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B_c Meson Studies

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Abstract Our studies on the meson B_c are briefly reviewed. The decays of the meson B_c to a *P*-wave charmonium χ_c or h_c are reported comparatively precisely because they are fresh results obtained recently.

The B_c meson is a particularly interesting hadron. It contains two different heavy quarks i.e. has two heavy flavor precisely, so it has nor direct strong, neither electromagnetic decays, and due to the fact that CKM matrix elements $V_{cb} << V_{cs}$ and the phase space of $b \to c+q(l)+\bar{q}'(\bar{\nu}_l)$ is much greater than that of $c \to s+q(\bar{l})+\bar{q}'(\nu)$, it happens that it has comparative weak decay rates of the two heavy flavors c and b, thus the two heavy quarks c, $\bar{b}$ in the meson B_c are in competition regarding subsequent decays, and it has very rich sizable decays than those of the mesons $B^{\pm}$, B^0, B_s, $D^{\pm}$, D^0, D_s etc.

It is interesting to point out that its properties, such as mass,lifetime, branching ratios and spectrum etc. of various decay channels may be used as a 'window' to study the two heavy flavors in the meantime with the meson only. It also is the heaviest and the latest discovered a double heavy meson predicted by the Standard Model (SM) (the t-quark is so heavy that its lifetime so short and there is no time long enough to form any hadron). Furthermore, of its weak decays, those to the final states containing a J/ψ, which is used to be an experimental signature especially in a hadronic environment, are expected to yield a sizable branching fraction, thus it is comparatively easy to be observed. Indeed, the meson B_c has a lot unique interesting properties.

The B_c meson (like B-baryons and doubly-heavy baryons) is too massive to produce at e^+e^- colliders such as the B factories running at the $\Upsilon(4S)$, and LEP-I (at *Z* peak), as stimated by Refs.[1,2]. LEP-I at *Z* peak had recorded only a few B_c candidates[3], so one cannot conclude B_c discovery. While CDF at Tevatron, having approximate 100 pb^{-1} of data in Run-I, isolated a sample of 23 B_c decays with the cascade decays $B_c \to J/\psi+l+\nu_l$ and $J/\psi \to \mu^-\mu^+$, and estimated its mass m_{B_c}= 6.40± 0.39±0.13GeV, the lifetime $\tau_{B_c} = 0.46^{+0.18}_{-0.16} \pm 0.03$ ps, as well as the 'relative' production rate $\sigma(B_c)/\sigma(b) \approx 2\times 10^{-3}$ etc[4].

B_c-meson production[1,2,5,6], spectroscopy[7,8] and various decay channels[9—13] were widely computed and results are consistent with the first observation of CDF. Now the further experimental studies of the meson are planned in Run-II at Tevatron and at LHC etc. Considering to the detectors BTeV and LHCB, in addition to CDF, D0, ATLAS and CMS, are designed specially for B-physics and so numerous $B_c^{\pm}$ events more than $10^8—10^{10}$ per year may be produced at Tevatron and at LHC, more rare decays of it become accessible and many interesting decays will be well-studied in the near future.

We should note here that it was in Refs. [1,2] that to realize first that the fragmentation functions for double heavy mesons can be calculable in perturbative QCD (pQCD) framework, and the 'decay constant' (or the wave function at origin in the potential model language) of the meson will be responsible for the non-perturbative effects in it. The reason, why fragmentation function of a double heavy meson is calculable, is that a gluon, which produces a pair of heavy quarks so must be time-like and hard in full possible phase space, is always needed, and the phase space should be integrated out by definition of the fragmentation function, while the rest part of the fragmentation function can be attributed to the decay constant. It is very different from the situation for light mesons and for (single) heavy mesons, where the gluon produces a pair of light quarks so only in a part of the phase space it is hard i.e. only in this part of the phase space pQCD is applicable, thus the fragmentation functions for light mesons and for (single) heavy mesons cannot be calculable in the framework of pQCD.

