_1}e^{-a_{21}\phi_2} + \theta\bar{\theta}a_{21}e^{-(a_{11}+a_{12})\phi_1}e^{-(a_{21}+a_{22})\phi_2} = 0,$$
$$\partial_+\partial_-\phi_2 - \theta\bar{\theta}\psi_+^{(1)}\psi_-^{(1)}a_{12}^2e^{-a_{12}\phi_1}e^{-a_{22}\phi_2} + \theta\bar{\theta}a_{12}e^{-(a_{11}+a_{12})\phi_1}e^{-(a_{21}+a_{22})\phi_2} = 0.$$

3　超 Drinfeld－Sokolov 构造

上一节给出分量场运动方程，这一节我们主要构造出 $osp(1|4)$Toda 分量场的解. 在特殊规范下定义 Drinfeld－Sokolov 线性系统：

$$\partial_+Q_+ = L_+Q_+, \quad \partial_-Q_+ = 0, \partial_-Q_- = Q_-L_-, \quad \partial_+Q_- = 0.$$

其中：$L_+ = \partial_+ K_+(x_+) + \overline{P}_+ + E_+$，　$K_\pm(x) = k_\pm^{(1)}(x)H_1 + k_\pm^{(2)}(x)H_2$，

$$P_+(x) = \rho_+^{(1)}(x)f_1 + \rho_+^{(2)}(x)f_2, P_-(x) = \rho_-^{(1)}(x)e_1 + \rho_-^{(2)}(x)e_2,$$

$$\overline{P}_+ = [\epsilon_+, P_+(x)] = \theta\rho_+^{(1)}(x)a_{12}e_2 + \theta\rho_+^{(2)}(x)a_{21}e_1,$$

$$\overline{P}_- = -[\epsilon_-, P_-(x)] = -\overline{\theta}\rho_-^{(1)}(x)a_{12}f_2 + \overline{\theta}\rho_-^{(2)}(x)a_{21}f_1.$$

$k_\pm^{(1)}(x_\pm), k_\pm^{(2)}(x_\pm), \rho_\pm^{(1)}(x)$分别是任意的玻色手征函数和反手征函数. $\rho_\pm^{(2)}(x_\pm)$是任意的费米手征函数和反手征函数. 为了得到方程的解，在李超代数 $osp(1|4)$的基础上引入手征矢量

$$\mu^{(i)}(x_+) = \langle\lambda^{(i)} | Q_+(x_+), \qquad \overline{\mu}^{(i)}(x_-) = Q_-(x_-) | \lambda^{(i)}\rangle,$$

$$\xi^{(i)}(x_+) = \langle\lambda^{(i)} | e_i e^{P_+(x_+)} Q_+(x_+), \overline{\xi}^{(i)}(x_-) = Q_-(x_-) e^{P_-(x_-)} f_i | \lambda^{(i)}\rangle.$$

于是构造解如下：

$$e^{\phi^i(x)} = \mu^{(i)}(x_+) D \overline{\mu}^{(i)}(x_-),$$

$$\psi_+^{(i)}(x) = \frac{\xi^{(i)}(x_+) D \overline{\mu}^{(i)}(x_-)}{\mu^{(i)}(x_+) D \overline{\mu}^{(i)}(x_-)},$$

$$\psi_-^{(i)}(x) = (-1)^{(i-1)} \frac{\mu^{(i)}(x_+) D \overline{\xi}^{(i)}(x_-)}{\mu^{(i)}(x_+) D \overline{\mu}^{(i)}(x_-)}.$$

上面 D 是作用在表示空间上的任意常数矩阵. 下面将来证明这确实是分量式运动方程的解. 通过直接计算可得到

$$\partial_+ \partial_- \phi^{(i)} = \partial_+ \partial_- \ln(\mu^{(i)}(x_+) D \overline{\mu}^{(i)}(x_-)) = \frac{\det\begin{pmatrix} \mu^{(i)}(x_+) D\overline{\mu}^{(i)}(x_-) & \partial_+ \mu^{(i)}(x_+) D\overline{\mu}^{(i)}(x_-) \\ \mu^{(i)}(x_+) D\partial_- \overline{\mu}^{(i)}(x_-) & \partial_+ \mu^{(i)}(x_+) D\partial_- \overline{\mu}^{(i)}(x_-) \end{pmatrix}}{(\mu^{(i)}(x_+) D\overline{\mu}^{(i)}(x_-))^2},$$

