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Energy Level Statistics in the $SO(6)$ Limit of Super Symmetry $U(6/4)$ in IBFM*

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Abstract The energy levels of the $SO(6)$ limit of super symmetry $U(6/4)$ belonging to odd-A nucleus are calculated with the interacting boson-fermion model. Its emphasis is to study the statistical properties of the nearest neighbor spacing distribution(NSD) and the spectral rigidity (Δ_3). And the factors that affect the properties of level statistics are also investigated. The calculated results indicate that the finite boson number N effect is prominent. When N has a value close to a realistic one, both the interaction strength of subgroup $SO^{BF}(5)$ and the spin play an important role in the energy level statistics. However, in the case of N close to infinite, the statistics all tend to be Poisson type.

Key words interacting boson-fermion model, the $SO(6)$ limit of super symmetry $U(6/4)$, odd-A nucleus, level statistics, energy levels unfolding, nearest neighbor spacing distribution, spectral rigidity

1 Introduction

It is of important significance to do quantum energy level statistics for nuclei with random-matrix theory(RMT)[1]. The structure and property of nuclei may be further studied with RMT. Recently, many theoretical and experimental energy level statistics of nuclei with different models have been done. For example, Shu et al. theoretically studied the statistics of the energy levels in $U(5)$、$O(6)$ and $SU(3)$ dynamical symmetries in the interacting boson model[2]; Cheng et al. discussed the experimental energy levels of odd-odd nucleus ^{86}Nb using the axially symmetric rotor plus quasi-particle model[3]. In this paper, we calculate the theoretical energy levels in the $SO(6)$ limit of super symmetry $U(6/4)$ belonging to odd-A nucleus in the interacting boson-fermion model[IBFM]. On one hand we study the statistics of the energy levels of odd-A nucleus briefly discussed previously, on the other hand IBFM can be tested by RMT.

2 Hamiltonian

More than twenty years ago, Arima and Iachello put forward the interacting boson model(IBM)[4]. In IBM, valence nucleus pairs are treated as bosons. It is a very effective phenomenological model for describing the low-lying collective properties of middle-heavy even-even nuclei. On the basis of IBM, it is obtained that the interacting boson-fermion model which adapts to collective properties of odd-A nucleus by coupling the freedom of single nucleus on the system of bosons. In this model, the Hamiltonian of odd-A nucleus is written as

$$H = H_B + H_F + V_{BF}, \tag{1}$$

where H_B is the boson Hamiltonian of even-even nucleus core, which is the same as IBM. While H_F is the single fermion Hamiltonian, and V_{BF} is the interacting component between bosons and fermion.

The biggest symmetry group is the direct product of boson

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symmetry group $U^B(6)$ and fermion symmetry group $U^F(m)$, that is $U^B(6)\times U^F(m)$, where $m=\sum(2j+1)$, and j is the orbit angular momentum of single nucleus. Many boson-fermion dynamical symmetries are produced when the boson freedom couples with pseudo-orbit freedom or pseudo-spin freedom of fermion. As to different broken symmetries of $U^B(6)\times U^F(m)$, the dynamical super symmetries are given by enbedding $U^B(6)\times U^F(m)$ into super symmetry $U(6/m)$.

The super symmetry $U(6/4)$ has been widely studied since it was given as the first super symmetry by Iachello in 1980[5—9], which was used to explain the Os-Ir isotopes successfully. To super symmetry $U(6/4)$, the single particle occupies the orbit $j=3/2$ and the biggest symmetry group of fermion is $U^F(4)$. Therefore, there are two boson-fermion group chains including $SO(6)$. The state vector of the $SO(6)$ is labeled as

$$\left|\begin{matrix} U^B(6) & U^F(4) & SO^B(6) & SO^{BF}(6) & SO^{BF}(5) & SO^{BF}(3) \\ [N] & [M] & \sum & (\sigma_1\sigma_2\sigma_3) & (\tau_1\tau_2) & \nu_\Delta J \end{matrix}\right\rangle, \tag{2}$$

where, every quantum number labels irreducible representation of the corresponding subgroup respectively. As to the odd-A nucleus, M is equal to 1. In the $SO(6)$ symmetry, the Hamiltonian is expressed by Casimir operator as:

$$H = AC_{2SO^B(6)} + BC_{2SO^{BF}(6)} + CC_{2SO^{BF}(5)} + DC_{2SO^{BF}(3)}, \tag{3}$$

So the energy eigenvalue is:

$$E = A\sum\left(\sum+4\right) + B[\sigma_1(\sigma_1+4)+\sigma_2(\sigma_2+2)+\sigma_3^2] + C[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)] + DJ(J+1), \tag{4}$$

therefore, we can calculate the energy levels of the $SO(6)$ symmetry under different spins and further perform the statistics of the quantum energy levels.

