第 29 卷 第 9 期 高 能 物 理 与 核 物 理 Vol. 29, No. 9
2005 年 9 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Sep., 2005

Prediction and Formation Mechanism of Triaxial Superdeformed Nuclei for $A\sim 80$*

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Abstract The three dimensional Total Routhian Surface (TRS) calculations are carried out for 64 nuclei in the $70\leqslant A\leqslant 90$ region to find triaxial superdeformed nuclei. A total of 12 nuclei are predicted to have triaxial superdeformation in which the neutron rotational energy plays a key role and the neutron shell energy plays additional role in the formation of triaxial superdeformed nuclei.

Key words triaxial superdeformation, $A\sim 80$ region, formalism mechanism

1 Introduction

Usually, the shape of a deformed nucleus is supposed to be an ellipsoid with small hexadecapole deformation. In the Nilsson model, the frequency of harmonic oscillator is described as

$$\omega_{\rm k}=\omega_0\left[1-\frac{2}{3}\varepsilon_2\cos(\gamma+k\frac{2\pi}{3})\right],\qquad k=1,2,3,\quad (1)$$

where ε_2 is the quadrupole deformation parameter and γ the triaxial deformation parameter. Let the half axis of a nucleus in x, y, z direction be a, b, c, respectively, then we get the relations between a, b, c and ε_2, γ from the condition $a\omega_1=b\omega_2=c\omega_3$:

$$\varepsilon_2 = \frac{\sqrt{9(bc+ac-2ab)^2+27(bc-ac)^2}}{2(ab+ac+bc)},\quad (2)$$

$$\gamma = \arctan\left(\frac{\sqrt{3}(b-a)}{a+b-2ab/c}\right),\quad (3)$$

and

$$\begin{aligned} a &= Rf[\varepsilon_2\cos\gamma+\sqrt{3}\varepsilon_2\sin\gamma+3]^{-1},\\ b &= Rf[\varepsilon_2\cos\gamma-\sqrt{3}\varepsilon_2\sin\gamma+3]^{-1},\qquad (4)\\ c &= Rf[3-2\varepsilon_2\cos\gamma]^{-1}, \end{aligned}$$

here $f=[-2\varepsilon_2^3\cos(3\gamma)-9\varepsilon_2^2+27]^{1/3}$ and R is the radius of a sphere whose volume is equal to the ellipsoid volume. For example, when $a:b:c=1:1:2$ (axial ellipsoid), ε_2 and γ will take the volume of 0.6 and 0°, respectively. Such nuclei, i.e., $c:a\sim 2:1$ and $a\sim b$, are called axial superdeformed nuclei. While $a\neq b$, the triaxial deformation γ will not be zero. For instance, if $a:b:c=3:4:6$, then (ε_2,γ) will be $(0.577, 30°)$. Such nuclei which have large γ deformation and large quadrupole deformation are called triaxial superdeformed (TSD) nuclei. From Eq. (3) we can see that γ is sensitive to the difference of a and b.

Until now, many data for axial superdeformed nuclei are accumulated but the data for triaxial superdeformed nuclei are quite few, and all of them are found

Received 21 January 2005, Revised 30 March 2005
* Supported by National Natural Science Foundation of China (10235020, 10305019, 10475115, 10475026), Natural Science Foundation of Zhejiang Province (Y604027) and Chinese Academy of Science Knowledge Innovation Project (KJCX1-N11)
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in Lu, Hf, Ta, Er and Zr[1—8]. Among the discovered TSD nuclei, most of them are located in the $A\sim160$ region, and only one nucleus is in the $A\sim80$ region. The triaxial behavior of ^{163}Lu has been further confirmed experimentally by the discovery of the wobbling mode[9]. However, one is not able to get the size of the triaxiality from this mode, and a theoretical model is still needed to get more information on this aspect. The triaxial superdeformed nuclei in the $A\sim160$ region has been predicted[10]. From that work triaxial superdeformation for proton configuration [660]1/2 in several other nuclei, besides the four discovered ones, were predicted. It is also pointed out that the shapes may co-exist for other certain configurations and a nucleus may have triaxial superdeformation in different quasi-particle configurations.