令 Δ 表示上式右边行列式，$G = Q_+ D Q_-$，现在来计算 Δ：

$$\Delta = \langle\lambda^{(i)} | Q_+ D Q_- | \lambda^{(i)}\rangle\langle\lambda^{(i)} | L_+ Q_+ D Q_- L_- | \lambda^{(i)}\rangle - \langle\lambda^{(i)} | Q_+ D Q_- L_- | \lambda^{(i)}\rangle\langle\lambda^{(i)} | L_+ Q_+ D Q_- | \lambda^{(i)}\rangle,$$

$$\Delta = \langle\lambda^{(i)} | G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | L_+ G L_- | \lambda^{(i)}\rangle - \langle\lambda^{(i)} | G L_- | \lambda^{(i)}\rangle \cdot \langle\lambda^{(i)} | L_+ G | \lambda^{(i)}\rangle,$$

$$\begin{aligned}\Delta = {} & \langle\lambda^{(i)} | G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | \overline{P}_+ G \overline{P}_- | \lambda^{(i)}\rangle - \langle\lambda^{(i)} | \overline{P}_+ G | \lambda^{(i)}\rangle \cdot \\ & \langle\lambda^{(i)} | G \overline{P}_- | \lambda^{(i)}\rangle + \langle\lambda^{(i)} | G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | \epsilon_+ G \overline{P}_- | \lambda^{(i)}\rangle - \\ & \langle\lambda^{(i)} | \epsilon_+ G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | G \overline{P}_- | \lambda^{(i)}\rangle + \langle\lambda^{(i)} | G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | \overline{P}_+ G\epsilon_- | \lambda^{(i)}\rangle - \\ & \langle\lambda^{(i)} | \overline{P}_+ G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | G \overline{P}_- | \lambda^{(i)}\rangle + \langle\lambda^{(i)} | G | \lambda^{(i)}\rangle \cdot \\ & \langle\lambda^{(i)} | \epsilon_+ G\epsilon_- | \lambda^{(i)}\rangle - \langle\lambda^{(i)} | \epsilon_+ G | \lambda^{(i)}\rangle\langle\lambda^{(i)} | G\epsilon_- | \lambda^{(i)}\rangle.\end{aligned}$$

将 $\overline{P}_\pm, \epsilon_\pm$ 的定义代入上面式子中，当 $i = 1$ 时

$$\begin{aligned}\Delta = {} & -\theta\overline{\theta}\rho_+^{(2)}(x)\rho_-^{(2)}(x)a_{21}^2[\langle\lambda^{(1)} | G | \lambda^{(1)}\rangle\langle\lambda^{(1)} | e_1 G f_1 | \lambda^{(1)}\rangle - \\ & \langle\lambda^{(1)} | e_1 G | \lambda^{(1)}\rangle\langle\lambda^{(1)} | G f_1 | \lambda^{(1)}\rangle] + \\ & \theta\overline{\theta}\rho_+^{(2)}(x)a_{21}[\langle\lambda^{(1)} | G | \lambda^{(1)}\rangle\langle\lambda^{(1)} | e_1 G[f_1, f_2] | \lambda^{(1)}\rangle - \\ & \langle\lambda^{(1)} | e_1 G | \lambda^{(1)}\rangle\langle\lambda^{(1)} | G[f_1, f_2] | \lambda^{(1)}\rangle] -\end{aligned}$$

$$\theta\bar{\theta}\rho_-^{(2)}(x)a_{21}[\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|[e_1,e_2]Gf_1|\lambda^{(1)}\rangle - \langle\lambda^{(1)}|[e_1,e_2]G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|Gf_1|\lambda^{(1)}\rangle] - \theta\bar{\theta}[\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|[e_1,e_2]G[f_1,f_2]|\lambda^{(1)}\rangle - \langle\lambda^{(1)}|[e_1,e_2]G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|G[f_1,f_2]|\lambda^{(1)}\rangle],$$

$$\Delta = \theta\bar{\theta}\rho_+^{(2)}(x)\rho_-^{(2)}(x)a_{21}^2\frac{1}{2}\langle\Lambda^{(1)}|G\otimes G|\Lambda^{(1)}\rangle - \theta\bar{\theta}\rho_+^{(2)}(x)a_{21}\frac{1}{2}\langle\Lambda^{(1)}|G\otimes G|\Xi^{(12)}\rangle - \theta\bar{\theta}\rho_-^{(2)}(x)a_{21}\frac{1}{2}\langle\Xi^{(12)}|G\otimes G|\Lambda^{(1)}\rangle - \theta\bar{\theta}\frac{1}{2}\langle\Xi^{(12)}|G\otimes G|\Xi^{(12)}\rangle.$$