3 Method of level statistics

At first, we take the unfolding process for a given spectrum $\{E_i\}$, separating it into the fluctuation part and the smoothed average part. The number of the levels below E is written as

$$N(E) = \bar{N}_{av}(E) + N_{fl}(E), \tag{5}$$

where $N(E)=\sum_i\theta(E-E_i)$, $N(E)$ is named as energy level staircase function and θ unit step function. When $E\geqslant E_i$, $\theta=1$, else $\theta=0$. Then we fix the $\bar{N}_{av}(E)$ semiclassically by taking a smooth polynomial function to fit the staircase function $N(E)$. Finally, we obtain the unfolded spectrum $\{X_i\}$, where $X_i=\bar{N}_{av}(E)$.

According to the random-matrix theory (RMT), the nearest neighbor spacing distribution (NSD) and the spectral rigidity (Δ_3)[10] were used to do energy level statistics for nucleus. The nearest neighbor spacing is defined as $S_i = X_i - X_{i-1}$. The normal nearest neighbor spacing is $s_i = S_i/D$, where $D=\langle S_i\rangle$. The distribution $P(s)$ is defined in such a way that $P(s)\mathrm{d}s$ is the probability for s_i within the infinitesimal interval $[s, s+\mathrm{d}s]$, which measures the level repulsion and short-range correlations between levels. For a regular system, it is expected to behave like the Poisson statistics

$$P(s) = \mathrm{e}^{-s}, \tag{6}$$

while, if the system is chaotic, a Wigner distribution

$$P(s) = (\pi/2)\exp(-\pi s^2/2), \tag{7}$$

is expected, which is consistent with the GOE statistics. Between these two kinds of systems, the Brody distribution

$$P(s) = \alpha(1+\omega)s^{\omega}\exp(-\alpha s^{(1+\omega)}), \tag{8}$$

where $\alpha = \left\{\Gamma\left(\frac{\omega+2}{\omega+1}\right)\right\}^{\omega+1}$ should be obeyed. Obviously, $\omega=0$ corresponds to the Possion distribution, and $\omega=1$ corresponds to the GOE distribution.

The spectral rigidity(Δ_3) that is a measure of the deviation of the unfolded staircase function from a straight line is defined as

$$\Delta_3(L) = \left\langle \min_{A,B}\frac{1}{L}\int_{-L/2}^{L/2}[n(x)-Ax-B]^2\mathrm{d}x\right\rangle, \tag{9}$$

where $n(x)$ is the staircase function for an unfolded spectrum in the interval $[-L/2, x]$ and the average $\langle\cdots\rangle$ is taken over a suitable energy interval over x in a given ensemble. A and B are fitting parameters. For Possion distribution,

$$\Delta_3(L) = L/15, \tag{10}$$

and for GOE distribution,

$$\Delta_3(L) = (\ln L - 0.0687)/\pi^2. \tag{11}$$

$\Delta_3(L)$ measures the long-range correlations between levels.

4 Numerical results and discussion

The properties of odd-A ^{181}Os can suitably be explained by the $SO(6)$ limit of super symmetry $U(6/4)$[5], in which the boson number is 14. At first, we choose the boson number 14 of uncertain nucleus to do level statistics so that the theoretical level statistics have practical significance. The positive parity is chosen and all the degenerate states are taken into consideration just as one single state. From Fig.1 to Fig.2, the calculated results indicate that the degree of chaoticity depends on the parameters and boson number of Hamiltonian.

From Fig.3, the level statistics show that $J = 15/2$ is a critical spin for the boson number 14. When the spin increases, initially there is a steady increase in chaoticity and then a maximum of onset of chaos is reached around $J = 15/2$. For spins above $J = 15/2$, we see a rapid decrease in chaoticity with increasing spin. This is consistent with the conclusions from papers[3,11], which are calculated with IBM and the axially symmetric rotor plus quasi-particle model respectively. It is explained that when the Coriolis interaction is comparable to the pairing interaction, the degree of chaoticity seems to be maximal. When spin J is increased, the Coriolis interaction is bigger than pairing interaction, which makes the decoupling of the quasiparticles from the core and their spin alignment along the rotation axis, so the dynamical system trends to a regular system.

From Fig.4, the calculation indicates that when boson number is not big, the level statistics is mainly decided by the interaction strength of $SO^{BF}(5)$. When it is smaller or larger, the level statistics trend to Poisson, while it is some value in between, the degree of chaoticity reaches to a maximal value. This indicates that the interaction strength between the single particle and even-even core affects the level statistics of odd-A nucleus greatly.

