An Effective Formulation to Solve Nuclear Many-Body Problems*

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Abstract The second order self-energies of hadrons are calculated from $\hat{S}_2$ matrix, and then an effective method to solve nuclear many-body problems is summarized. In addition, the density-dependent relativistic many body methods are discussed.

Key words nuclear matter, self-energy, scattering matrix

1 Introduction

Although quantum field theory has succeeded greatly in the scope of particle physics, people have met some difficulties when the methods of the quantum field theory are applied to nuclear systems. The ground state of the nuclear system is filled with interacting nucleons, which is different from the vacuum in perturbation theory. In addition, The coupling constants of strong interaction between nucleons are far larger than the fine structure constant in the quantum electrodynamics, so the perturbation method could not be performed perfectly on nuclear systems. At last, the nucleons and mesons consist of quarks and gluons, and are not fundamental components in the level of modern knowledge. All these factors determine that the perturbation method in quantum field theory can only be generalized to solve nuclear many-body problems approximately and effectively.

Walecka and his group attempted to solve the nuclear many-body problems in the framework of quantum field theory, and developed the method of relativistic mean-field approximation[1—3]. In this method, the field operators of the scalar meson and vector meson are replaced with their expectation values in the nuclear matter, respectively. Therefore, the calculation is simplified largely. Until now, there have been some excellent review articles giving a more detailed description on this topic[3—7]. In the earliest relativistic mean-field theory, the resultant compression modulus is almost 550MeV[1], which is far from the experimental data range of 200—300MeV. To solve this problem, the nonlinear self-coupling terms of scalar mesons are introduced to produce the proper equation of state of nuclear matter[8]. No doubt, additional parameters would give more freedoms to fit the saturation curve of nuclear matter. Zimanyi and Moszkowski developed the derivative scalar coupling model yielding a compression modulus of 225MeV without any additional parameter[9].

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With the bare nucleon-nucleon interaction, the properties of symmetric nuclear matter at various densities can be determined in the relativistic Brueckner-Hartree-Fock calculation. For each value of the density, the relativistic mean-field equations are solved and the corresponding coupling constants are fixed to the results of Brueckner calculation. Therefore, a relativistic mean-field model with density-dependent coupling constants is obtained[10, 11]. Meanwhile, the Debye screening masses of mesons in the nuclear matter are calculated in the relativistic mean-field approximation[12], and it shows all the screening meson masses increase with the nucleon number density. With the meson masses replaced by their corresponding screening masses in Walecka-1 model, the saturation properties of the nuclear matter are fitted reasonably, and then a density-dependent relativistic mean-field model is proposed. The nonlinear self-coupling terms of the mesons are not included in both of these density-dependent relativistic mean-field models. In the nuclear matter, the screening meson masses increase with the density of the nuclear matter, and it is equivalent to the statement that the coupling constants decrease with the density increasing while the masses of mesons retain constant. At this point, these two models are consistent with each other.

The relativistic mean-field results may be derived by summing the tadpole diagrams self-consistently in nuclear matter, retaining only the contributions from nucleons in the filled Fermi sea in the evaluation of the self-energy and energy density, i.e., the relativistic mean-field method is consistent to the relativistic Hartree approximation[3]. It is correct only in the framework of the original Walecka model. In the most popular Walecka-2 model, in which the nonlinear self-coupling terms of the scalar meson are included, whether the relativistic mean-field method is still consistent to the relativistic Hartree approximation has not be studied. In Sect. 2, we obtained the self-energies of hadrons by calculating the second order $\hat{S}$ matrix in the nuclear matter, then an effective method to solve nuclear many-body problems is evaluated. In Sect. 3, the effective nucleon mass in the nuclear matter is discussed with the nonlinear self-coupling terms of the scalar meson included in the Lagrangian density. In Sect. 4, the density-dependent relativistic mean-field methods are discussed. The summary is given in Sect. 5.

2 The self-energies of hadrons in the nuclear matter

According to Walecka-1 model, the nucleons ψ interact with scalar mesons σ through a Yukawa coupling $\bar{\psi}\psi\sigma$ and with neutral vector mesons ω that couple to the conserved baryon current $\bar{\psi}\gamma_\mu\psi$. the Lagrangian density can be written as

$$\begin{aligned}\mathscr{L} = {} & \bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu - M_N)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \\ & \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\sigma\bar{\psi}\sigma\psi - \\ & g_\omega\bar{\psi}\gamma_\mu\omega^\mu\psi, \end{aligned} \tag{1}$$

with M_{N}, m_σ and m_ω the nucleon, scalar meson and vector meson masses, respectively, and $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ the vector meson field tensor.

