

Spontaneous Leptogenesis in Brans-Dicke Cosmology*

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Abstract The role of the auxiliary scalar field ϕ of BD theory in baryon number asymmetry is discussed in this paper. We consider a derivative coupling of this gravitational scalar field to the baryon current J^{μ}_{B} or the current of the baryon number minus lepton number $J^{\mu}_{\text{B-L}}$ based on a series of works of R. Morganstern about the Brans-Dicke cosmology. We find that the spontaneous baryogenesis by this coupling is able to yield sufficient baryon asymmetry $n_{\text{B}}/s \sim 10^{-10}$ provided the time of the grand unification is a little bit earlier. In addition, Davoudiasl et al have recently introduced a new type of interaction between the Ricci scalar R and the baryon current J^{μ} , $\partial_{\mu} R J^{\mu}$ and also proposed a mechanism for baryogenesis, the gravitational baryogenesis. However, the Einstein equation tells us that $\dot{R} = 0$ in the radiation-dominated epoch of the standard FRW cosmology. In this paper we reconsider the feasibility of having gravitational baryogenesis with such a form of interaction in radiation-filled BD cosmology. We show that \dot{R} does not vanish in this case and the required baryon number asymmetry can also be achieved.

Key words leptogenesis, scalar coupling, Brans-Dicke cosmology

1 Introduction

The origin of the baryon number asymmetry remains a big puzzle in cosmology and particle physics. Conventionally, it is argued that this asymmetry is generated from an initial baryon symmetric phase dynamically as long as the following conditions are satisfied^[1]: (1) baryon number non-conserving interactions; (2) C and CP violations; (3) out of thermal equilibrium. When the CPT is violated dynamically, however the baryon number asymmetry can be generated in thermal equilibrium^[2]. In connecting to dark energy, a class of models of spontaneous leptogenesis^[3–5] are investigated recently by introducing a interaction between the dynamical dark energy scalars and the ordinary matter. Specifically, a

derivative coupling of the quintessence scalar field Q to the baryon or lepton current is under consideration:

$$\mathcal{L}_{\text{int}} \sim \partial_{\mu} Q J^{\mu}. \quad (1)$$

One silent feature of this scenario for baryogenesis is that the present accelerating expansion and the generation of the matter and antimatter asymmetry of our universe is described in a unified way.

Recently, a new mechanism called gravitational baryogenesis in thermal equilibrium has been proposed by Davoudiasl et al^[6]. They introduced explicitly an interaction between the Ricci scalar curvature with derivative and the baryon number current:

$$\mathcal{L} = \frac{1}{M^2} \partial_{\mu} R J^{\mu}. \quad (2)$$

And the baryon number asymmetry is given by

Received 16 March 2005, Revised 31 May 2005

*Supported by National Natural Science Foundation of China (10273017)

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$$\frac{n_B}{s} \sim \frac{\dot{R}}{M^2 T}, \quad (3)$$

which shows that n_B/s is determined by the value of \dot{R} , however the Einstein equation, $R = 8\pi G T^\mu{}_\mu = 8\pi G(1 - 3\omega)\rho$, tells us that $\dot{R} = 0$ in the radiation-dominated epoch of the standard Friedmann-Robertson-Walker (FRW) cosmology. Davoudiasl et al in Ref. [6] have considered three different possibilities of obtaining a non-vanishing \dot{R} which include the effects of trace anomaly, reheating and introducing a non-thermal component with $\omega > 1/3$ dominant in the early universe. In the brane-world scenario Shiromizu and Koyama in Ref. [7] provided another example for $\dot{R} \neq 0$. A generalized form of the derivative coupling of the Ricci scalar to the ordinary matter

$$\mathcal{L}_{\text{int}} \sim \partial_\mu f(R) J^\mu. \quad (4)$$

is also discussed in Ref. [8]. Taking $f(R) \sim \ln R$, they have shown that $\partial_\mu f(R) \sim \partial_\mu R/R$ does not vanish and the required baryon asymmetry can also be generated in the early universe.