As for the decays of the meson B_c, there are three 'components': c-quark decay, $\bar{b}$-quark decay and c, $\bar{b}$-annihilation[10]. We pointed out that i) Since $J/\psi \to l\bar{l}$ is the best signature in a hadronic collider, and the branching ratio of $B_c \to J/\psi+\cdots$ is quite great, so to observe B_c is the best through $B_c \to J/\psi+\cdots$. ii) Of the three component the c-decay is the biggest, so the decays $B_c \to B_s(B_s^*)+l+\nu_l$ and $B_c \to B_s(B_s^*)+h$ (here h indicates a light hadron) are very sizable[11,10]. Therefore, one may study B_s in hadronic colliders through B_c decay, i.e. to select those B_s just from B_c decay only. The advantages to study B_s through B_c are i) Since B_c is positively charged, the B_s produced through B_c decays can be tagged precisely and comparatively easy i.e. we are definitely sure it is B_s, if it comes from B_c, and it is $\bar{B}_s$ if it comes from $\bar{B}_c$. ii) With the fine vertex detector, which is installed in each modern detector now, B_c lifetime is long enough, one can track the vertexes of the B_c production and decay points in the detector in the concerned events, so one can powerfully reject the various backgrounds.

We know that it is very interesting to study CP violation through B_s- $\bar{B}_s$ mixing and B_s (or $\bar{B}_s$) itself in hadronic colliders because it is an important complement to the study of CP violation at B-factories. Now, so many $\cong 10^9$, even 10^{10}, B_c mesons will be produced in a year, at least such as at LHC $\sqrt{s}$ =14TeV, to study CP violation in B_s with the B_c ($\bar{B}_c$) events indeed is practicable.

The decays of the meson B_c to a P-wave chamonium, such as the semileptonic decays $B_c \to \chi_c\ (h_c) + l+\nu_l$ (here χ_c are the spin triplet states $(c\bar{c})[^3P_J]$ and h_c the singlet $(c\bar{c})[^1P_1]$) and two-body nonleptonic decays $B_c \to \chi_c(h_c)+h$, are estimated very recently[12,13].The decays are specially interesting: The cascade decays, the decays of B_c to a *P*-wave charmonium with a sizable radiative decay $\chi_c[{}^3P_{1,2}]\to J/\psi+\gamma$ accordingly, may contribute a substantial background to the observation of the meson B_c through the semileptonic decays $B_c\to J/\psi+l+\nu_l$ and $J/\psi\to\mu^+\mu^-$, that is just the way for CDF to observe the meson. Considering to the fact that due to its difficulties in production and detection, $h_c[{}^1P_1]$, the P-state singlet charmonium, has not been confirmed yet. If the decays $B_c\to h_c+\cdots$ are sizable, then we will have a new 'window' to look at h_c. Finally it is also useful knowledge on the decay $B_c\to\chi_c[{}^3P_0]+h$, if one would like to apply the principle, as pointed in Ref.[14], through the interference of a resonance (here $\chi_c[{}^3P_0]$) and an according 'direct' channel, to observing potential CP violations.

Since the momentum recoil in B_c-decays can be very great, even relativistic, the recoil effects should be treated carefully. To consider the recoil effects properly, the so-called generalized instantaneous approximation proposed firstly in Ref.[9] is adopted in the estimate.

Now let me report the latest estimate on the B_c decays to a *P*-wave charmonium briefly. As for the semileptonic decays $B_c\to X_c+l^++\nu_l$, the T-matrix element:

$$T=\frac{G_F}{\sqrt{2}}V_{ij}\bar{u}_{\nu_l}\gamma_\mu(1-\gamma_5)v_l\langle X_c(p',\varepsilon)|\,J^\mu_{ij}|\,B_c(p)\rangle\ , \tag{1}$$

where X_c denotes χ_c or h_c, V_{ij} is the Cabibbo-Kobayashi-Maskawa(CKM) matrix element and J^μ is the charged current responsible for the decay and p, p' are the momenta of initial state B_c and final state X_c respectively. Thus accordingly the unpolarized one:

$$\overline{\sum}|\,T\,|^2=\frac{G^2{}_F}{2}|\,V_{ij}|^2 l^{\mu\nu}h_{\mu\nu}\ , \tag{2}$$