The TSD nucleus in $A\sim80$, ^{86}Zr, was discovered in 1998. TSD ^{86}Zr seems not to be an accident appearing in the $A\sim80$ region. Other TSD nuclei must also exist in this region. In this paper, we attempt to predict the TSD nuclei near $A\sim80$ by TRS (Total Routhian Surface) calculations and to give the corresponding formation mechanism. In Sect. 2 a brief description of the three-dimensional TRS theory, which is used to determine the nuclear deformation, is included. The prediction of TSD nuclei in the $A\sim80$ region is presented in Sect. 3. The discussion of the formation mechanism of TSD nuclei is devoted in Sect. 4 and in Sect. 5 the summary is degiven.

2 A brief description of the TRS model

The Hamiltonian of quasi-particles moving in a quadrupole deformed potential rotating around the x-axis with a frequency ω can be written as

$$H^{\omega}=H_{\rm s.p.}(\varepsilon_2,\varepsilon_4,\gamma)-\lambda N+\Delta(P+P^{+})-\omega J_x, \quad (5)$$

where $H_{\rm s.p.}$ denotes the deformed Hamiltonian of single particle motion, the second term on the right hand side is the chemical potential, the third term is the pairing interaction and the last term stands for the Coriolis forces. The modified-harmonic-oscillator (MHO) potential with the parameters κ and μ for the mass region taken from Ref. [11] is employed in the present calculation. The pairing-gap parameter is determined empirically by $\Delta=0.9\Delta_{\rm o.e.}$, and $\Delta_{\rm o.e.}$ is taken from experimental odd-even mass difference[12]. As an approximation, we did not take into account the deformation and rotation dependence of pairing.

The total routhian surface, namely, the total energy in the rotating frame as a function of ε_2, γ, and ε_4, of a (Z,N) nucleus for a fixed quasi-particle configuration c.f. can be calculated by

$$\begin{aligned}E^{\rm c.f.}(\varepsilon_2,\varepsilon_4,\gamma;\omega)=&E_{\rm ld}(\varepsilon_2,\varepsilon_4,\gamma)+\\&E_{\rm corr}(\varepsilon_2,\varepsilon_4,\gamma;\omega=0)+\\&E_{\rm rot}(\varepsilon_2,\varepsilon_4,\gamma;\omega)+\\&\sum_{i\in{\rm c.f.}}{\rm e}_i^{\omega}(\varepsilon_2,\varepsilon_4,\gamma),\end{aligned} \quad (6)$$

where $E_{\rm ld}$ is the liquid-drop model energy[13], $E_{\rm corr}$ is the quantum-effect correction to the energy, which includes both the shell[14] and pairing corrections[15]. The collective rotational energy $E_{\rm rot}$ can be microscopically calculated as the energy difference between the expectation values of H^{ω} with and without rotation, by using the wave function for the quasi-particle vacuum configuration[16]. The last term of Eq. (2) is the sum of quasi-particle energies belonging to the configuration c.f., which generates the deformation drive. All of the terms in Eq. (2) depend on (Z,N) numbers which are not written explicitly. The equilibrium deformations of nucleus are calculated by minimizing the total routhian energy of Eq. (2) with respect to ε_2, ε_4, and γ. Here we take the hexadicupole deformation ε_4 as a free parameter in order to get better results.

In the real process of minimizing the total routhian, we minimize the total routhian as a function of ε_4 for each point (ε_2,γ) and get two surfaces $E^{\rm c.f.}(\varepsilon_2,\gamma)$ and $\varepsilon_4(\varepsilon_2,\gamma)$ first. Then from the surface of $E^{\rm c.f.}(\varepsilon_2,\gamma)$ we can find the minimum $E^{\rm c.f.}_{\rm min}$ and corresponding $\varepsilon_{2\rm min}$ and $\gamma_{\rm min}$. From $\varepsilon_4(\varepsilon_2,\gamma)$, $\varepsilon_{2\rm min}$ and $\gamma_{\rm min}$, we can determine the value of $\varepsilon_{4\rm min}$. Based on above steps, we can find the equilibrium deformations $\varepsilon_{2\rm min}$, $\varepsilon_{4\rm min}$ and $\gamma_{\rm min}$ which possibly exist in the nucleus.