As is well known, Brans-Dicke (BD) theory of gravity is one of the simplest extensions/modifications of Einstein's general relativity^[9–11]. Compared with general relativity, as well as the metric tensor of space-time which describes the geometry there is an auxiliary scalar field ϕ which also describes the gravity. The role of the gravitational scalar in BD theory has been much exploited to achieve inflation, quintessence^[12–20] and k -essence^[21]. The main constraints on this gravitational scalar come from the solar system experiment^[22]. In addition, the testing of BD theory using stellar distances, the CMB temperature and polarization anisotropy have also been discussed in Ref. [23, 24]. In this regard, here we focus particularly on the scalar of the BD theory to model baryo(lepto)genesis behavior by combining both the spontaneous symmetry breaking and its gravitational origin in BD cosmology.

2 Spontaneous leptogenesis

We consider a derivative coupling of the auxiliary

gravitational scalar ϕ to the ordinary matter. So we have now an effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{c}{M^2} \partial_\mu \phi J^\mu, \quad (5)$$

where M is the cutoff scale which, for example, could be Planck mass M_{pl} or the scale of grand unification theory M_{GUT} , and c is the coupling constant which characterizes the strength of gravitational scalar interacting with ordinary matter in the standard model of the electroweak theory. Taking $J^\mu = J_B^\mu$, during the evolution of the spatial flat FRW universe, \mathcal{L}_{eff} in the Eq. (5) gives rise to an effective chemical potential μ_b for baryons:

$$\frac{c}{M^2} \partial_\mu \phi J^\mu \rightarrow \frac{c}{M^2} \dot{\phi} n_B = \frac{c}{M^2} \dot{\phi} (n_b - n_{\bar{b}}), \quad (6)$$

$$\mu_b = c \frac{\dot{\phi}}{M^2} = -\mu_{\bar{b}}. \quad (7)$$

In thermal equilibrium the baryon number asymmetry is given by (when $T \gg m_b$)

$$n_B = \frac{g_b T^3}{6} \left[\frac{\mu_b}{T} + \mathcal{O}\left(\frac{\mu_b}{T}\right)^3 \right] \approx c \frac{g_b \dot{\phi} T^2}{6M^2}, \quad (8)$$

where g_b counts the internal degrees of freedom of the baryon. Using the familiar expression for entropy density

$$s = \frac{2\pi^2}{45} g_* T^3, \quad (9)$$

we arrive at the final expression for the baryon to entropy ratio:

$$\frac{n_B}{s} \approx \frac{15c}{4\pi^2} \cdot \frac{g_b \dot{\phi}}{g_* M^2 T}, \quad (10)$$

here $\dot{\phi}$ in Eq. (10) can be obtained by solving the equation of motion of auxiliary scalar ϕ given below

$$(2\omega - 3) \left(\frac{\ddot{\phi}}{\phi} + 3H \frac{\dot{\phi}}{\phi} \right) + \frac{8\pi}{\phi} T_m = \frac{c}{M^2} [\dot{n}_B + (\dot{\phi} + 3H)n_B], \quad (11)$$

where H is the Hubble constant and T_m is the energy-momentum scalar of the ordinary matter in the universe.

In applying the BD theory to cosmology, we may write down the flat Robertson-Walker line element as

$$ds^2 = -dt^2 + a^2(t) \tilde{g}_{ij} dx^i dx^j. \quad (12)$$

Where i, j run from 1 to 3, $a(t)$ is the scale of the non-compact 3-dimensional flat space. The action of

BD theory reads

$$I = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(\phi \cdot R + \omega g^{\mu\nu} \frac{\nabla_\mu \phi \nabla_\nu \phi}{\phi} \right) + \int d^4x \sqrt{-g^4} \cdot L_m, \quad (13)$$

here ω is the parameter of BD theory; Then the corresponding field equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi}{\phi} T_{\mu\nu} - \frac{1}{\phi} (g_{\mu\nu} \phi^{;\alpha}{}_{;\alpha} - \phi_{;\mu;\nu}) - \frac{\omega}{\phi^2} \phi_{;\mu} \phi_{;\nu} + \frac{1}{2} \frac{\omega}{\phi^2} g_{\mu\nu} \nabla_\sigma \phi \nabla^\sigma \phi. \quad (14)$$

The field equation for ϕ reads

$$\square^2 \phi = \phi^{;\mu}{}_{;\mu} = \frac{8\pi}{-3+2\omega} T_m. \quad (15)$$