Finally, we have calculated the level statistics in each set of parameters for $N = 60$, and $N = 180$, where N is the boson number of uncertain nucleus. The results for these different N are plotted in Fig. 1 and Fig. 2, respectively. The results of

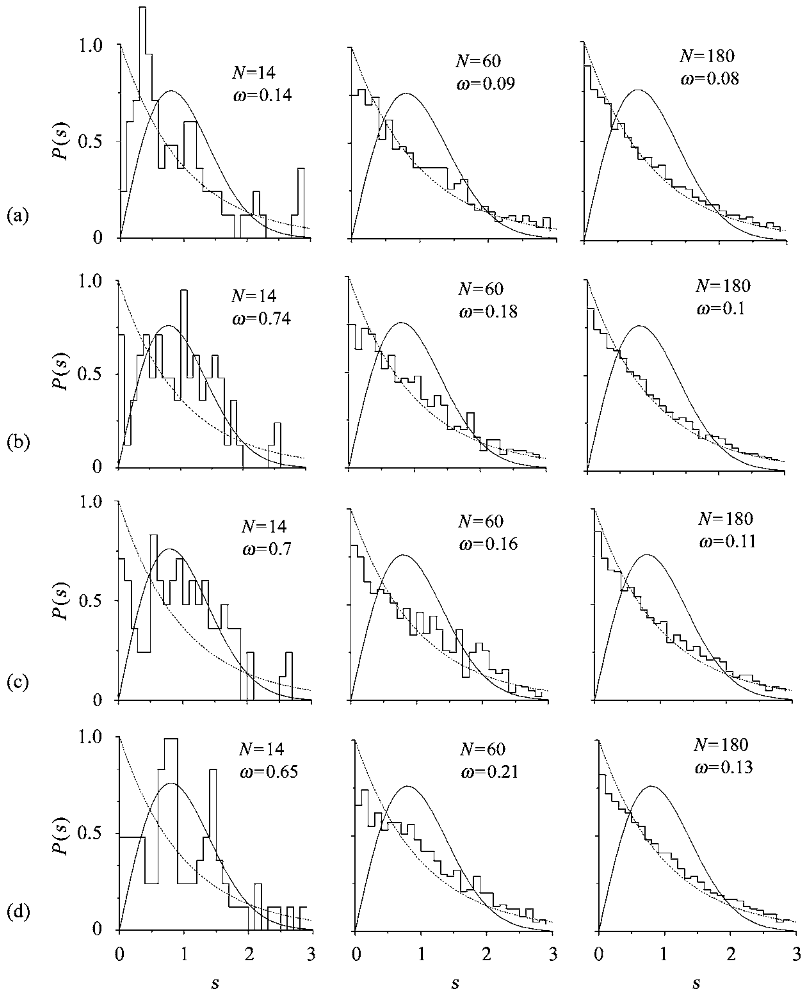

Fig.1. Nearest-neighbor spacing distribution $P(s)$ of the states $J = 3/2$ with four sets of Parameters: (a) $A = -0.5, B = 0.07, C = 0.2, D = 0.001$; (b) $A = -0.5, B = 0.07, C = 0.15, D = 0.001$; (c) $A = -0.5, B = 0.07, C = 0.1, D = 0.001$; (d) $A = -0.5$, $B = 0.07$, $C = 0.05$, $D = 0.001$. In all figures, the sold lines and dashed lines describe the GOE and Poisson atatistics, respectively.

