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Breakdown of Entanglement during the Teleportation*

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Abstract The teleportation may become an important means for remote distance communications in the future, and the mechanism is based on entanglement of quantum states. But the entanglement is fragile. As the state is disturbed by the environment the entanglement may be broken down. In this work, we choose the electron-positron pair in an entangled state of spin 0 as an example to investigate the rate of breaking down of the entanglement by the Compton scattering with the background radiation photons or Bremstrahlung with strong magnetic fields of some astronomical objects which the electron or positron passes by. Since the spin projection of single electron (positron) is not physically measurable and the electron beams cannot keep its shape for long because of the Coulomb repulsion among the charged particles in the beam, the only way is to shoot one electron-positron pair each time and continuously repeat the processes. With all the restraints this study has only pedagogic meaning, but may shed light on further studies where other information messagers are chosen.

Key words teleportation, entanglement, electron-positron pair, Compton scattering, Bremstrhlung scattering

The quantum teleportation might become an important means for remote communication in the future[1—3]. Therefore it is necessary to investigate its reliability. The mechanism is based on entanglement of quantum states. This means may apply to other fields. Possibilities were discussed for optical and atomic physics, for example[4—8] and also investigated for quantum computer[9—11]. The microscopic particles, such as electron-positron, photons or even atoms can be the messangers which carry information. they are produced from a parent particle, for instance, η, or other mesons of spin-0 (due to the helicity suppression, this production rate is low), and the pair of electron and positron reside in a state with total angular momentum $J=0$. That is an entangled state. Later the two particles (e^+ and e^-) fly away back to back in the center-of-mass frame of the parent particle. The quantum mechanics principle requests that no matter how far they are separated, even for a space-like separation, if there is no disturbance, these two particles remain in the entangled state. The state is described as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2), \qquad (1)$$

Where the subscripts 1 and 2 refer to particle 1 and 2 respectively. If we use the helicity to replace the spin projections with respect to a fixed direction, one has:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_1|+\rangle_2 - |-\rangle_1|-\rangle_2), \qquad (2)$$

where $|+\rangle$ and $|-\rangle$ correspond to the eigenstates with helicity being + or −.

If the states of the two separated particles are not disturbed, the entanglement will be retained forever, as the quantum principle demands. However, electrons and positrons are charged, thus they must interact with photons. In fact, the entanglement is fragile as generally expected. On the long route of the flight, due to the interaction with photon, the spin projections of the electron (positron) may flip (as well as the helicities) and then the entanglement is broken down.

Now let us investigate the processes which can break

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down the entanglement, i. e. flip the helicities of either electron or positron. First, our universe is a huge black body whose background radiation corresponds to 2.7K and can be described by the distribution function:

$$\rho = N\mathrm{e}^{-\frac{\hbar w}{kT}}. \tag{3}$$

As a reasonable approximation, here we only need the Maxwell distribution function which is simpler. In the natural unit system, $\hbar$ and the Boltzman constant k are set to be unity. By a Compton scattering with the background photon, the helicity of the electron (positron) has a certain probability to be flipped. One can generally expect that this effect should be very small by our common . knowledge.

The second source is more serious. If the electron (or positron) passes a neighborhood of a huge astronomical object whose magnetic field is very strong, for example, neutron star, supernova or even a cluster of stars, etc., the helicity may be flipped by their strong magnetic field. It is a Bremsstrahlung scattering case where the flipping probability is proportional to the magnetic field B of the astronomical object.

Moreover, as well known, the spin projection of an individual electron is not experimentally measurable, moreover one can neither use a beam of electrons because the electrons would diverge by the Coulomb interaction among each other. Therefore, we shoot the electron-positron pairs one by one with a necessary time interval by which the individual electron can be identified, for each pair the electron and positron are entangled. We will evaluate the helicity flipping probability in the strong magnetic field by the quantum field theory, namely determine the fraction of electrons (positrons) which do not retain the original entanglement with their partners flying in opposite direction.