Therefore, the fundamental equations of BD cosmology may be taken as

$$H^2 = \frac{8\pi}{3} \left[\frac{\rho}{\phi} - \frac{\omega}{16\pi} \frac{\dot{\phi}^2}{\phi^2} - \left(\frac{3H}{8\pi} \frac{\dot{\phi}}{\phi} \right) \right]; \quad (16)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} \frac{(-3+\omega)\rho + 3\omega p}{(-3+2\omega)\phi} + \frac{\omega}{3} \frac{\dot{\phi}^2}{\phi^2} + H \frac{\dot{\phi}}{\phi}; \quad (17)$$

$$-\ddot{\phi} - 3 \frac{\dot{a}}{a} \dot{\phi} = \frac{8\pi}{-3+2\omega} (1-3w)\rho. \quad (18)$$

Here w is the parameter of the state equation of the ordinary matter ($w = p/\rho$). The exact solutions to BD cosmologies in flat Friedmann universes had been drawn by R. Morganstern from the early 70's^[25–27]. He found that as $a \rightarrow 0$ the expansion parameter a has the same $t^{1/3}$ behavior for all curvature k in radiation-filled BD cosmology^[26]. Therefore, we now turn to investigate the feasibility of having spontaneous baryogenesis in the early universe with BD gravity in the case of $w = 1/3$, so we have

$$\rho \propto a^{-4} \quad \text{and} \quad \dot{\phi} \propto a^{-3}. \quad (19)$$

In the research of BD cosmology, a power law form for both the scale-factor and scalar field has ever been applied to investigate the dynamics of a self-interacting BD field to account for the acceleration of the universe at late epochs $t \rightarrow t_0$ in Ref. [28]. More importantly, the exact general solutions to BD cosmology for $p = \rho/3$ found in Ref. [26] has also demonstrated a limiting power-law behavior for early epochs $t \rightarrow 0$. For these reasons, we also consider a power-law form for the scale factor as an approximation to find its asymptotic behavior in the early

universe,

$$a = a_{\text{in}} \left(\frac{t}{t_{\text{in}}} \right)^n. \quad (20)$$

Thus we have $H = n/t$, then Eq. (18) of $w = 1/3$ immediately yields the power law form for the scalar field $\dot{\phi} = \dot{\phi}_{\text{in}}(t/t_{\text{in}})^{-3n}$ and $\phi = \dot{\phi}_{\text{in}}(t/t_{\text{in}})^{1-3n} \cdot t_{\text{in}} \frac{1}{1-3n}$. Giving this result, now we can write down the parameterized Friedmann equation of (16) in a radiation-dominant epoch,

$$n^2 t^{-2} = \frac{8\pi(1-3n)\rho}{3\dot{\phi}_{\text{in}} t_{\text{in}}} \cdot \left(\frac{t_{\text{in}}}{t} \right)^{1-3n} - (n-3n^2)t^{-2} - \frac{\omega(1-3n)^2 t^{-2}}{6}. \quad (21)$$

On the other hand, the radiation is in equilibrium so that $\rho = \frac{\pi^2}{30} g_* T^4$ can be available in the final calculation. Therefore, when the interaction of the spontaneous baryogenesis is decoupled, a simple relation between the age and temperature of the universe is obtained,

$$t^{-1-3n} = \frac{8\pi(1-3n)}{3\dot{\phi}_{\text{in}}[n-2n^2+\omega(1-3n)^2/6]} \cdot (t_{\text{in}})^{-3n} \frac{\pi^2}{30} g_* T^4. \quad (22)$$

Substituting Eq. (22) into the power law form for the scalar field, we obtain

$$\dot{\phi} = \dot{\phi}_{\text{in}}(t/t_{\text{in}})^{-3n} = \frac{8\pi(1-3n)}{3[n-2n^2+\omega(1-3n)^2/6]} \cdot t \frac{\pi^2}{30} g_* T^4. \quad (23)$$