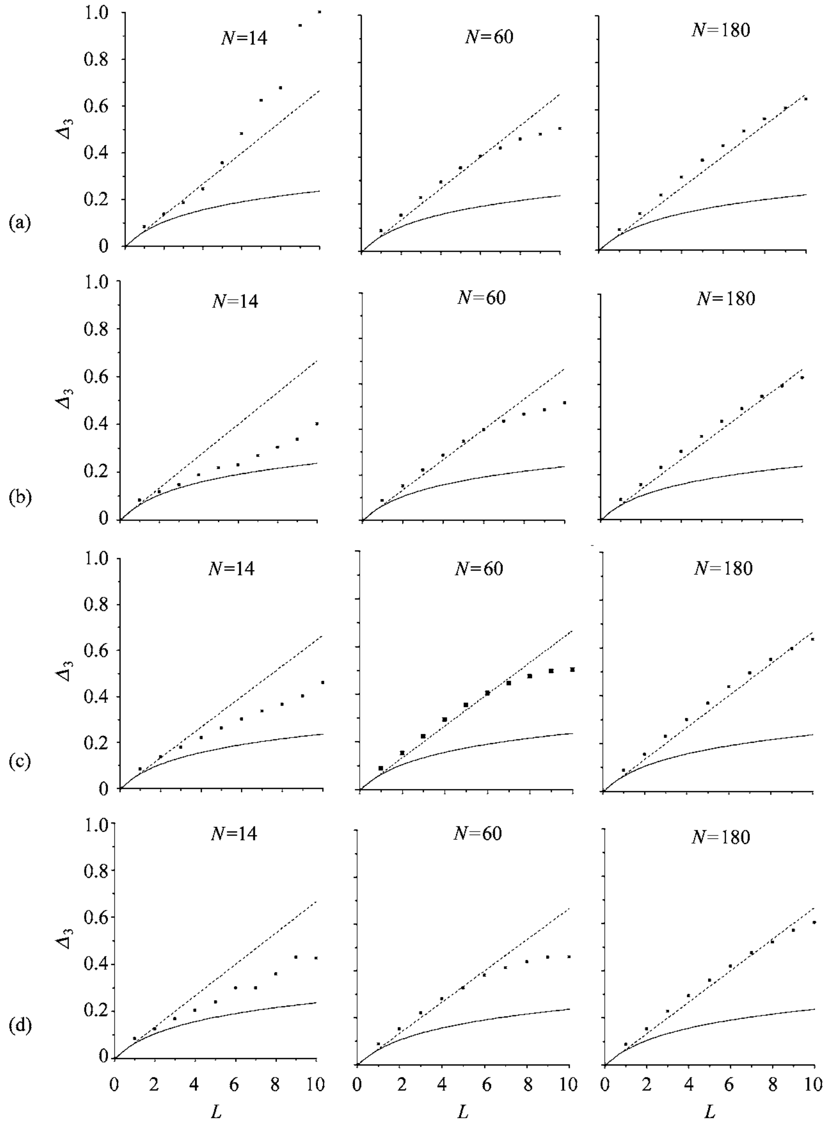

Fig.2.　The same as Fig.1 but for the spectral rigidity.

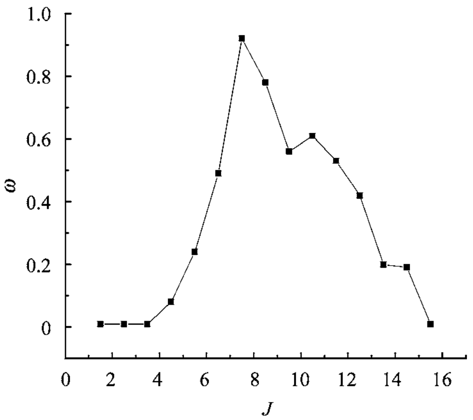

Fig.3.　Quantum measures of chaos versus spin for $A = -0.5, B = 0.07, C = 0.01, D = 0.001$ and $N = 14$.

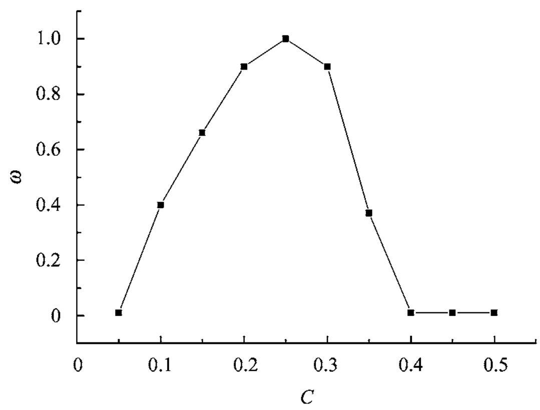

Fig.4.　Quantum measures of chaos versus interaction strength of $SO(5)$ for $A = -0.5, B = 0.5, D = 0.001, N = 14$ and $J = 3/2$.

level statistics indicate that when the boson number N increases, the statistics trend to regular distribution independently of the interaction parameters. So, when the boson number $N \to \infty$, the statistics trend to a Poisson type, which is consistent with the conclusion of paper[2].

5 Conclusion

We study the energy level statistics of the states in the $SO(6)$ limit of super symmetry $U(6/4)$ belonging to odd-A nucleus. The calculated results indicate that the finite boson number N effect is prominent. When N has a value close to a realistic one, both the interaction strength of subgroup $SO^{BF}(5)$ and the spin play an important role in the energy level statistics. However, in the case of N close to infinity, the statistics tend to a Poisson type.

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IBFM模型 $U(6/4)$超对称 $SO(6)$极限的能谱统计*

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摘要 用相互作用玻色子费米子模型(IBFM)计算了奇 A 核 $U(6/4)$超对称 $SO(6)$极限的理论能谱,对不确定核的最近邻能级间距分布和能谱刚性度进行了研究,并对影响能谱统计特征的因素进行了讨论.结果表明,有限的玻色子数 N 的大小显著地影响能谱统计.当 N 接近于真实核的玻色子数时,子群 $SO^{BF}(5)$的作用强度和自旋对能谱统计起重要作用.然而,当 N 趋于无穷大时,能谱统计总是趋于 Poisson 分布.

关键词 相互作用玻色子费米子模型 $U(6/4)$超对称 $SO(6)$极限 奇 A 核 能谱统计 能谱展示 能谱的最近邻能级间距 能谱刚性度

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