In general, we believe that a beam cannot be taken as the messangers of information, but need some other agents, mainly the neutral photons or neutrinos. Better than electron-positron pairs, photons do not carry charges, they interact with other photons via loops, so the cross sections are much suppressed and as well as for neutrinos. The corresponding calculations are more complex and as the first step, we just investigate the electron-positron pair case. Concretely, we calculate the probability of the helicity flipping and estimate the distance the particle can fly when the flipping probability is larger than a certain value. The processes are either via the Compton scattering with the background radiation photons or Bremsstrahlung scattering with the strong magnetic field of a huge astronomical object. Indeed, this investigation for the electron-positron pairs has only pedagogic meaning, but can help us to further study the reliability of the teleportation when other kinds of messangers are employed.

(1) Via the Compton scattering

The electron (or positron) scatters with a background photon with energy w_i and its helicity may flip. We are going to evaluate the probability of the flip[12].

The differential cross section for the spin flip can be written as[13]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{2}\sum_{\varepsilon_i,\varepsilon_f} |\mathscr{M}|^2\left(\frac{\omega_f}{\omega_i}\right)^2 \tag{4}$$

and

$$\begin{aligned}\mathscr{M} = {} & \bar{u}(p_f, s_f(-))\left[\not{\varepsilon}_f \frac{1}{\not{p}_i + \not{k}_i - m_\mathrm{e}}\not{\varepsilon}_i + \right. \\ & \left. \not{\varepsilon}_i \frac{1}{\not{p}_i - \not{k}_f - m_\mathrm{e}}\not{\varepsilon}_f\right] u(p_i, s_i(+)) = \\ & \sum_{s_i', s_f'(-)} \bar{u}(p_f, s_f')\frac{1-\gamma_5\not{s}_f}{2}\left[\not{\varepsilon}_f \frac{1}{\not{p}_i + \not{k}_i - m_\mathrm{e}}\not{\varepsilon}_i + \right. \\ & \left. \not{\varepsilon}_i \frac{1}{\not{p}_i - \not{k}_f - m_\mathrm{e}}\not{\varepsilon}_f\right] \cdot \frac{1+\gamma_5\not{s}_i}{2} u(p_i, s_i'(+)),\end{aligned}$$

where p_i, k_i, p_f, k_f and $s_i, s_f, \varepsilon_i, \varepsilon_f$ are the 4-momenta and polarizations of the incoming electron, photon and outgoing electron, photon, and the symbols (−) and (+) stand for the helicities being − and + respectively. The helicity projectors select proper helicity state of the incoming and outgoing electrons. Without losing generality, in this equation, we choose $h_i = s_i = +$ and $h_f = s_f = -$. In fact, because the energy of the background photons is very small, the direction of flight of the electron is almost unchanged after collisions, and so is its energy, i. e. $p \cong p'$.

The probability of flipping the helicity of the electron is

$$\begin{aligned}P = {} & \int 2E \cdot 2wv\sigma(E, w)\mathrm{e}^{-\frac{w}{kT}}\mathrm{d}^3 kts = \\ & \int 2E \cdot 2w\sigma(E, w)\mathrm{e}^{-\frac{w}{kT}}\mathrm{d}^3 kLs,\end{aligned} \tag{5}$$

where t is the time of flight in the space, L is the travel distance of the electron in the space. $2E2\omega \cdot v$ corresponds to the relative flux of the electron and photon when we adopt the normalizations

$$\{b, b^+\} = (2\pi)^3 2E\delta^3(\boldsymbol{P} - \boldsymbol{P}'), \text{for fermion;} \tag{6}$$

$$[a, a^+] = (2\pi)^3 2w\delta^3(\boldsymbol{K} - \boldsymbol{K}'), \text{for boson.} \tag{7}$$

The distribution $\mathrm{e}^{(-\frac{w}{kT})}\mathrm{d}^3 k$ stands for the density of the background photons and s is an effective cross section for real collisions between the electron and photon. Obviously, by the quantum mechanics (QM), s can be approximated as λ^2

where λ is to De Broglie wavelength of the electron. Thus the flipping probability is proportional to the time of flight, the cross section, and the energy of electron. Here a factor of 2 ~ 3 corresponding to the photon degeneracy and other properties may be missing, but it does not affect our estimate of magnitude.

If we take the corresponding parameters as $m_e = 0.51$ MeV, $T = 2.7\mathrm{K}$[14] and kinetic energy of the electron to be 250MeV, when we select the flipping probability to be 50%, i.e. $P = 0.5$, our numerical results show that the travel distance is $L = 5.0 \times 10^8$ly.