As far as the radiation-filled BD cosmology is concerned, R. Morganstern has found the limiting behavior of the exact solution for the expansion parameter is of $t^{1/3}$. Considering the spontaneous baryogenesis takes place at the early times of the universe, when Eq. (21) possesses an asymptotic solution as $n \rightarrow 1/3$, so it requires that

$$-\omega(1-3n)^2/6 = n-2n^2. \quad (24)$$

in the limiting behavior. The equal sign of the above equation would come into existence only when $n = 1/3$ in our model. However, to say the least, Eq. (24) is still a fine approximation when we consider the exact solution to radiation-filled BD cosmology which possesses an asymptotic behavior of $n \rightarrow 1/3$ at the early epoches. Therefore, the ratio of baryon

number to entropy in this case is given by

$$\frac{n_B}{s} \approx \frac{15c}{4\pi^2} \cdot \frac{8\pi^3 g_b}{90M^2} \cdot \frac{(1-3n)}{n-2n^2+\omega(1-3n)^2/6} \cdot tT^3, \quad (25)$$

where t and T refer to the age and temperature of the universe, respectively.

In the scenario of spontaneous baryogenesis, the baryon asymmetry is generated in thermal equilibrium. This requires that baryon number violating interactions occur rapidly ($\Gamma_B > H$). However, if the B-violating interactions keep in equilibrium until $\dot{\phi} \rightarrow 0$, the final baryon asymmetry will be zero. Denoting the epoch when the B-violating interactions freeze out by T_D (corresponding to t_D), i.e., $\Gamma_B(t_D) = H(T_D)$, the final baryon number asymmetry is obtained

$$\frac{n_B}{s} \approx 1c \cdot \frac{(1-3n)}{n-2n^2+\omega(1-3n)^2/6} \cdot \frac{t_D T_D^3}{M^2}. \quad (26)$$

In the above numerical calculations, we have used $g_b \sim \mathcal{O}(1)$. Furthermore, we may a priori assume that $\frac{(1-3n)}{n-2n^2+\omega(1-3n)^2/6} \sim \mathcal{O}(10^{-3})$ for considering $|\omega| > 500$. However, as we shall see in the last part of this section, it will not change our result much if we assume this expression takes the value in the order $\sim 10^{-3}$. After the interaction of the spontaneous baryogenesis decouples the universe will continue to evolve into a radiation-dominated epoch until the primordial nucleosynthesis takes place. But we must note that thereafter the evolutionary behavior of the scale factor will deviate from the $n \rightarrow 1/3$ law along with the increasing of the time t . As a very good approximation, we can recall the relation of equilibrium thermodynamics that $\frac{t_D}{t} = \left(\frac{a_D}{a}\right)^{4/4n} = \left(\frac{\rho_D}{\rho}\right)^{-\frac{1}{4n}} = \left(\frac{\pi^2 g_*/30T_D^4}{\pi^2 g_*/30T^4}\right)^{-\frac{1}{4n}}$ to obtain the evolution of the temperature. To make our discussion to be as independent as possible of the cosmological evolution models, here we choose the values of T_{GUT} at the grand unification phase transitions epoch to be the input parameters in our model. Firstly, we choose $T_{\text{GUT}} \sim 10^{15} \text{GeV}$ considered from particle physics. Then the induced baryon number asymme-

try in radiation-filled BD cosmology is

$$\frac{n_B}{s} \approx 10^{-3} c \cdot \frac{t_{\text{GUT}} T_{\text{GUT}}^{\frac{1}{n}} T_D^{3-\frac{1}{n}}}{M^2} \Big|_{n \rightarrow 1/3} \approx 1c \times 10^{42} \text{GeV}^3 \cdot \frac{t_{\text{GUT}}}{M_{\text{pl}}^2}, \quad (27)$$

here we have taken the cut-off factor $M \approx M_{\text{pl}}$. Then yielding a sufficient $\frac{n_B}{s} \approx 10^{-10}$ would require that $t_{\text{GUT}} \sim 10^{-14} \text{GeV}^{-1} \sim 10^{-38} \text{s}$. This value is not very far away from the Planck time ($t_{\text{pl}} \sim 10^{-43} \text{s}$) but still can be achieved, it just needs the grand unification phase transition to take place a little bit earlier comparing with that of Einstein's cosmology. If we consider that the index of the power law n will show a preference of a little departure from $1/3$ along with the decrease of the temperature until T_D , then the requirement on the time when the grand unification becomes to take place would be further released to be $> 10^{-38} \text{s}$. Along this way, the spontaneous baryogenesis using the BD scalar is also possible to yield sufficient n_B/s at a radiation-dominated epoch.