The momentum-space propagators for the scalar meson, vector meson and the nucleon take the forms of[3]

$$\mathrm{i}\Delta(p) = \frac{-1}{p^2 - m_\sigma^2 + \mathrm{i}\varepsilon}\ , \tag{2}$$

$$\mathrm{i}D_{\mu\nu}(p) = \frac{g_{\mu\nu}}{p^2 - m_\omega^2 + \mathrm{i}\varepsilon}\ , \tag{3}$$

$$\mathrm{i}G_{F\alpha\beta}(p) = (\gamma_\mu p^\mu + M_{\mathrm{N}})_{\alpha\beta}\left(\frac{-1}{p^2 - M_{\mathrm{N}}^2 + \mathrm{i}\varepsilon}\right). \tag{4}$$

As the effect of Fermi sea is considered, an on-shell part

$$\begin{aligned}\mathrm{i}G_{D\alpha\beta}(p) = {} & (\gamma_\mu p^\mu + M_{\mathrm{N}})_{\alpha\beta} \times \\ & \left(-\frac{\mathrm{i}\pi}{E(p)}\delta(p^0 - E(p))\theta(p_{\mathrm{F}} - |\boldsymbol{p}|)\right), \end{aligned} \tag{5}$$

is included in the nucleon propagator besides the Feynman propagator in Eq. (4), where $E(p) = \sqrt{\boldsymbol{p}^2 + M_{\mathrm{N}}}$, and p_{F} is the Fermi momentum of nucleons.

Since the vector meson couples to the conserved baryon current, the longitudinal part in the propagator of the vector meson will not contribute to physical quantities[13]. Therefore, only the transverse part in the propagator of the vector meson is written in Eq. (3).

According to the Feynamn diagrams shown in Figs. 1 and 2, the self-energies of the nucleon, the scalar and vector meson in the nuclear matter can be calculated with the Feynman rules in Ref. [3]. It should be noticed that there is a more factor of (i) in each of the propagators of the hadrons in our manuscript than those propagators in Ref. [3].

In this section, we will calculate the self-energies of hadrons in the nuclear matter from Wick's theorem of quantum field theory.

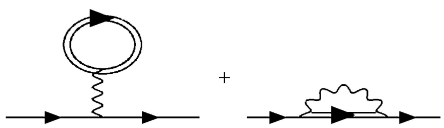

Fig. 1.　Feynman diagrams for the second order self-energy of the nucleon in nuclear matter calculated in the Walecka model. The double solid lines denote the nucleon propagators defined in the ground state of nuclear matter.

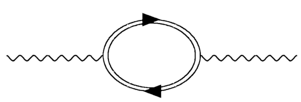

Fig. 2.　Feynman diagram for the second order self-energies of the scalar or vector meson in nuclear matter calculated in the Walecka model. Same case as in Fig. 1.

The momentum-space propagator of the nucleon takes the form of Feynman propagator[13]. Therefore, the Pauli blocking effect of Fermi sea is excluded in the propagator of the nucleon, and the on-shell propagator is no use in the following calculations.

The interaction Hamiltonian can be expressed as

$$\mathscr{H}_I = g_\sigma \bar{\psi}\sigma\psi + g_\omega \bar{\psi}\gamma_\mu \omega^\mu \psi \ . \tag{6}$$

In the second order approximation, only

$$\hat{S}_2 = \frac{(-\mathrm{i})^2}{2!}\int \mathrm{d}^4 x_1 \int \mathrm{d}^4 x_2 T\left[\mathscr{H}_\mathrm{I}(x_1)\mathscr{H}_\mathrm{I}(x_2)\right], \tag{7}$$

in the S-matrix should be calculated in order to obtain the self-energy corrections of the nucleon and mesons.

2.1　The nucleon self-energy in nuclear matter

The second order self-energy of the nucleon coupling to the scalar meson is discussed firstly.

In order to obtain the second order self-energy correction of the nucleon in Fermi sea, only the normal ordering product

$$N\left[\bar{\psi}(x_1)\sigma\overbrace{(x_1)\psi(x_1)\bar{\psi}(x_2)\sigma}(x_2)\psi(x_2)\right], \tag{8}$$

in the Wick's expansion of the time-ordered product in Eq. (7) should be considered, where the overbrace "$\overbrace{\quad}$" denote the contraction of a pair of field operators.