where $h_{\mu\nu}$ is the hadronic tensor and $l^{\mu\nu}$ the leptonic tensor. The later ($l_{\mu\nu}$) is easy to compute whereas the former $h_{\mu\nu}$ have the general feature as:

$$h_{\mu\nu}=-\alpha g_{\mu\nu}+\beta_{++}(p+p')_\mu(p+p')_\nu+\beta_{+-}(p+p')_\mu(p-p')_\nu+\beta_{-+}\}(p-p')_\mu(p+p')_\nu+ \\ \beta_{--}(p-p')_\mu(p-p')_\nu+i\gamma\varepsilon_{\mu\nu\rho\sigma}(p+p')^\rho(p-p')^\sigma\ . \tag{3}$$

With straightforward calculations, the relevant differential decay rate is obtained:

$$\frac{d^3\Gamma}{dxdy}=|\,V_{ij}|^2\frac{G^2{}_FM^5}{32\pi^3}\left\{\alpha\frac{(y-m_l^2/M^2)}{M^2}+2\beta_{++}\left[2x\left(1-\frac{M'^2}{M^2}+y\right)-4x^2-y+\frac{m_l^2}{4M^2}\times\right.\right. \\ \left.\left(8x+\frac{4M'^2-m_l^2}{M^2}-3y\right)\right]+4(\beta_{+-}+\beta_{+-})\frac{m_l^2}{M^2}\left(2-4x+y-\frac{2M'^2m_l^2}{M^2}\right)+ \\ \left.4\beta_{--}\frac{m_l^2}{M^2}\left(y-\frac{m_l^2}{M^2}\right)-\gamma\left[y\left(1-\frac{M'^2}{M^2}-4x+y\right)+\frac{m_l^2}{M^2}\left(1-\frac{M'^2}{M^2}+y\right)\right]\right\}\ , \tag{4}$$

where $x\equiv E_l/M$ and $y\equiv(p-p')^2/M^2$, M is the mass of B_c meson, M' is the mass of final state X_c. The coefficient functions α, β_{++}, γ can be formulated in terms of form factors. Note that in the above differential decay width, the mass of the lepton m_l is kept precisely, so the formula is also available τ-lepton.

(1) If X_c is $h_c([{}^1P_1])$ state, the vector current matrix element can be written as:

$$\langle X_c(p',\varepsilon)|\,V_\mu|\,B_c(p)\rangle\equiv r\varepsilon^*_\mu+s_+(\varepsilon^*\cdot p)(p+p')_\mu+s_-(\varepsilon^*\cdot p)(p-p')_\mu\ , \tag{5}$$

and the axial vector current matrix element:

$$\langle X_c(p',\varepsilon)|A_\mu|B_c(p)\rangle \equiv \mathrm{i}v\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(p+p')^\rho(p-p')^\sigma, \tag{6}$$

where p and p' are the momenta of B_c and h_c respectively, ε is the polarization vector of h_c.

(2) If X_c is $\chi_c([^3P_0])$ state, the axial current

$$\langle X_c(p')|A_\mu|B_c(p)\rangle \equiv \mu_+(p+p')_\mu+\mu_-(p-p')_\mu, . \tag{7}$$

The vector matrix element vanishes.

(3) If X_c is $\chi_c([^3P_1])$ state:

$$\langle X_c(p',\varepsilon)|V_\mu|B_c(p)\rangle \equiv l\varepsilon^*_\mu+c_+(\varepsilon^*\cdot p)(p+p')_\mu+c_-(\varepsilon^*\cdot p)(p-p')_\mu, \tag{8}$$

and

$$\langle X_c(p',\varepsilon)|A_\mu|B_c(p)\rangle \equiv \mathrm{i}q\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}(p+p')^\rho(p-p')^\sigma . \tag{9}$$