3 The prediction of the triaxial superdeformed nuclei

Before calculating $A\sim80$ nuclei, the TRS method is checked and compared with the discovered TSD nucleus, ^{86}Zr. Our calculated result, $(\varepsilon_2,\gamma)=(0.455, 16.8)$, is very coincidental with the result in Ref. [8] , indicating the reliability of the method for the calculation in the $A\sim80$ mass region.

In the following, the progress used to determine the deformation of a nucleus will be described in detail with the example of ^{80}Kr. From the fact that the γ-ray energy within a superdeformed band in $A\sim80$ is much higher than that in $A\sim160$, we get that the superdeformed nuclei in $A\sim80$ rotate much faster than those in $A\sim160$ because the rotational frequency, ω, is approximately half the γ-ray energy. Thus, when we predict the shape of a nucleus in $A\sim80$, the ω must be larger. In this paper, we fixed the ω as $0.1\omega_0$, where $\omega_0=41/\sqrt[3]{A}$MeV. In the three-dimensional calculation, ε_4 from -0.04 to 0.10 is divided into 11 points. The total routhian energy in each $(\varepsilon_2\cos(\gamma+30°),\varepsilon_2\sin(\gamma+30°))$ point will be minimized with respect to the corresponding 11 points. Fig. 1(a) shows a contour plot of the total routhian surface in which each point corresponds to the same ω but different ε_4. In Fig. 1(a), there is a local minimum marked by "+" which has the deformation $(\varepsilon_2,\gamma)=(0.393,28.8°)$. The hexadecupole parameter ε_4, corresponding to the local minimum in Fig. 1(a), is determined by Fig. 1(b) which is the counter plot of ε_4. The value of ε_4 in each grid point in Fig. 1(b) is got from the minimization of the total routhian against ε_4. The symbol "+" point in Fig. 1(b), which corresponds to the minimum in Fig. 1(a), has a value of ε_4 0.030. Thus, the deformation of ^{80}Kr at $\omega=0.1\omega_0$ is determined as $(\varepsilon_2,\gamma,\varepsilon_4)=(0.393,28.8°,0.030)$. During the calculation, we do not add the quasi-particle energy (the last item in Eq. (6)) to the total routhian energy, because at such a high rotational frequency, one or two pairs of particles are broken and their contributions are automatically included as a part of the the rotational energy (see Sect. 4 for details).

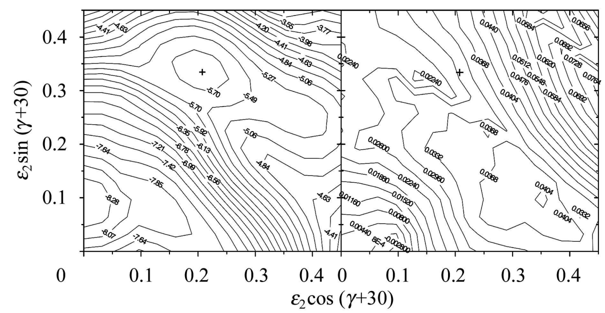

Fig. 1. The shape determination of ^{80}Kr. (a) The counter plot of total routhian. The unit of the number in the surface is MeV. "+" indicates a local minimum whose deformation is $(\varepsilon_2,\gamma)=(0.393,28.8°)$; (b) The counter plot of ε_4. The hexadecupole deformation at symbol "+", which has the same position as "+" in (a), is 0.030.

Following the step described above, we analyzed all of the β stable even-even nuclei in the $70\leqslant A\leqslant90$ region, totally 64 nuclei. The predicted TSD nuclei are listed in Table 1. In this paper, we call the deformation with $\varepsilon_2>0.35$ and $10°\leqslant\gamma\leqslant50°$ as triaxial superdeformation. So, in Table 1, only the TSD nuclei under the deformation condition, $\varepsilon_2>0.35$ and $10°\leqslant\gamma\leqslant50°$, are listed.