On the other hand, if the B-violating interactions conserve B-L, the asymmetry generated will be erased by the electroweak Sphaleron^[29]. Hence, now we turn to leptogenesis^[30, 31]. We take J^μ in Eq. (5) to be $J^\mu_{\text{B-L}}$. Doing the calculations with the same procedure as above for $J^\mu = J^\mu_{\text{B}}$ we have the final asymmetry of the baryon number minus lepton number:

$$\frac{n_{\text{B-L}}}{s} \approx 1c \cdot \frac{(1-3n)}{n-2n^2+\omega(1-3n)^2/6} \cdot \frac{t_D T_D^3}{M^2}. \quad (28)$$

The asymmetry $n_{\text{B-L}}$ in (28) will be converted to baryon number asymmetry when electroweak Sphaleron B+L interaction is in thermal equilibrium which happens for temperature in the range of $10^2 \sim 10^{12} \text{GeV}$. T_D in Eq. (28) is the temperature below which the B-L interactions freeze out.

In the Standard Model of the electroweak theory, B-L symmetry is exactly conserved, however many models beyond the standard model, such as Left-Right symmetric model predict the violation of the B-L symmetry. In this paper we take an effective Lagrangian approach and parameterize the B-L violation by higher dimensional operators. There are many operators which violate B-L symmetry, how-

ever at dimension 5 there is only one operator,

$$\mathcal{L}_L = \frac{2}{f} l_L l_1 \chi \chi + H.c., \quad (29)$$

where f is a scale of new physics beyond the Standard Model which generates the B-L violations, l_L and χ are the left-handed lepton and Higgs doublets respectively. When the Higgs field gets a vacuum expectation value $\langle \chi \rangle \sim v$, the left-handed neutrino receives a majorana mass $m_\nu \sim \frac{v^2}{f}$.

In the early universe the lepton number violating rate induced by the interaction in Eq. (29) is^[32]

$$\Gamma_L \sim 0.04 \frac{T^3}{f^2}. \quad (30)$$

Since Γ_L is proportional to T^3 , for a given f , namely the neutrino mass, B-L violation will be more efficient at high temperature than at low temperature. Requiring this rate be larger than the universe expansion rate $\sim n/t \sim \frac{8n\pi(1-3n)}{3[n-2n^2+\omega(1-3n)^2/6]} \cdot \frac{1}{\phi} \frac{\pi^2}{30} g_* T^4$ until the temperature T_D , we have

$$f \sim \left(\frac{0.04}{n} t_D T_D^3 \right)^{\frac{1}{2}} \Big|_{n \rightarrow 1/3} \sim (0.12 t_D T_D^3)^{\frac{1}{2}}. \quad (31)$$

Therefore, we obtain a T_D -dependent lower limit on the neutrino mass:

$$m_\nu \sim \frac{v^2}{(0.12 t_D T_D^3)^{\frac{1}{2}}} \sim \frac{6 \times 10^4 \text{GeV}^2}{(0.12 t_D T_D^3)^{\frac{1}{2}}} \sim 1 \text{eV}, \quad (32)$$

where we have used the energy scale of electroweak SSB phase transition of about 246GeV. Taking three neutrino masses to be approximately degenerated, i.e., $m_1 \sim m_2 \sim m_3 \sim \tilde{m}$ and defining $\Sigma = 3\tilde{m}$, one can see that for $t_D T_D^3 \sim 3 \times 10^{28} \text{GeV}^2$, three neutrinos are expected to have masses \tilde{m} around $\mathcal{O}(1 \text{eV})$. The current cosmological limit comes from WMAP^[33] and SDSS^[34]. The analysis of Ref. [33] gives $\Sigma < 0.69 \text{eV}$. The analysis from SDSS shows, however that $\Sigma < 1.7 \text{eV}$ ^[34]. These limits on the neutrino masses require $t_D T_D^3$ be larger than $5.7 \times 10^{29} \text{GeV}^2$ and $9.2 \times 10^{28} \text{GeV}^2$. The almost degenerate neutrino masses required by the leptogenesis of this model will induce a rate of the neutrinoless double beta decays accessible for the experimental sensitivity in the near future^[35]. Interestingly, a recent study^[36] on the cosmological data showed a preference for neutrinos with degenerate masses in this range. Therefore, we may