(4) If X_c is $\chi_c([^3P_2])$ state:

$$\langle X_c(p',\varepsilon)|A_\mu|B_c(p)\rangle \equiv k\varepsilon^*_{\mu\nu}p^\nu+b_+(\varepsilon^*_{\rho\sigma}p^\rho p^\sigma)(p+p')_\mu+b_-(\varepsilon^*_{\rho\sigma}p^\rho p^\sigma)(p-p')_\mu, \tag{10}$$

and

$$\langle X_c(p',\varepsilon)|V_\mu|B_c(p)\rangle \equiv \mathrm{i}h\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu\alpha}p_\alpha(p+p')^\rho(p-p')^\sigma . \tag{11}$$

The form factors r, s_+, s_-, v, u_+, u_-, l, c_+, c_-, k, b_+, b_- and h are functions of the momentum transfer $t=(p-p')^2$ and from our earlier experiences in Ref. [9] we know that all of them can be straightforwardly calculated with the approach, the generalized instantaneous approximation.

In order to treat the possible great recoil effects in the decays properly, we need divide the relative 4-momentum q into two parts, $q_\parallel$ and $q_\perp$, a parallel (time-like) part and an orthogonal one to p respectively, as done in Ref. [9]:

$$q^\mu = q^\mu_{p/\!/} + q^\mu_{p\perp},$$

where $q^\mu_{p/\!/} \equiv (p\cdot q/M_P^2)p^\mu$ and $q^\mu_{p\perp} \equiv q^\mu - q^\mu_{p/\!/}$. Correspondingly, we have two Lorentz invariant variables:

$$q_p = \frac{p\cdot q}{M_P},\quad q_{pT} = \sqrt{q_p^2-q^2} = \sqrt{-q^2_{p\perp}} .$$

The covariant 'volume element' of the relative momentum k can be written in an invariant form:

$$\mathrm{d}^4k = \mathrm{d}k_p k^2_{pT}\mathrm{d}k_{pT}\mathrm{d}s\mathrm{d}\phi , \tag{12}$$

where ϕ is the azimuthal angle, $s=(k_pq_p-k\cdot q)/(k_{pT}q_{pT})$.

With the generalized instantaneous approximation, finally one may find that there are two independent functions ξ_1 and ξ_2 appearing in all the form factors in the relevant current matrix elements and defined as follows:

$$\varepsilon^\lambda(L)\cdot\varepsilon_0\xi_1 \equiv \int\frac{\mathrm{d}^3q'_{p'\perp}}{(2\pi)^3}\psi'^*_{n1Mz}(q'_{p'T})\psi_{n00}(q_{pT}),$$

$$\varepsilon^\alpha_\lambda(L)\xi_2 \equiv \int\frac{\mathrm{d}^3q'_{p'\perp}}{(2\pi)^3}\psi'^*_{n1Mz}(q'_{p'T})\psi_{n00}(q_{pT})q'^\alpha_{p'\perp} , \tag{13}$$

where

$$\varepsilon_{0\mu} \equiv \frac{p_\mu - \frac{p.p'}{M'^2}P'_\mu}{\sqrt{\frac{(p.p')^2}{M'^2} - M^2}}$$

describes the polarization along recoil momentum $\boldsymbol{p'}$, and the P-wave (L=1) orbital 'polarization' $\varepsilon_\alpha^\lambda(p)$ in moving (p^2=m^2):

$$\varepsilon_\alpha^{\pm1}(p) = \mp\sqrt{\frac{3}{8\pi}}\left(f_\alpha^1(p) \pm \mathrm{i} f_\alpha^2(p)\right), \qquad \varepsilon_\alpha^0(p) = \mp\sqrt{\frac{3}{4\pi}} f_\alpha^3(p),$$

with

$$f_\mu^i(p) = \frac{-m^2 g_{i\mu} - m(Eg_{i\mu} - p_i g_{0\mu}) + p_i p_\mu}{m(E+m)}, \quad (i=1,2,3;\ \mu=0,1,2,3).$$

In general, the matrix elements for the weak-binding systems may depend on only two 'universal' functions, which are generated directly by the overlapping integrations of the initial and final wave function: one is the integration without the relative momentum $q'_{p'\perp}$ being inserted between the two wave functions i.e. ξ_1, and the one with the relative momentum $q'_{p'\perp}$ being inserted linearly i.e. ξ_2.

