

## A Possible Halo-Like Proton 2p States of $^{13}\text{N}^*$

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**Abstract** Assuming a single proton 2p state structure of  $^{13}\text{N}$ , we study the halo-like structure of  $^{13}\text{N}$  by calculating proton- $^{13}\text{N}$  elastic scattering at the incident energy of 1 GeV in the framework of Glauber multiple scattering theory. The theoretical prediction of the differential cross section is obtained. Unlike  $^{13}\text{C}$ , there are no experimental data available at present to examine this prediction. However, the result could be a guide to the coming experimental measurement, particularly, for a possible experiment at the Institute of Modern Physics, the Chinese Academy of Sciences, Lanzhou, China. Recalling our previous theoretical prediction for  $^{13}\text{C}$  halo-like structure and its experimental verifications, our present results evidently show that a possible halo-like proton skin of  $^{13}\text{N}$  in ground state most likely exists. Therefore, an immediate experimental measurement of p- $^{13}\text{N}$  elastic scattering at the energy of 1 GeV is urgently demanded.

**Key words** nuclear structure, halo-like phenomena, high energy scattering, Glauber theory

The recent discovery of neutron halo phenomena<sup>[1-3]</sup> is one of the most interesting events in nuclear physics. Many experimental and theoretical efforts have been devoted to the investigation of the neutron-rich nuclei  $^6\text{He}$ ,  $^8\text{He}$ ,  $^{11}\text{Li}$ ,  $^{11,12,14}\text{Be}$ ,  $^{14}\text{B}$ , and  $^{14}\text{C}$ <sup>[4-7]</sup> and the proton-rich nuclei  $^8\text{B}$ <sup>[8]</sup>,  $^{11}\text{C}$ ,  $^{12}\text{N}$ <sup>[9]</sup>,  $^{17}\text{F}$  and  $^{17}\text{Ne}$ <sup>[10-13]</sup> to establish the existence of the proton halo or skin in these nuclei.

Theoretically, we consider in this short note if the proton-halo-like single particle 2p state with the occupation number less than 1 exists in the ground state of the  $^{13}\text{N}$  nucleus. The reason is that  $^{13}\text{N}$  is a mirror nucleus of  $^{13}\text{C}$  and the analysis of the charge and magnetic form factors of elastic electron scattering on  $^{13}\text{C}$ , magnetic moments of  $^{13}\text{C}$  and  $^{13}\text{N}$ , and also  $\log ft$  for  $\beta$ -decay of  $^{13}\text{N}$  into  $^{13}\text{C}$  clearly indicate a necessity to take into account the 2p shell in the ground state of nuclei  $^{13}\text{C}$  and  $^{13}\text{N}$ <sup>[14,15]</sup>.

In order to get a deeper insight into the structure of neutron-rich nuclei, the halo phenomenon has been the subject of

numerous studies during the last decades. Various experimental methods were applied, such as the beta-decay measurements following in-beam polarization by optical pumping, and the investigation of momentum distribution of the reaction products after fragmentation of the halo nuclei impinging on a nuclear target. It should be pointed out that obtaining the precise information on the radial structure and the size of halo nuclei from experiments on total interaction cross sections and momentum distributions may inherently be limited by incomplete knowledge of the rather complicated reaction mechanisms under study. Systematic errors in the determination of the matter radii seem to appear from uncertainties in describing re-scattering of the target nucleons on the nuclear constituents of the projectile, uncertainties in taking the final state interaction into account, etc. Therefore, in order to avoid these uncertainties and to attain more detailed information upon the radial shape of the halo nuclei and their nuclear matter radii, another method, namely the method of small-

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angle proton elastic scattering at intermediate energy, has recently been applied<sup>[16]</sup> which is particularly good for studying the spatial structure of halo nuclei since the contribution to the cross section from the scattering on the halo nucleons is concentrated just at small scattering angle, i. e., at small momentum transfer.

Our previous research of Ref. [17] confirmed that the neutron 2p state in  $^{13}\text{C}$  is the single-particle halo-like state with a large root-mean-square radius  $\langle r_{2p,1/2}^2 \rangle^{1/2} = 11.02\text{fm}$ . Remind  $\langle r_{1p,1/2}^2 \rangle^{1/2} = 3.40\text{fm}$ . At the same time, many papers<sup>[18,19]</sup> analyzed the magnetic form factor of elastic electron scattering on  $^{13}\text{C}$ , and the experimental elastic magnetic form factor was found to be reproducible when the 2p shell is taken into account. As mentioned,  $^{13}\text{N}$  is a mirror nucleus of  $^{13}\text{C}$ . For confirmation of the existence of the halo-like proton 2p state in the structure of  $^{13}\text{N}$ , we investigate elastic scattering of protons with the intermediate energy of 1GeV on nucleus  $^{13}\text{N}$  in this paper using the Glauber theory of diffractive multiple scattering<sup>[20]</sup> and the same parameters as that used in  $^{13}\text{C}$  calculations. Therefore, this paper is a parameter free calculation.