(2) Via the Bremsstrahlung

Instead of colliding with a single photon, the electron interacts with the magnetic field. According to the standard field theory, it is a Bremsstrahlung process. Here, as an approximation, we simplify the picture that the magnetic field near an astronomical object is constant and uniform as

$$\boldsymbol{B} = B\hat{z}. \tag{8}$$

Thus we can write

$$A^{\mathrm{ext}} = \left(0, \frac{1}{2}\boldsymbol{B} \times \boldsymbol{r}\right) = \left(0, 0, -\frac{1}{2}Br\sin\theta, 0\right). \tag{9}$$

A Fourier transformation can bring A^{ext} in the configuration space into the momentum space, and it is written as

$$A^{\mathrm{ext}} = \left(0, 0, -4\pi B\sin\theta \int \frac{\mathrm{d}^4 q}{(2\pi)^3}\delta(q^0)\frac{1}{\boldsymbol{q}^4}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}, 0\right), \tag{10}$$

where θ is a constant angle between $\boldsymbol{B}$ and the direction of $\hat{z}$ which can be arbitrarily chosen.

Then the differetial scattering cross section is

$$\mathrm{d}\sigma = \frac{4\alpha^2 m_e^2 B^2}{\pi |\boldsymbol{p}_i| |\boldsymbol{p}_f + \boldsymbol{k} - \boldsymbol{p}_i|^8} \times \sum_{\varepsilon} |\mathscr{M}^2 w_f |\boldsymbol{p}_f| \mathrm{d}w_f \mathrm{d}\Omega_\gamma \mathrm{d}\Omega_e, \tag{11}$$

where

$$\begin{aligned}\mathscr{M} = {} & \bar{u}(p_f, s_f)\left[\not{\varepsilon}\frac{1}{\not{p}_f + \not{k} - m_e}\gamma^2 + \gamma^2\frac{1}{\not{p}_i - \not{k} - m_e}\not{\varepsilon}\right]u(p_i, s_i) = \\ & \sum_{s_i', s_f'} \bar{u}(p_f, s_f')\frac{1 - \gamma_5 \not{s}_f}{2}\left[\not{\varepsilon}\frac{1}{\not{p}_f + \not{k} - m_e}\gamma^2 + \gamma^2\frac{1}{\not{p}_i - \not{k} - m_e}\not{\varepsilon}\right]\frac{1 + \gamma_5 \not{s}_i}{2}u(p_i, s_i').\end{aligned} \tag{12}$$

The flipping probability now is

$$P = \frac{2Ev\sigma t}{s} = \frac{2E\sigma L}{s}. \tag{13}$$

It looks different from that for the Compton scattering case. Because the cross section of the Compton scattering is for a collision of an electron with a photon, but in the case of Bremsstrahlung, it is a scattering of an electron with a field. The fluxes are different in the two cases. By a dimensional analysis we can see the validity of the formula. One more reason is that for the Bremsstrahlung the energy is conserved, but not the 3-momentum.

Thus in the integration, we have only $2\pi\delta(\sum E)$ which corresponds to the time interval T, thus the volume part $2\pi\delta(\sum \boldsymbol{p})$ is missing. To be explicit, one can write a better expression as

$$P = 2Ev\sigma\frac{1}{V}ts\frac{1}{l}. \tag{14}$$

The extra $\frac{1}{l}$ comes from the line density of the virtual photon flight ($e^- + \gamma^* \rightarrow e^- + \gamma$) on the route of the electron (the factor 2ω is for the real photon in the Compton scattering case). One can approximate $V \propto \lambda^3$, $l \propto \lambda$ where λ is to De Broglie wavelength of the electron.

If we still take $P = 0.5$, and the magnetic field is as strong as $B = 3.5\mathrm{T}$, we can get the relationship between E_i and $L = v \cdot t$ as shown in Table.1.

Table 1. The relationship between E_i and L.

E_i/MeV	1	5	10	100	200	250
L/km	3.8	0.2	1.2	0.013	0.078	0.1

If one changes the magnetic filed strength, for example, the magnetic field of the Earth is much weaker than 1T, i.e. only with order of gauss, the penetration length would be much longer. For a comparison, we present the relationship between the magnetic field strength B and the penetration length L in Table 2.