Table 1. Predicted TSD nuclei in the $70\leqslant A\leqslant90$ region for rotational frequency of $0.1\hbar\omega_0$ and conditions of $\varepsilon_2>0.35$ and $10°\leqslant\gamma\leqslant50°$.

nucleus	ε_2	γ	ε_4	nucleus	ε_2	γ	ε_4
^{72}Ni	0.448	12.6°	0.048	^{80}Kr	0.394	28.6°	0.030
^{74}Ni	0.393	21.3°	0.023	^{86}Zr	0.471	19.0°	0.043
^{76}Zn	0.404	21.3°	0.029	^{88}Mo	0.500	14.7°	0.055
^{76}Ge	0.403	30.7°	0.027	^{90}Mo	0.375	40.0°	0.037
^{78}Se	0.396	32.7°	0.027	^{90}Ru	0.483	23.1°	0.045
^{80}Se	0.351	36.1°	0.022				

The location of the predicted nuclei among the nuclei in the $70\leqslant A\leqslant90$ region is shown in Fig. 2. In this figure, solid circles represent the predicted TSD nuclei, open circles axial SD nuclei, cross symbols the nuclei in which we did not find superdeformation. Obviously, regular in this figure is that when $N=42,44,46$, most of the nuclei have superdeformation. Especially, for nuclei of $N=44,46$, most of them have triaxial superdeformation. Apparently, the neutron properties control the formation of axial superdeformation and triaxial superdeformation. How and why do the neutron properties control the formation mechanism of TSD nuclei?

4 Formation mechanism of TSD nuclei in $A\sim 80$

In Fig. 2, it is obvious that the neutron numbers of the most predicted TSD nuclei are 44 and 46. Only ^{90}Mo is an exception. This phenomenon indicates that the neutron property governs the formation of TSD nuclei.

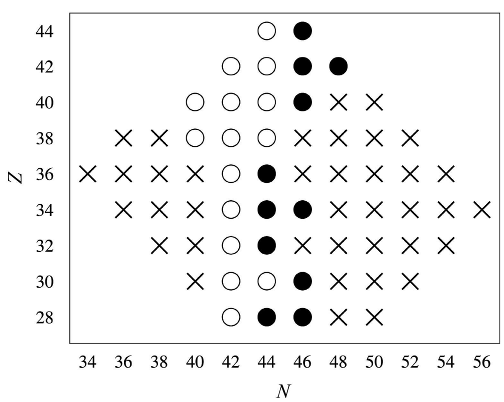

Fig. 2. The prediction of triaxial superdeformed nuclei.
The solid circle, open circle and cross represent the predicted triaxial, axial, non-superdeformed nuclei, respectively. It is shown that most of the predicted TSD nuclei are located in $N=44,46$.

In order to discuss the mechanism in detail, the ^{80}Kr, predicted to have triaxial superdeformation, is selected. According to the Eq. (6), most parts of the total routhian energy, $E_{\rm rot}$, $E_{\rm shell}$, $E_{\rm pair}$ and $E_{\rm sum}(=E_{\rm rot}+E_{\rm shell}+E_{\rm pair})$, are plotted in Fig. 3. Fig. 3(a) and Fig. 3(b) show the TRS elements of proton and neutron in ^{80}Kr, respectively. The energy scale of the contour lines is 0.28MeV.

In Fig. 3(a1), the surface of proton rotational energy is flat and the deformation of the local minimum is small. Therefore, the proton rotating energy cannot affect the formation of TSD shape. In Fig. 3(a2), although the proton shell correction energy has two deep minimums, the proton pairing correction energy, shown in Fig. 3(a3), has two high peeks near the minimum in Fig. 3(a2) and canceled the minimum of shell correction energy. The sum of the three types of energy, $E_{{\rm sum}(p)}$, shown in Fig. 3(a4), is flat in the central part of the surface. This means that the proton properties in ^{80}Kr are helpless to form TSD nuclei.