According to Glauber Multiple scattering theory, the amplitude of proton elastic scattering off nucleus can be written in eikonal form as the following

$$F_{\hat{f}}(q) = \frac{ik}{2\pi} H_{\text{cm}}(q) \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} \times \langle \psi_f | 1 - \prod_{j=1}^A [1 - \Gamma_j] | \psi_i \rangle, \quad (1)$$

where  $\psi_{i,f}$  are the nuclear wave functions of the initial (final) state,  $H_{\text{cm}}(q)$  denotes the center-of-mass correction factor and the  $\Gamma_j(\mathbf{b} - \mathbf{s}_j)$  stands for the single particle profile function

$$\Gamma_j(\mathbf{b} - \mathbf{s}_j) = \frac{1}{2\pi i k} \int d^2\mathbf{q} e^{-i\mathbf{q}(\mathbf{b}-\mathbf{s}_j)} f_j(q), \quad (2)$$

with  $f_j(q)$  being the hadron-hadron two body scattering amplitude. The  $\mathbf{k}$  in Eq.(1) is the wave vector of the projectile proton,  $\mathbf{b}$  the impact parameter,  $\mathbf{q}$  the momentum transfer. Since the effect of the correlation of the odd proton with the core  $^{12}\text{C}$  is negligibly small, the amplitude in Eq.(1) can be represented as

$$F_{\hat{f}}(q) = \frac{ik}{2\pi} H_{\text{cm}}(q) \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}} \{ \delta_{\hat{f}} - [ \delta_{\hat{f}} - \langle \psi_p | \psi_p \rangle - \langle \psi_p | \Gamma_{\text{pp}}(\mathbf{b} - \mathbf{s}_p) | \psi_p \rangle ] \} \times F_{\text{DIS}}(^{12}\text{C}), \quad (3)$$

with

$$F_{\text{DIS}}(^{12}\text{C}) = \langle \psi_{\text{core}}(^{12}\text{C}) | \prod_{i=1}^{A-1} [1 - \Gamma_i(\mathbf{b} - \mathbf{s}_i)] | \times \psi_{\text{core}}(^{12}\text{C}) \rangle. \quad (4)$$

The wave function of  $^{13}\text{N}$  is taken to be a simple product of single odd proton wave function  $\psi_p$  in the  $p$  sub-shell and the nuclear wave function of the  $^{12}\text{C}$  core of the target  $^{13}\text{N}$ ,  $\psi_{\text{core}}(^{12}\text{C})$ . The normalization is

$$\langle \psi_{^{13}\text{N}} | \psi_{^{13}\text{N}} \rangle = 1.0. \quad (5)$$

In Eq. (3),  $\langle \psi_p | \Gamma_{\text{pp}}(\mathbf{b} - \mathbf{s}_p) | \psi_p \rangle$  is the matrix element of the incident proton scattering off the odd proton in the target  $^{13}\text{N}$ , while the factor  $F_{\text{DIS}}(^{12}\text{C})$  (see Eq.(4)) denotes the proton elastic scattering off the  $^{12}\text{C}$  core since the nucleus  $^{13}\text{N}$  has been regarded as a  $^{12}\text{C}$  core plus the odd proton.

Suppose the odd proton in  $^{13}\text{N}$  can stay either in 1p state or in 2p state so that its single particle wave function can be written as

$$| \psi_p \rangle = (\eta_{1p})^{1/2} | \psi_{1p} \rangle + (\eta_{2p})^{1/2} | \psi_{2p} \rangle. \quad (6)$$

with  $\eta_{1p(2p)}$  being mixed probability amplitude. Then, the matrix element of  $\langle \psi_p | \Gamma_{\text{pp}} | \psi_p \rangle$  can be written as

$$\begin{aligned} \langle \psi_p | \Gamma_{\text{pp}} | \psi_p \rangle &= \eta_{1p} \langle \psi_{1p} | \Gamma_{\text{pp}} | \psi_{1p} \rangle + \\ &\eta_{2p} \langle \psi_{2p} | \Gamma_{\text{pp}} | \psi_{2p} \rangle + \\ &(\eta_{1p} \eta_{2p})^{1/2} \langle \psi_{1p} | \Gamma_{\text{pp}} | \psi_{2p} \rangle + \\ &(\eta_{1p} \eta_{2p})^{1/2} \langle \psi_{2p} | \Gamma_{\text{pp}} | \psi_{1p} \rangle. \end{aligned} \quad (7)$$