这里为了计算方便，我们需要准备一些关于李超代数 $osp(1|4)$ 张量积表示知识. 对此，定义权态

$$\begin{aligned}|\Lambda^{(1)}\rangle &= |\lambda^{(1)}\rangle\otimes f_1|\lambda^{(1)}\rangle - f_1|\lambda^{(1)}\rangle\otimes f_1|\lambda^{(1)}\rangle,\\ \langle\Lambda^{(1)}| &= \langle\lambda^{(1)}|\otimes\langle\lambda^{(1)}|e_1 - \langle\lambda^{(1)}|e_1\otimes\langle\lambda^{(1)}|.\\ |\Xi^{(12)}\rangle &= (f_2\otimes 1 + 1\otimes f_2)|\Lambda^{(1)}\rangle = \\ &\quad |\lambda^{(1)}\rangle\otimes[f_1,f_2]|\lambda^{(1)}\rangle + [f_1,f_2]|\lambda^{(1)}\rangle\otimes|\lambda^{(1)}\rangle,\\ \langle\Xi^{(12)}| &= \langle\Lambda^{(1)}|(e_2\otimes 1 + 1\otimes e_2) = \\ &\quad \langle\lambda^{(1)}|[e_1,e_2]\otimes\langle\lambda^{(1)}| + \langle\lambda^{(1)}|\otimes\langle\lambda^{(1)}|[e_1,e_2].\end{aligned}$$

$$(H_i\otimes 1 + 1\otimes H_i)|\Lambda^{(1)}\rangle = (2\delta_{i1} - a_{i1})|\Lambda^{(1)}\rangle, \langle\Lambda^{(1)}|\Lambda^{(1)}\rangle = -2.$$

从上面可以看出，$|\Lambda^{(1)}\rangle$ 是李超代数 $osp(1|4)$ 张量积表示的最高权态. 相应地，$|\Xi^{(12)}\rangle$ 是张量积表示的次高权态. 这里引入态 $\sqrt{2}\otimes|\lambda^{(2)}\rangle^{\otimes(-a_{21})}$ 由于 $(H_i\otimes 1 + 1\otimes H_i)\sqrt{2}\otimes|\lambda^{(2)}\rangle^{\otimes(-a_{21})} = (2\delta_{i1} - a_{i1})\sqrt{2}\otimes|\lambda^{(2)}\rangle^{\otimes(-a_{21})}$ 而且它的归一化常数 $(\sqrt{2}\otimes\langle\lambda^{(2)}|^{\otimes(-a_{21})})(\sqrt{2}\otimes|\lambda^{(2)}\rangle^{\otimes(-a_{21})}) = -2$. 因此，$|\Lambda^{(1)}\rangle = \sqrt{2}\otimes|\lambda^{(2)}\rangle^{\otimes(-a_{21})}$. 至此，通过计算可以得到如下结果

$$\begin{aligned}\frac{1}{2}\langle\Lambda^{(1)}|G\otimes G|\Lambda^{(1)}\rangle &= -\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle^2,\\ \frac{1}{2}\langle\Lambda^{(1)}|G\otimes G|\Xi^{(12)}\rangle &= -a_{21}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|Gf_2|\lambda^{(2)}\rangle,\\ \frac{1}{2}\langle\Xi^{(12)}|G\otimes G|\Lambda^{(1)}\rangle &= -a_{12}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle,\\ \frac{1}{2}\langle\Xi^{(12)}|G\otimes G|\Xi^{(12)}\rangle &= a_{21}\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle - a_{21}^2\langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle\langle\lambda|Gf_2|\lambda^{(2)}\rangle.\end{aligned}$$

另一方面可以得到

$$[-\theta\bar{\theta}\psi_+^{(2)}\psi_-^{(2)}a_{21}^2e^{-2\phi_1}e^{-2\phi_2} - \theta\bar{\theta}a_{21}e^{-\phi_1}]e^{2\phi_1} = \theta\bar{\theta}a_{21}^2[-\rho_+^{(2)}\rho_-^{(2)}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle^2 + \rho_+^{(2)}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|Gf_2|\lambda^{(2)}\rangle + \rho_-^{(2)}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle + \langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|Gf_2|\lambda^{(2)}\rangle] - \theta\bar{\theta}a_{21}\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle.$$