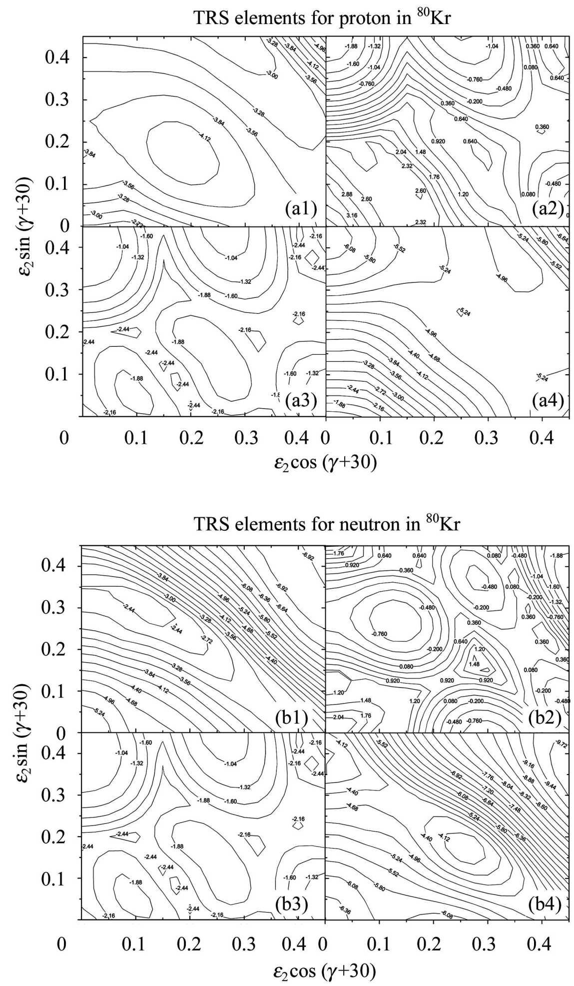

Fig. 3. The formation mechanism of ^{80}Kr.
(a) and (b) are for protons and neutrons, respectively. (a1), (b1) are for rotating energy, (a2), (b2) for shell correction energy, (a3), (b3) for pairing energy, (a4), (b4) are the sum of the previous three items. It is shown that neutron rotating energy plays a key role in the formation of the predicted TSD ^{80}Kr. See text for details.

However, the neutron properties, shown in Fig. 3(b), are different from Fig. 3(a). The neutron rotational energy, shown in Fig. 3(b1) decreases sharply with increasing large ε_2 deformation and therefore has a strong driving effect towards large elongation deformation. In Fig. 3(b2), the neutron shell correction has two minimums but they are canceled by the pairing energy shown in Fig. 3(b3). Thus, the driving force in large quadrupole deformation remains. Summing the three neutron parts of the total routhian energy, $E_{\mathrm{sum}(n)}$, we obtain Fig. 3(b4). This figure is much similar to Fig. 3(b1), having small driving effect to spherical deformation and large driving effect to large quadrupole deformation. This is very important for ^{80}Kr to form TSD shape. Fig. 4 shows the sum of liquid drop energy and $E_{\mathrm{sum}(n)}$. A large quadrupole and triaxial minimum appear on this surface. Since $E_{\mathrm{sum}(p)}$ is flat in this region, the minimum shown in Fig. 4 exists also in the total routhian surface, see Fig. 1. In the formation of TSD shape, the rotational energy plays a crucial role. Because the neutron shell correction energy also decreases sharply in large deformation, it also has an additional role to form TSD shape.

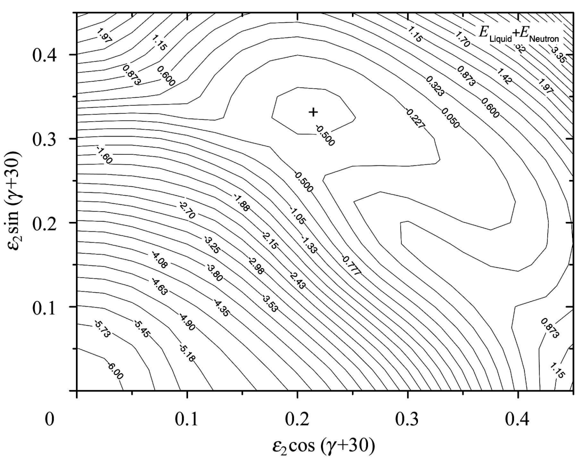

Fig. 4. The counter plot of the sum of liquid drop energy, neutron rotating energy, shell correction energy and pair correction energy. This plot shows the local minimum marked by "+" which is close to the local minimum in Fig. 1(a).