The reason is that if the integration is that with higher power of the relative momentum $q'_{p'\perp}$ than linearly being inserted, it will be the relativistic corrections to ξ_1 or ξ_2. For instance, those with $q'^2_{p'\perp}, q'^4_{p'\perp}, \ldots$being inserted will be attributed to the corrections to ξ_1 and those with $q'^3_{p'\perp}, q'^5_{p'\perp}, \ldots$ inserted to the corrections to ξ_2. Therefore there are only such two `universal' functions ξ_1, ξ_2 in the form factors and the decays.

In the case for the decays from an S wave state to another S wave state, due to the fact that the wave functions are normalized, ξ_1 must be much greater than ξ_2, even when the recoil momentum is great. Thus ξ_2 may be ignored safely i.e. under quite satisfied accuracy it is enough to consider ξ_1 only. Furthermore, as found in Ref.[9], the 'universal' function ξ (just ξ_1 here) may be directly related to the Isgur-Wise function appearing in heavy flavor effective theory (HFET) for heavy meson decays since spin-flavor symmetry [19], if letting the mass of c-quark approach to infinity. Whereas in the present case, for the decays from B_c to a P-wave charmonium, it is different: for the decays from an S-wave state to a P-wave state, as the common and familiar cases in the atomic and nuclear transitions, the function ξ_2 is dominant in the region of a small recoil momentum but suppressed 'once' to compare with the normalization of the wave functions, and it will decrease slowly as the momentum recoil is increasing; whereas, the function ξ_1, as expected, is zero or tiny when the recoil momentum is zero or small, but it may become comparable even greater than ξ_2 as the momentum recoil is increasing. We should note that there is no conflict with our knowledge about the atomic and nuclear transitions, where the functions ξ_1 are always ignored due to the fact that the momentum recoil in all possible region is very small i.e. only the function ξ_2 'acts'. Now for the decay of B_c to a *P*-wave charmonium, the momentum recoil may be great even relativistic, we do not have any reason to ignore one of the two 'universal'functions ξ_1 and ξ_2 to obtain correct results, instead, we need to keep two of them precisely. In order to see the feature clearly, we specially show the two function ξ_1 and ξ_2 precisely, i.e. put the curves of their values vs. momentum transfer into a figure (Fig.1). One may see from Fig.1 that in a quite great region we have $\xi_1 \geq \xi_2$. Note specially that since the integration ξ_1 approaches to zero when recoil momentum vanishes, so ξ_1 cannot be obtained by boosting the final state wave function as done in the cases with small recoil for the function ξ_2.

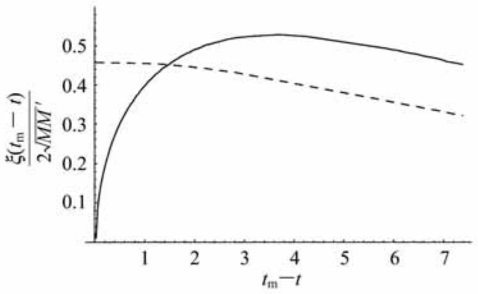

Fig.1. The functions ξ_1 and ξ_2 vs.t_m-t. They were computed through overlapping-integrations of the wave functions for $\chi_c(h_c)$ and B_c (see Eq.(38)).The solid line is of ξ_1, the dashed one is of ξ_2.

For the decays of B_c meson to a *P*-wave charmonium, the precise dependence for each form factor appearing in the weak current matrix element on the functions ξ_1, ξ_2 is too complicated to show here in present short talk, so we do not but one can find it in Ref. [13].

With the form factors the rates of the semileptonic decays and the spectrum of the charged lepton can be calculated straightforwardly numerically.

As for the non-leptonic decay modes $B_c \to \chi_c(h_c)$ +h (caused by the decay b→c), the following effective Lagrangian L_{eff}, in which QCD corrections are involved, was adopted in the estimate:

$$L_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}\left[c_1(\mu)Q_1^{cb} + c_2(\mu)Q_2^{cb}\right] + \mathrm{h.c.}\right\} + \text{Penguin operators}. \quad (14)$$

The Wilson coefficients $c_i(\mu)$ were evaluated at the energy scale μ and Q_1^{cb} and Q_2^{cb} (CKM favoured only) are four-quark operators.