Given the wave functions of single particle,  $\psi_{1p}$  and  $\psi_{2p}$ , one can evaluate the matrix element  $\langle \psi_p | \Gamma_{\text{pp}} | \psi_p \rangle$  using Eq.(2) and Eq.(7). The numerical calculations of the matrix elements of  $\langle \psi_{\text{core}}(^{12}\text{C}) | \prod_{i=1}^{A-1} [1 - \Gamma_i(\mathbf{b} - \mathbf{s}_i)] \times | \psi_{\text{core}}(^{12}\text{C}) \rangle$  can also be easily performed as we have done in Ref.[21], i. e.

$$F_{\text{DIS}}(^{12}\text{C}) = \langle \psi_{\text{core}}(^{12}\text{C}) | \prod_{i=1}^{A-1} [1 - \Gamma_i(\mathbf{b} - \mathbf{s}_i)] \times | \psi_{\text{core}}(^{12}\text{C}) \rangle = 1 - \det | \langle \psi_{nljm} | 1 - \Gamma_i | \psi_{nljm} \rangle |. \quad (8)$$

In Eq. (8),  $\psi_{nljm}$  is single particle wave function with  $nljm$  being an appropriate set of single particle quantum numbers which can completely describe a single nucleon state. Eq. (8) denotes the contribution from  $^{12}\text{C}$  core of  $^{13}\text{N}$  target nucleus, as a distortion factor for the incident proton, to the  $p$ - $^{13}\text{N}$  elastic scattering. We emphasize that our approach in calculating the distortion factor, Eq.(8), includes all scattering terms which exist, one body through  $(A-1)$  body scattering terms (always with the eikonal restriction that no nucleon is struck more than once), since  $\psi_{\text{core}}(^{12}\text{C})$  is described

by a Slater determinant which considers the Pauli correlation of nucleons in  $^{12}\text{C}$ , and we use the determinant method to calculate the distortion factor  $F_{\text{DIS}}(^{12}\text{C})$ . In one word, not any higher order terms of the scattering amplitudes are omitted in our present calculations. All the terms are included.

The amplitude of the proton-nucleon scattering  $f_i(q)$  in Eq. (2) can be parameterized in the form

$$f_j(q) = \frac{ik\sigma_{pp}}{4\pi} [1 - i\rho_{pN}] e^{-q^2\beta_{pp}^2/2}. \quad (9)$$

where  $\sigma_{pp}$  is the total cross section of the pp interaction,  $\rho_{pp}$  is the ratio of the real part of the pp amplitude to its imaginary part, and  $\beta_{pp}$  is the slope parameter of the pp two body scattering amplitude. These parameters are taken here to be

$$\sigma_{pp} = \sigma_{pn} = 4.40\text{fm}^2, \quad (10)$$

$$\rho_{pp} = \rho_{pn} = -0.25, \quad (11)$$

$$\beta_{pp}^2 = \beta_{pn}^2 = 0.30\text{fm}^2. \quad (12)$$

Substituting Eq. (9) into Eq. (2), we arrive at

$$\Gamma_j(\mathbf{b} - \mathbf{s}_j) = \frac{1}{2\pi i k} \int d^2\mathbf{q} e^{-i\mathbf{q}(\mathbf{b}-\mathbf{s}_j)} f_j(q) = \frac{\sigma_{pp}(1 - i\rho_{pp})}{4\pi\beta_{pp}^2} e^{-(\mathbf{b}-\mathbf{s}_j)^2/2\beta_{pp}^2}. \quad (13)$$