至此,分量式运动方程的第一个解得到验证.当 $i=2$ 时,

$$\begin{aligned}\Delta = & \theta\bar{\theta}\rho_+^{(1)}(x)\rho_-^{(1)}(x)a_{12}^2[\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|e_2Gf_2|\lambda^{(2)}\rangle - \\ & \langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|Gf_2|\lambda^{(2)}\rangle] - \\ & \theta\bar{\theta}\rho_+^{(1)}(x)a_{12}[\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|e_2G[f_1,f_2]|\lambda^{(2)}\rangle - \\ & \langle\lambda^{(2)}|e_2G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|G[f_1,f_2]|\lambda^{(2)}\rangle] + \\ & \theta\bar{\theta}\rho_-^{(1)}(x)a_{12}[\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|[e_1,e_2]Gf_2|\lambda^{(2)}\rangle - \\ & \langle\lambda^{(2)}|[e_1,e_2]G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|Gf_2|\lambda^{(2)}\rangle] - \\ & \theta\bar{\theta}[\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|[e_1,e_2]G[f_1,f_2]|\lambda^{(2)}\rangle - \\ & \langle\lambda^{(2)}|[e_1,e_2]G|\lambda^{(2)}\rangle\langle\lambda^{(2)}|G[f_1,f_2]|\lambda^{(2)}\rangle],\end{aligned}$$

$$\begin{aligned}\Delta = & \theta\bar{\theta}\rho_+^{(1)}(x)\rho_-^{(1)}(x)a_{12}^2\frac{1}{2}\langle\Lambda^{(2)}|G\otimes G|\Lambda^{(2)}\rangle - \\ & \theta\bar{\theta}\rho_+^{(1)}(x)a_{12}\frac{1}{2}\langle\Lambda^{(2)}|G\otimes G|\Xi^{(21)}\rangle - \\ & \theta\bar{\theta}\rho_-^{(1)}(x)a_{12}\frac{1}{2}\langle\Xi^{(21)}|G\otimes G|\Lambda^{(2)}\rangle + \theta\bar{\theta}\frac{1}{2}\langle\Xi^{(21)}|G\otimes G|\Xi^{(21)}\rangle.\end{aligned}$$

同样,引入了李超代数 $osp(1|4)$ 最高权 $|\lambda^{(2)}\rangle$ 的张量积表示:

$$\begin{aligned}|\Lambda^{(2)}\rangle &= |\lambda^{(2)}\rangle\otimes f_2|\lambda^{(2)}\rangle + f_2|\lambda^{(2)}\rangle\otimes f_2|\lambda^{(2)}\rangle, \\ \langle\Lambda^{(2)}| &= \langle\lambda^{(2)}|\otimes\langle\lambda^{(2)}|e_2 + \langle\lambda^{(2)}|e_2\otimes\langle\lambda^{(2)}|. \\ |\Xi^{(21)}\rangle &= (f_1\otimes 1 + 1\otimes f_1)|\Lambda^{(2)}\rangle = \\ & |\lambda^{(2)}\rangle\otimes[f_1,f_2]|\lambda^{(2)}\rangle + [f_1,f_2]|\lambda^{(2)}\rangle\otimes|\lambda^{(2)}\rangle, \\ \langle\Xi^{(21)}| &= \langle\Lambda^{(2)}|(e_1\otimes 1 + 1\otimes|\lambda^{(2)}\rangle, \\ \langle\Xi^{(21)}| &= \langle\Lambda^{(2)}|(e_1\otimes 1 + 1\otimes e_1) = \\ & -\langle\lambda^{(2)}|[e_1,e_2]\otimes\langle\lambda^{(2)}| - \langle\lambda^{(2)}|\otimes\langle\lambda^{(2)}|[e_1,e_2].\end{aligned}$$