Because the coefficients of 'penguin' operators in the effective Lagraingen are small in comparison with the two main ones c_1 and c_2, we are restricted ourselves to focus only on those decay modes, where up to the preliminary accuracy, the contributions from penguin terms can be neglected, although it is pointed out in the Ref. [18] that the penguin may have interference with the main ones and can course an increase about (3—4)% in the total width. Moreover, we also restricted ourselves only to consider the decay modes where the weak annihilation contribution is small due to precise reasons such as the helicity suppression etc.1) , namely we do not take into account the contributions from the weak annihilation here. In the estimate we adopt the factorization assumption, so the relevant coefficients in the combinations as $a_1=c_1+\kappa c_2$ and/or $a_2=c_2+\kappa c_1$ (here $\kappa=1/N_c$ and N_c numbers of color) appeare in the amplitudes. For non-leptonic decays, a factor $\langle 0|A_\mu|M(p)\rangle = \mathrm{i} f_M p_\mu$ or $\langle 0|V_\mu|V(p,\varepsilon)\rangle = f_V\, M_V \varepsilon_\mu$ can be factorized out from the whole amplitude, and the rest factor being the weak current matrix elements $\langle \chi_c|V_\mu|B_c\rangle$ and/or $\langle \chi_c|A_\mu|B_c\rangle$, which are the same as those in the semileptonic decays. The coefficients in the combination a_1, a_2 are due to the weak currents being Fierz-reordered. In the numerical calculation, we choose $a_1=c_1$ and $a_2=c_2$, i.e., we take $\kappa=0$ in the spirit of the large N_c limit, and QCD correction coefficients c_1 and c_2 are computed at the energy scale of m_b.

With the relations between the currents and form factors obtained in the semileptonic decays, finally the factorized amplitudes for nonleptonic decays are formulated in terms of the form factors and the decay constants by definitions: $\langle 0|A_\mu|M(p)\rangle = \mathrm{i} f_M p_\mu$ and $\langle 0|V_\mu|V(p,\varepsilon)\rangle = f_V M_V \varepsilon_\mu$. Then the widths for the two-body nonleptonic decays are computed straightforwardly.

To give the precise values for the decays, the parameters are chosen as follows: $\lambda=0.24\mathrm{GeV}^2$, $\alpha=0.06\mathrm{GeV}$, $\Lambda_{QCD}=0.18\mathrm{GeV}$, $a=e=2.7183$, $V_0=-0.93\mathrm{GeV}$, $V_{bc}=0.04$[21], $m_1=1.846\mathrm{GeV}$, $m_2=5.243\mathrm{GeV}$. The masses:

$$M_{B_c}=6.33\mathrm{GeV}, \quad M'=3.50\mathrm{GeV},$$

corresponding radial wave-functions of B_c meson and the P-wave charmonium χ_c, h_c are computed with the potential model in B.S. equation framework numerically. Note again we only carry out the lowest order calculations without considering the splitting caused by L-S and S-S couplings etc.

As mentioned above, the numerically computed functions $\xi_1(t_m-t)$ and $\xi_2(t_m-t)$ i.e. two overlap integrals of the wave functions of initial and final states are put in Fig.1. Where $t_m=(M-M')^2$, $t=(P-P')^2$. The function ξ_2 corresponds to the 'normal' one from a S-wave state to a P-state, which can obtained by 'boosting' the final state wave function simply as done sometimes, if the recoil is not too great. Whereas the function ξ_1 is special. It appears in the concerned decays properly and its value is null at the point when the recoil exactly vanishes, hence it cannot be obtained by boosting of the final state wave-function at all.

1) We will consider the contribution from penguin and weak annihilation carefully elsewhere.

Table 1. The semileptonic decay widths (in the unit 10^{-15} GeV).