To confirm that the reason is also effective for other nuclei in the $A\sim80$ region, we analyze ^{78}Se which is predicted to have TSD shape and ^{86}Kr which is predicted to have no TSD shape as well. The results support our analysis for the formation mechanism of TSD nuclei that the rotational energy plays a key role and neutron shell energy plays an additional role in the formation of TSD nuclei.

It has been pointed out that the rotational energy is the difference between the expectation value of H^{ω} (Eq. (5)) with and without rotation. When the rotational frequency is high, the pairing of protons and/or neutrons will be broken and their angular momentum alignment will affect the rotational energy. In order to see the broken pair of protons and neutrons, the calculated quasi-particle routhians are presented in Fig. 5(a) for protons and Fig. 5(b) for neutrons. In Fig. 5(a), [431]3/2 orbit crosses over the [440]1/2 orbit at $\omega=0.082\hbar\omega_0$ for protons, while in Fig. 5(b), [420]1/2 orbit crosses over the [413]7/2 orbit for both $\alpha=\pm1/2$ at $\omega=0.09\hbar\omega_0$ for neutrons. Therefore, When ^{80}Kr rotates with $\omega=0.10\hbar\omega_0$, one proton pair and two neutron pairs are broken and this will take effect on the rotational energy.

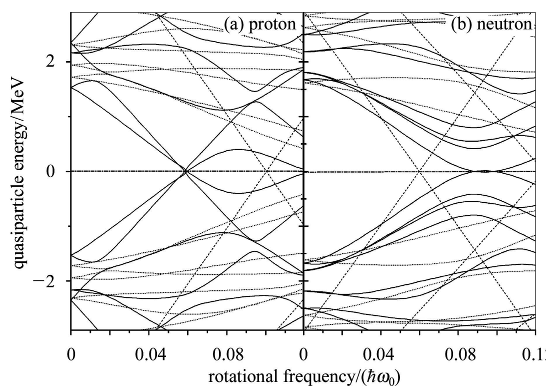

Fig. 5. The quasi-particle energy for protons (a) and neutrons (b) in ^{80}Kr. The following convection is used: solid lines: $(\pi=+,\alpha=+\frac{1}{2})$, dotted lines: $(\pi=+,\alpha=-\frac{1}{2})$, dash-dotted lines: $(\pi=-,\alpha=+\frac{1}{2})$ and dashed lines: $(\pi=-,\alpha=-\frac{1}{2})$.

Compared with the analysis of TSD nuclei in the $A\sim160$ region, the quadrupole deformation parameters of the predicted TSD nuclei in the $A\sim80$ region are larger than those in the $A\sim160$ region. And also, the formation mechanism is different between the two regions. In the $A\sim160$ region, the neutron shell correction energy controls the formation of TSD nuclei, while in the $A\sim80$ region, the rotating energy, which includes the effect of quasi-particle angular momentum alignments, controls the formation of TSD nuclei.

5 Summary

In summary, by fixing the rotational frequency ω at $0.1\hbar\omega_0$, we predict that 11 nuclei have triaxial

superdeformation under the condition of $\varepsilon_2 \geqslant 0.35$ and $10° \leqslant \gamma \leqslant 50°$ by the three dimensional TRS calculation in the $A\sim80$ region. Most of these TSD nuclei are located in the $N=44,46$ region, only ^{90}Mo is an exception. By analyzing the formation mechanism of TSD nuclei in the $A\sim80$ region, we find that the neutron rotational energy which includes the contribution of quasiparticle angular momentum alignment plays a key role to form TSD nuclei and the neutron shell energy plays an additional role.

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$A\sim80$核区三轴超形变的预言及形成机制*

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摘要 利用三维总位能面方法, 对$A\sim80$核区的64个原子核作了三轴超形变的存在性研究. 经过计算, 从中找到了12个原子核可能存在三轴超形变. 研究发现, 原子核中的中子转动能对三轴超形变的形成起主要作用, 中子壳修正对其形成也有一定的影响.

关键词 三轴超形变 $A\sim80$核区 形成机制

2005 – 01 – 21 收稿, 2005 – 03 – 30 收修改稿

*国家自然科学基金(10235020, 10305019,10475115, 10475026), 浙江省自然科学基金(Y604027)和中国科学院知识创新工程重点方向性项目(KJCX1-N11)资助

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