recall the final Eq. (26) for spontaneous baryogenesis by using the BD scalar. If the cut-off factor M takes the value from the grand unification scale 10^{16}GeV to the Planck scale 10^{19}GeV , then yielding a sufficient baryon asymmetry may require that $\frac{1}{3} \times 10^{-6} < \frac{(1-3n)}{n-2n^2+\omega(1-3n)^2/6} < 1/3$. It is quite easy to be achieved for a large number of BD parameter ω .

The experimental CPT test with a spin-polarized torsion pendulum^[37] puts strong limits on the axial vector background b_μ defined by $\mathcal{L} = b_\mu \bar{e} \gamma^\mu \gamma_5 e$ ^[38]:

$$|\mathbf{b}| \leq 10^{-28} \text{GeV}. \quad (33)$$

For the time component b_0 , the bound is relaxed to be at the level of 10^{-25}GeV ^[39]. In our model, assuming the auxiliary scalar couples to the electron axial current the same as Eq. (5), we can estimate the CPT-violation effect on the laboratory experiments. As we know, the inverse of the Newtonian gravitational constant G takes the average value of the scalar ϕ , in the spirit of BD theory. The variation of G in the present time is bounded as $\frac{\dot{G}}{G} < 1.2 \times 10^{-43} \text{GeV}$ by Ref. [40, 41]. Hence the induced CPT-violating b_0 is

$$b_0 \sim \frac{c}{M^2} \dot{\phi}_0 \sim \frac{c}{M_{\text{gut}}^2} \cdot \frac{-\dot{G}}{G^2} \leq -1.2 \times 10^{-37} \text{GeV}, \quad (34)$$

which is much below the current experimental limits.

3 Gravitational leptogenesis

In this section we will recall the gravitational baryogenesis^[6] in thermal equilibrium, which may fail to work in the framework of Einstein's cosmology just introduced in the beginning of this paper. So we investigate it again in BD cosmology. The effective Lagrangian of a CP-violating interaction between the derivative of the Ricci scalar curvature R and the baryon number(B) current J^μ still have the form of Eq. (2). Taking $J^\mu = J_B^\mu$, during the evolution of the spatial flat FRW universe, \mathcal{L}_{eff} in Eq. (2) gives rise to an effective chemical potential μ_b for baryons:

$$\begin{aligned} \frac{c}{M^2} \partial_\mu R J^\mu &\rightarrow \frac{c}{M^2} \dot{R} n_B = \frac{c}{M^2} \dot{R} (n_b - n_{\bar{b}}), \quad (35) \\ \mu_b &= c \frac{\dot{R}}{M^2} = -\mu_{\bar{b}}. \quad (36) \end{aligned}$$

We could directly give out the final expression for the baryon to entropy ratio in thermal equilibrium:

$$\frac{n_B}{s} \approx \frac{15c}{4\pi^2} \cdot \frac{g_b \dot{R}}{g_* M^2 T}, \quad (37)$$

\dot{R} in Eq. (37) can be obtained by solving the equation of motion of the Ricci scalar given below

$$\begin{aligned} \dot{R} = & 8\pi \left(\frac{\dot{T}_m}{\phi} - \frac{T_m \dot{\phi}}{\phi^2} \right) + 2\omega \left(\frac{\dot{\phi}\ddot{\phi}}{\phi^2} - \frac{\dot{\phi}^3}{\phi^3} \right) - \\ & 3 \left(\frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}\ddot{\phi}}{\phi^2} \right) - 9H \left(\frac{\ddot{\phi}}{\phi} - \frac{\dot{\phi}^2}{\phi^2} \right) - 9\dot{H} \frac{\dot{\phi}}{\phi}, \quad (38) \end{aligned}$$

$$\dot{R} = 6 \frac{\ddot{a}}{a} + 6 \frac{\dot{a}\ddot{a}}{a^2} - 12 \frac{\dot{a}^3}{a^3}. \quad (39)$$