Using Eqs. (3), (6), (7), (8), (10)–(13) we can easily evaluate the amplitude of the proton- $^{13}\text{N}$  elastic scattering,  $F_f(q)$ , and then we can get the differential cross section by definition of  $d\sigma/d\Omega = |F_f(q)|^2$ . In our numerical calculations, the wave functions of single particle are given by the harmonic oscillator potential<sup>[17,18]</sup> with the size parameter  $b_{1p} = 2.389\text{fm}$ ,  $b_{2p} = 2.753\text{fm}$ , and  $\eta_{1p} = 1.034$ ,  $\eta_{2p} = 0.133$ . It should be stressed that here  $\langle\psi_p|\psi_p\rangle \neq 1$ ,  $\langle\psi_{\text{core}}(^{12}\text{C})|\psi_{\text{core}}(^{12}\text{C})\rangle \neq 1$ , but Eq. (5) is held. Our theoretical prediction of differential cross section,  $d\sigma/d\Omega$  (mb/sr), for p- $^{13}\text{N}$  elastic scattering at the energy of 1GeV is shown in Fig. 1. Obviously, the differential cross section has a quite similar diffractive feature like p- $^{13}\text{C}$  elastic scattering at the energy of 1GeV. Therefore, some conclusions may be obtained by comparison with the study of p- $^{13}\text{C}$  elastic scattering.

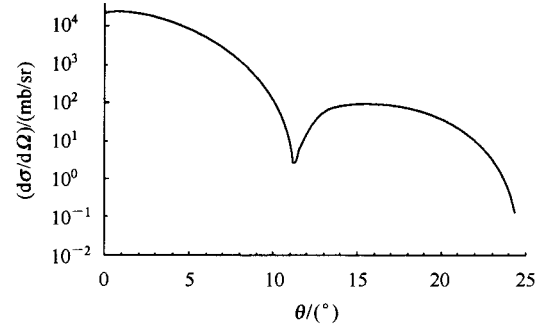


Fig. 1. Plot of our theoretical prediction for the differential cross section of the proton with energy of 1GeV elastically scattered off the  $^{13}\text{N}$  target with  $\eta_{1p} = 1.034$ ,  $\eta_{2p} = 0.133$ ,  $b_{1p} = 2.389\text{fm}$ ,  $b_{2p} = 2.753\text{fm}$ ,  $\sigma_{pp} = \sigma_{pn} = 4.40\text{fm}^2$ ,  $\rho_{pp} = \rho_{pn} = -0.25$ ,  $\beta_{pp}^2 = \beta_{pn}^2 = 0.30\text{fm}^2$ .

In summary, we study proton halo-like structure of  $^{13}\text{N}$  by calculating the p- $^{13}\text{N}$  elastic scattering at the incident energy of 1GeV in the framework of Glauber multiple scattering theory, and by assuming a single proton 2p state structure of  $^{13}\text{N}$  ground state. The theoretical prediction of the differential cross section for the proton- $^{13}\text{N}$  elastic scattering is obtained. Unfortunately, unlike  $^{13}\text{C}$ , there are no experimental data available at present to confirm this prediction. However, our result could be a guide to the coming experimental measurement, particularly, to a possible future experiment at the Institute of Modern Physics, Lanzhou, China. Recalling our theoretical predictions of  $^{13}\text{C}$  halo-like structure and its experimental verifications, our present results evidently show that a possible halo-like proton skin of  $^{13}\text{N}$  most likely exists. If the coming experimental data measured in the future would fall down on our predicted curve, we could have a halo-like  $^{13}\text{N}$  nucleus. If not, we should look for another explanation for the new observation. Therefore, an experimental measurement of p- $^{13}\text{N}$  elastic scattering at the energy of 1GeV is urgently demanded.

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 $^{13}\text{N}$  的一个可能的类晕的质子 2p 态 \*朱基珍<sup>2;1</sup> 周丽娟<sup>2</sup> 马维兴<sup>1,2,3;2</sup>

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**摘要** 假设 $^{13}\text{N}$ 是单粒子的2p态的结构,用Glauber多重散射理论计算了1GeV的质子在 $^{13}\text{N}$ 上弹性散射的微分截面,研究了 $^{13}\text{N}$ 的类晕结构.与 $^{13}\text{C}$ 的情况不同,虽然目前还没有关于p- $^{13}\text{N}$ 弹性散射微分截面的实验材料与我们的理论结果相比较,但是这个理论结果可以作为对未来的实验测量工作的一个指导,特别是对兰州近代物理研究所的晕核实验研究工作有参考和指导价值.参照先前我们对 $^{13}\text{C}$ 晕核结构的理论预言和进一步的实验证实,本文的计算结果清楚地表明: $^{13}\text{N}$ 可能存在着一个类晕的质子皮结构.因此,实验上测量p- $^{13}\text{N}$ 弹性散射的微分截面就是当前需要马上进行的一个实验研究工作.

**关键词** 原子核结构 原子核的类晕现象 高能散射 Glauber理论

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