用同样的方法引入态 $\sqrt{2}\otimes|\lambda^{(1)}\rangle^{\otimes(-a_{12})}$,并且容易证明 $|\Lambda^{(2)}\rangle=\sqrt{2}\otimes|\lambda^{(1)}\rangle^{\otimes(-a_{12})}$ 于是,通过计算可以得到如下结果:

$$\begin{aligned}\frac{1}{2}\langle\Lambda^{(2)}|G\otimes G|\Lambda^{(2)}\rangle &= \langle\lambda^{(1)}|G|\lambda^{(1)}\rangle, \\ \frac{1}{2}\langle\Lambda^{(2)}|G\otimes G|\Xi^{(21)}\rangle &= -a_{12}\langle\lambda^{(1)}|Gf_1|\lambda^{(1)}\rangle, \\ \frac{1}{2}\langle\Xi^{(21)}|G\otimes G|\Lambda^{(2)}\rangle &= -a_{21}\langle\lambda^{(1)}|e_1G|\lambda^{(1)}\rangle, \\ \frac{1}{2}\langle\Xi^{(21)}|G\otimes G|\Xi^{(21)}\rangle &= -a_{12}\frac{\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle^2}{\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle} + a_{12}^2\frac{\langle\lambda^{(1)}|e_1G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|Gf_1|\lambda^{(1)}\rangle}{\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle}.\end{aligned}$$

另一方面可以得到

$$\begin{aligned}&[\theta\bar{\theta}\psi_+^{(1)}\psi_-^{(1)}a_{12}^2e^{\phi_1}e^{-2\phi_2} - \theta\bar{\theta}a_{12}e^{-\phi_1}]e^{2\phi_2} = \\ &\theta\bar{\theta}a_{12}^2[\rho_+^{(1)}\rho_-^{(1)}\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle + \rho_+^{(1)}\langle\lambda^{(1)}|Gf_1|\lambda^{(1)}\rangle + \\ &\rho_-^{(1)}\langle\lambda^{(1)}|e_1G|\lambda^{(1)}\rangle +\end{aligned}$$

$$\langle\lambda^{(1)}|e_1 G|\lambda^{(1)}\rangle\langle\lambda^{(1)}|Gf_1|\lambda^{(1)}\rangle\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle^{-1}] -$$
$$\theta\bar{\theta}a_{12}\langle\lambda^{(1)}|G|\lambda^{(1)}\rangle^{-1}\langle\lambda^{(2)}|G|\lambda^{(2)}\rangle^2$$

至此,我们验证了两个分量场的解.其余 4 个方程也可用同样方法得到验证.由于明显形式解的求解过程太繁琐,作者没有给出分量场方程解的显式.

4 讨论

本文在李超代数 $osp(1|4)$ 及其最高权表示理论基础上,运用 Leznov-Saveliev 代数分析方法和 Drinfeld-Sokolov 构造给出了 $osp(1|4)$Toda 场运动方程矩阵元形式解.这里值得注意的是,当李超代数的 *Cartan* 矩阵非对角元素非负时,这种方法将不再适用.在这篇文章中并没有讨论这些解是否具有定域性周期性特点,这些我们将在以后的文章加以阐述.

参考文献(References)

1 Sorokin, Toppan. hep-th/9610038
2 Derjagin V B, Leznov A N, Sorin A. solv-int/9803010
3 Prata N G N. hep-th/9704851
4 Leznov A N, Saveliev M V. Lett. Math. Phys., 1979, **207**:489
5 CHAO L, HOU BoYu. Annals. Phys, 1994, **1-20**:230
6 CHAO L, QU C Z. Int. J. Phys. 1997, **36**:7
7 Leznov A N, Saveliev M V. Lett. Math. Phys., 1982, **6**:505
8 Leznov A N. Phys. Lett., 1978, **B 79**. 294; Commun. Math. Phys., 1980. **74**
9 YANG ZhangYing, ZHAO Liu, ZHEN Yi. High Energy Phys. and Nucl. Phys. (in Chinese), 1999, **24**:91
(杨战营,赵柳,甄翼. 高能物理与核物理,1999,**24**:91)
10 Olshannesky M A. Commun. Math. Phys., 1983, **88**:63
11 Kac V G. Adv. Math., 1977, **26**:85

Construction on the Solution of $osp(1|4)$Toda Model

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Abstract The Leznov - Saveliev algebraic analysis method and Drinfeld - Sokolov construction are applied to the supersymmetric case. In this approach, we obtained the solution of the $osp(1|4)$Toda model on the base of the Lie super algebra $osp(1|4)$and its highest weight by introducing chiral vectors. Therefore, we generalized this method to two rank case.

Key words Toda, super algebra, highest weight chiral

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