Obviously, there are two methods which may be used to find the behavior of \dot{R} of the asymptotic solution ($n \rightarrow 1/3$). The first simply involves substituting the power law approximation of Eq. (20) in the function of the curvature scalar (39) of the Robertson-Walker metric,

$$\dot{R} = 12n[(n-1)^2 - n^2] \cdot t^{-3}. \quad (40)$$

The second method can be used even when the explicit solutions are not known, and moreover it provides a convenient check on the asymptotic behavior of the exact solution (namely the assumed power law form (20) when $n \rightarrow 1/3$). By substituting the power law of the scale factor in Eq. (38) (which is resulted directly from the field equation in BD cosmology (14)), we again have

$$\dot{R} = -2(1-3n)^2 \omega \cdot t^{-3}. \quad (41)$$

On the other hand, here we should recall the relation (24) as a fine approximation at the earliest times. After that, we can compute the limiting behavior of \dot{R} using both methods, and of course we find that the results agree. Taking $n \rightarrow 1/3$, we immediately have $\dot{R} = 12t^{-3}/9$. Thus we obtain the ratio of baryon number to entropy, which is given by

$$\frac{n_B}{s} \approx \frac{15c}{4\pi^2} \cdot \frac{12g_b}{9g_* M^2 T} \cdot t^{-3}, \quad (42)$$

where t and T also refer to the age and temperature of the Universe. A rapidly violating of baryon number is still required in this case. In addition, if the B-violating interactions keep in equilibrium until $\dot{R} \rightarrow 0$, the final baryon asymmetry will be zero. Denoting

the epoch when the B-violating interactions freeze out by T_D (corresponding to t_D), i.e., $\Gamma_B(t_D) = H(T_D)$, the final baryon number asymmetry in this case may be given by

$$\frac{n_B}{s} \approx \frac{15c}{4\pi^2} \cdot \frac{12g_b}{9g_* M^2 T_D} \cdot t_D^{-3} \approx \frac{c}{2} \times 10^{-2} \cdot \frac{t_D^{-3}}{M^2 T_D}. \quad (43)$$

In the above numerical calculations, we have also used $g_b \sim \mathcal{O}(1)$ and $g_* \sim \mathcal{O}(100)$. Recalling the equilibrium thermodynamics $t_D \sim t_{\text{GUT}} \left(\frac{T_{\text{GUT}}}{T_D} \right)^{\frac{1}{n}}$ at a radiation-dominated epoch $w = 1/3$, then the final result will be given under the consideration of the limiting behavior of the exact solution in Ref. [26],

$$\begin{aligned} \frac{n_B}{s} \approx & \frac{c}{2} \times 10^{-2} \cdot t_{\text{GUT}}^{-3} \frac{(10^{15} \text{GeV})^{-\frac{3}{n}} T_D^{\frac{3}{n}}}{M^2 T_D} \Big|_{n \rightarrow 1/3} \sim \\ & \frac{c}{2} \times 10^{-137} \text{GeV}^{-9} \cdot t_{\text{GUT}}^{-3} \cdot \frac{T_D^8}{M^2}. \quad (44) \end{aligned}$$

Similarly, if the B-violating interactions conserve B-L, the asymmetry generated by gravitational CP-violation will also be erased by the electroweak Sphaleron^[29]. So we again have to turn to leptogenesis^[30, 31]. We take J^μ in Eq.(35) to be $J^\mu_{\text{B-L}}$. Doing the calculations with the same procedure as above for $J^\mu = J^\mu_{\text{B}}$ we have the final asymmetry of the baryon number minus lepton number in a similar way to spontaneous leptogenesis

$$\frac{n_{\text{B-L}}}{s} \approx \frac{c}{2} \times 10^{-137} \text{GeV}^{-9} \cdot t_{\text{GUT}}^{-3} \cdot \frac{T_D^8}{M^2}. \quad (45)$$

Taking $c \sim \mathcal{O}(1)$, $n_{\text{B-L}}/s \sim 10^{-10}$ and $M \sim 10^{16} \text{GeV}$, for the desired result is obtained when $T_D < 10^{15} \text{GeV}$, then the time t_{GUT} when the grand unification is about taking place may be required to be

$$t_{\text{GUT}} < 10^{-37} \text{s}. \quad (46)$$

That is to say, the gravitational baryogenesis in BD cosmology also requires the grand unification phase transition to take place a little bit earlier comparing with that of Einstein's cosmology.