	$\Gamma(B_c\to{}^1P_1 l\nu_l)$	$\Gamma(B_c\to{}^3P_0 l\nu_l)$	$\Gamma(B_c\to{}^3P_1 l\nu_l)$	$\Gamma(B_c\to{}^3P_2 l\nu_l)$
e(μ)	2.509	1.686	2.206	2.732
τ	0.356	0.249	0.346	0.422

The obtained lepton energy spectra for the decays $B_c\to\chi_c+e(\mu)+\nu$ are shown in Fig.2, where the mass of charged lepton is ignored, and that for the decays $B_c\to\chi_c+\tau+\nu$, where the mass of charged lepton τ cannot be ignored, is shown in Fig.3, where $|\boldsymbol{p}_l|$ is the momentum of lepton. The difference between Fig.2 and Fig.3 is due to the sizable mass of τ-lepton. For the semileptonic decays, the corresponding widths are put in Table 1.

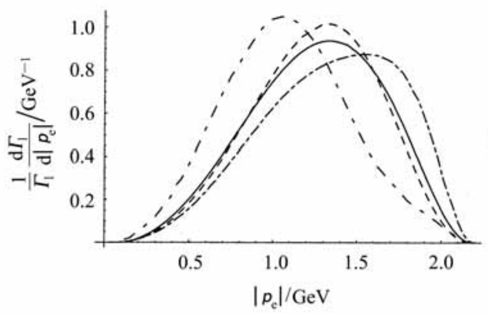

Fig.2. Lepton energy spectrum for $B_c\to\chi_c+e(\mu)+\nu$, where the solid line is the result of $^1P_1$ state, dotted-blank-dashed line is of $^3P_0$, dashed line is line is of $^3P_0$, dotted-dashed line is $^3P_2$.

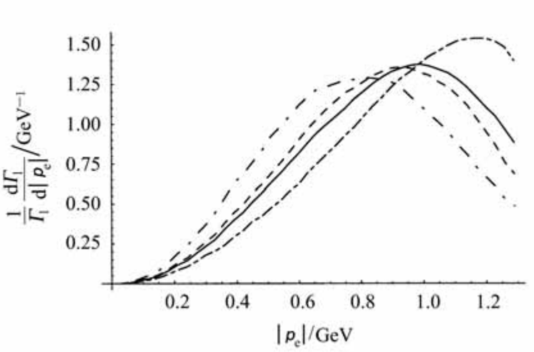

Fig.3. Lepton energy spectrum for $B_c\to\chi_c+\tau+\nu_\tau$, where the solid line is the result of $^1P_1$ state, dotted-blank-dashed line is of $^3P_0$, dashed line is of $^3P_1$, dotted-dashed line is $^3P_2$.

As for the non-leptonic two-body decays $B_c\to\chi_c(h_c)+h$, we only evaluate some typical channels, whose widths are relatively larger, and let me put the results in Table 2.

In the numerical calculations for non-leptonic decays, we choose $a_1=c_1$ and $a_2=c_2$, i.e., $\kappa=0$, and c_1 and c_2 are evaluated at the energy scale of m_b.

The values of the decay constants: $f_{\pi^+}=0.131$GeV, $f_{\rho^+}=0.208$GeV, $f_{a1}=0.229$GeV, $f_{K^+}=0.159$GeV, $f_{K^{*+}}=0.214$GeV, $f_{D_s}=0.213$GeV, $f_{D_s^*}=0.242$GeV, $f_{D^+}=0.209$GeV, $f_{D^{*+}}=0.237$GeV are adopted by fitting decays of B and D mesons.

If comparing the results in Table 1 with the decays of B_c to *S*-wave charmonium J/ψ and η_c e.g. ($\Gamma(B_c\to J/\psi+l+\nu)\sim25\times10^{-15}$ GeV [9,11], one can realize the semileptonic decays of B_c to the *P*-wave charmonium states in magnitude are about tenth of the decay $B_c\to J/\psi+l+\nu_l$. As for the two-body nonleptonic decays, due to the fact that the recoil momentum is fixed properly in a given decay, the $B_c\to\chi_c(h_c)+h$ can be so greater as twentieth of the *S*-wave decay $B_c\to J/\psi(\eta_c)+h$.