Table 2. The relationship between B and L when $P = 0.5$ and $E_i = 250$MeV are taken.

B/T	3.5	35	350	3500
L/m	103	1.03	0.01	1.0×10^{-4}

(3) Speculation by the Quantum mechanics

In fact, Just by the QM, one can evaluate how deep an electron can penetrate into a magnetic field of B without flipping its helicity (the time when the first flip occurs). The expression can be found in any textbook of QM, for example, that by Zeng[15].

$$P = \sin^2\theta \sin^2 wt \tag{15}$$

where θ is the angle between magnetic field and electron flying direction and $\omega = \frac{eB}{2m_e}$. This expression is the probability of one flip in the magnetic field for time duration t. We can estimate the travel distance in the field when the first flip occurs and find that the typical time is 10^{-11}s and the distance for an average velocity is about no more than $10^{-3}m$, at 1T.

This result looks quite different from that we obtained by evaluating the cross section of the Bremstrahlung scattering, but the reason is obvious. Eq. (15) is derived for extremely non-relativistic cases and the kinetic energy of the electron is very small compared to its mass (0.5MeV). By a naive understanding, the smaller the kinetic energy of the incoming electron is, the larger the scattering cross section would be, and the increase may overtake the decrease of the relative velocity. The difference can be fully understood.

Indeed, in this work, we discuss possible mechanisms which break down the entanglement of an electron-positron pair as the electron and positron fly apart back to back in the outer space. First we consider the background radiation in the universe, namely helicity is flipped by a Compton scattering of the electron(positron) with a background photon. We calculate the probability of flipping the helicity of the electron and find that for an electron with kinetic energy as large as 250MeV, the electron may travel 10^8 light-years before it has 50% probability to flip its helicity. If one uses photons as the information messager, they interact with the background photons via loops and the process is much more suppressed compared with the electron case, so that can actually travel in the space without any constraint from the background radiation.

Then we discuss the situation that if the electron passes a strong magnetic field such as in the neighborhood of a large astronomical objects, where the magnetic field can be as strong as a few Teslas. We find that for an electron with kinetic energy being 250MeV, it can penetrate into the magnetic field by a few meters at about 1T before it flips its helicity.

Since the helicity is flipped, the entanglement is broken down and it cannot bring any information anymore or the information may be lost on the way[16]. We know that the entanglement is fragile, and in this work we observe that a strong magnetic field may easily break down the entangled state.

Because in this work we only need to evaluate the order of magnitude, we do not consider some details. For example, we take the De Broglie wavelength in the dimensional analysis and ignore some degeneracy factors of photon in the statistical function, etc. One can naturally expect that a numerical factor of 1 ~ 10 may be missing, but of course, it does not change our qualitative conclusion at all.

As we discussed above, this work has only pedagogic meaning because from the present point of view, electron-positron cannot be taken as the messangers of teleportation. However, on the other hand, the teleportation will find more applications in various fields, especially for the quantum computer, therefore the result of this work suggests that in the future, when the teleportation which is based on the quantum entanglement, is taken seriously, no matter what particles are chosen to be the messangeers, the problem of breaking down the entanglement by interactions with environment must be confronted[17]. Then one may have to consider more realistic situations.

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远程传输中纠缠态的破坏*

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摘要　将来的远程通信中,量子远程传输可能成为重要的手段之一.整个理论是基于量子纠缠态机制的.但是纠缠态是很脆弱的.当量子态被环境扰动时,纠缠态会被打破.在此文中,我们选择处在总自旋为0的纠缠态中的正负电子为例,研究纠缠态由于与背景辐射光子的康普顿散射或与电子经过的大天体的强磁场散射而破坏的概率.由于单个电子的自旋投影不是物理可观测量,而且,电子束由于带电粒子间的库仑斥力,不可能较长期保持成团的形状.因而只能每次只发射一对正负电子对,并不断重复这个过程.由于这些限制,此文研究只有理论上的意义,但是也许对将来如何选择信息传递信使的研究有帮助.

关键词　远程传输　纠缠　正-负电子对　康普顿散射　韧致辐射

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