In the same way, we investigate the CPT-violation effect on the laboratory experiments for the coupling between the curvature scalar and the electron axial current. Firstly, viewed from Taylor expansion, when a variable quantity evolves to a great number, the derivative of a function of this quantity would tend to be more and more smooth and flatter along with

the increasing of the rank of the derivative. Thus we have $\frac{d}{dt}\ddot{a} \ll \frac{d}{dt}\dot{a} \ll \frac{d}{dt}a$ at the present time (here the infinitesimal unit $\Delta t \sim 1\text{s}$ is chosen to obtain a large number for the age of the present universe relative to considered time interval). Secondly, the Hubble's parameter at the present time has been given as $H_0 \sim 10^{-17}\text{s}^{-1} \sim 2 \times 10^{-42}\text{GeV}$. Therefore the induced CPT-violating b_0 in this case may be estimated at

$$b_0 \sim \frac{c}{M^2}\dot{R}_0 \sim \frac{c}{M^2} \left[-6\frac{\ddot{a}\dot{a}}{a^2} + 6\frac{\ddot{a}}{a} - 12H^3 + 12\frac{\ddot{a}}{a} \cdot H \right] \Bigg|_{t \rightarrow t_0} \leq \frac{c}{M_{\text{gut}}^2} t [2.4 \times 10^{-83}\text{GeV}^2 \cdot \text{s}^{-1} + 1.2 \times 10^{-41}\text{GeV} \cdot \text{s}^{-2} - 9.6 \times 10^{-125}\text{GeV}^3] \leq 4.8 \times 10^{-122}\text{GeV}, \quad (47)$$

which also is much below the current experimental limits.

4 Conclusion

In summary, we propose in this paper a scenario of

spontaneous baryogenesis in BD cosmology by introducing a derivative coupling of the auxiliary scalar ϕ to the ordinary matter. Our model is able to explain the baryon number asymmetry $n_B/s \sim 10^{-10}$ without conflicting with the experimental tests on CPT. As a complementarity, we also investigate the scenario of gravitational baryogenesis in radiation-filled BD cosmology from a derivative coupling of the Ricci scalar to the ordinary matter introduced by Davoudiasl et al.^[6], in this case the current baryon asymmetry can also be achieved at early epoches just for a non-vanishing \dot{R} , which is different from the model in Einstein's cosmology. As a result, we should also keep in mind, to obtain a sufficient baryon asymmetry in both these two models in BD cosmology, the time when the grand unification is about taking place would be required to be a little bit earlier comparing with that of Einstein's cosmology.

After this work was firstly submitted a related independent work^[42] appeared in the archives, which has similar motivations.

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Brans-Dicke 宇宙中的轻子数生成*

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摘要 讨论了 Brans-Dicke 理论中的辅助性标量 ϕ 在宇宙的重子数不对称起源中所扮演的可能角色. 在 Morganstern 关于 Brans-Dicke (BD) 宇宙学的一系列工作的基础上, 讨论了这个引力性标量场的导数和重子流 J^{μ}_{B} 或者“重子-轻子”流 $J^{\mu}_{\text{B-L}}$ 的耦合效应. 发现由这个耦合形成的自发重子数生成机制, 只要其中大统一发生的时标足够提前, 就可以产生足够的重子数不对称 $n_{\text{B}}/s \sim 10^{-10}$. 此外, 也研究了由 Davoudiasl 引进的里奇标量 R 的导数和重子流之间的相互作用耦合在辐射为主的 BD 宇宙中的重子数生成效应. 结果表明, 在此情况下因为 \dot{R} 不再为零也同样可以获得重子数的不对称, 从而不同于爱因斯坦宇宙框架下的相关结论.

关键词 轻子数生成 标量耦合 Brans-Dicke 宇宙学

2005 - 03 - 16 收稿, 2005 - 05 - 31 收修改稿

*国家自然科学基金(10273017)资助

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