Table 2. The two-body non-leptonic decay widths (in unit 10^{-15} GeV).

Channel	Γ	$\Gamma(\alpha_1=1.132)$	Channel	Γ	$\Gamma(\alpha_1=1.132)$
$^1P_1\pi^+$	α_1^2 0.569	0.729	$^1P_1\rho$	α_1^2 1.40	1.79
$^3P_0\pi^+$	α_1^2 0.317	0.407	$^3P_0\rho$	α_1^2 0.806	1.03
$^3P_1\pi^+$	α_1^2 0.0815	0.104	$^3P_1\rho$	α_1^2 0.331	0.425
$^3P_2\pi^+$	α_1^2 0.277	0.355	$^3P_2\rho$	α_1^2 0.579	0.742
$^1P_1A_1$	α_1^2 1.71	2.19	$^1P_1K^+$	α_1^2 4.26×10^{-3}	5.46×10^{-3}
$^3P_0A_1$	α_1^2 1.03	1.33	$^3P_0K^+$	α_1^2 2.35×10^{-3}	3.02×10^{-3}
$^3P_1A_1$	α_1^2 0.671	0.859	$^3P_1K^+$	α_1^2 0.583×10^{-3}	0.747×10^{-3}
$^3P_2A_1$	α_1^2 1.05	1.34	$^3P_2K^+$	α_1^2 1.99×10^{-3}	2.56×10^{-3}
$^1P_1K^*$	α_1^2 7.63×10^{-3}	9.78×10^{-3}	$^1P_1D_s$	α_1^2 2.32	2.98
$^3P_0K^*$	α_1^2 4.43×10^{-3}	5.68×10^{-3}	$^3P_0D_s$	α_1^2 1.18	1.51
$^3P_1K^*$	α_1^2 2.05×10^{-3}	2.63×10^{-3}	$^3P_0D_s$	α_1^2 0.149	0.191
$^3P_2K^*$	α_1^2 3.48×10^{-3}	4.77×10^{-3}	$^3P_2D_s$	α_1^2 0.507	0.650
$^1P_1D_s^*$	α_1^2 1.99	2.56	$^1P_1D^+$	α_1^2 0.0868	0.111
$^3P_0D_s^*$	α_1^2 1.48	1.89	$^3P_0D^+$	α_1^2 0.0443	0.0568
$^3P_1D_s^*$	α_1^2 2.21	2.83	$^3P_1D^+$	α_1^2 0.00610	0.00782
$^3P_2D_s^*$	α_1^2 2.68	3.44	$^3P_2D^+$	α_1^2 0.0209	0.0267
$^1P_1D^{*+}$	α_1^2 0.0788	0.101			
$^3P_0D^{*+}$	α_1^2 0.0567	0.0726			
$^3P_1D^{*+}$	α_1^2 0.0767	0.0983			
$^3P_2D^{*+}$	α_1^2 0.0972	0.124			

The first observation of B_c by CDF group is through the semileptonic decay $B_c\to J/\psi+l+\nu_l$ and $J/\psi\to\mu^+\mu^-$, hence, with the observation as a reference and the results above, the conclusions may be obtained [12,13]: The decays of B_c to a charmonium X_c are accessible in Run-II at Tevatron and at LHC, especially when concerning the particular detector for *B*-physics BTeV and LHCB at the two colliders. The cascade decays i.e. the decays $B_c\to\chi_c[^3P_{1,2}]+l+\nu_l$ with an according one of the radiative decays $\chi_c[^3P_{1,2}]\to J/\psi+\gamma$ followed, may substantially affect the observation by CDF group, especially, when the efficiency of detecting a photons is not very ideal. With sizable branching ratio, the decays $B_c\to h_c+l+\nu_l$ and/or $B_c\to h_c+h$ may open a fresh 'window' to observe the charmonium state $h_c[^1P_1]$potentially, especially the charmonium state $h_c[^1P_1]$ has not been well-established experimentally yet.

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