When a nucleon with momentum k and spin δ is considered in the nuclear matter, its field operator $\psi(k,\delta,x)$ and conjugate field operator $\bar{\psi}(k,\delta,x)$ can be expressed as

$$\begin{aligned}\psi(k,\delta,x) \ = \ & A_{k\delta}U(k,\delta)\exp(-\mathrm{i}k\cdot x)+ \\ & B^\dagger_{k\delta}V(k,\delta)\exp(\mathrm{i}k\cdot x),\end{aligned} \tag{9}$$

and

$$\begin{aligned}\bar{\psi}(k,\delta,x) \ = \ & A^\dagger_{k\delta}\bar{U}(k,\delta)\exp(\mathrm{i}k\cdot x)+ \\ & B_{k\delta}\bar{V}(k,\delta)\exp(-\mathrm{i}k\cdot x)\end{aligned} \tag{10}$$

respectively. In the calculation of the self-energy of the nucleon with the momentum k and the spin δ, a pair of the nucleon field operator and the conjugate operator in the normal ordering product of Eq. (8) should be replaced with Eqs. (9) and (10), while the other pair of the nucleon field operator and the conjugate operator connected with the underbrace "$\underbrace{\quad}$" in the following normal ordering products denote the nucleon in the Fermi sea, and would be replaced with their expansion forms of a complete set of solutions to the Dirac equation, respectively.

$$\begin{aligned}& N\left[\bar{\psi}(x_1)\sigma\overbrace{(x_1)\psi(x_1)\bar{\psi}(x_2)\sigma}(x_2)\psi(x_2)\right] \rightarrow \\ & 2\sigma\overbrace{(x_1)\sigma}(x_2)\times \\ & \Big\{N\left[\bar{\psi}(k,\delta,x_1)\psi(k,\delta,x_1)\bar{\psi}\underbrace{(x_2)\psi}(x_2)\right]+ \\ & N\left[\bar{\psi}(k,\delta,x_1)\psi\underbrace{(x_1)\bar{\psi}}(x_2)\psi(k,\delta,x_2)\right]\Big\}.\end{aligned} \tag{11}$$

Suppose there are no antinucleons in the ground state of nuclear matter and the Fermi sea is filled with interacting nucleons, only the positive-energy components are considered in the expansion forms of the nucleon field operator and its conjugation. The expectation value of $\hat{S}_2$ in the nuclear matter on the first term in Eq. (11) can be written as

$$\begin{aligned}\langle G|\hat{S}_2|G\rangle \;=\; & \mathrm{i}g_\sigma^2(2\pi)^4\delta^4(p_1+k_1-p_2-k_2)\times \\ & \sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \bar{U}(k,\delta)U(k,\delta)\mathrm{i}\Delta(0)\bar{U}(p,\lambda)U(p,\lambda),\end{aligned}$$

where $k_1=k_2=k$, and $p_1=p_2=p=(E(p),\boldsymbol{p})$, and $\theta(x)$ is the step function.

According to Dyson equation, the nucleon propagator in the nuclear matter can be derived as

$$\begin{aligned}&\frac{\mathrm{i}}{\not{k}-M_\mathrm{N}-\varSigma_1^\sigma+\mathrm{i}\varepsilon}=\frac{\mathrm{i}}{\not{k}-M_\mathrm{N}+\mathrm{i}\varepsilon}+\frac{\mathrm{i}}{\not{k}-M_\mathrm{N}+\mathrm{i}\varepsilon}\times \\ &\mathrm{i}\frac{g_\sigma^2}{m_\sigma^2}\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ &\bar{U}(p,\lambda)U(p,\lambda)\frac{\mathrm{i}}{\not{k}-M_\mathrm{N}+\mathrm{i}\varepsilon},\end{aligned}\tag{12}$$

then the second order self-energy of the nucleon in the nuclear matter from the first term in Eq. (11) can be written as

$$\begin{aligned}\varSigma_1^\sigma \;=\; & -\frac{g_\sigma^2}{m_\sigma^2}\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \bar{U}(p,\lambda)U(p,\lambda)=-\frac{g_\sigma^2}{m_\sigma^2}\rho_\mathrm{S}\ ,\end{aligned}\tag{13}$$

with

$$\rho_\mathrm{S}=\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|),\tag{14}$$

the scalar density of protons or neutrons.

The second order self-energy of the nucleon relevant to the second term in Eq. (11) can be obtained similarly

$$\begin{aligned}\varSigma_2^\sigma \;=\; & g_\sigma^2\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \left[U(p,\lambda)\ \mathrm{i}\varDelta(k-p)\bar{U}(p,\lambda)\right]= \\ & -g_\sigma^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \left[\frac{\not{p}+M_\mathrm{N}}{2M_\mathrm{N}}\frac{1}{(k-p)^2-m_\sigma^2}\right].\end{aligned}\tag{15}$$

Correspondingly, the normal ordering products relevant to the self-energy of the nucleon coupling to the vector meson in the calculation of the $\hat{S}_2$ matrix can be written as

$$\begin{aligned}&N\left[\bar{\psi}(x_1)\gamma_\mu\overbrace{\omega^\mu(x_1)\psi(x_1)\bar{\psi}(x_2)\gamma_\nu\omega^\nu}(x_2)\psi(x_2)\right]\rightarrow \\ &2\overbrace{\omega^\mu(x_1)\omega^\nu}(x_2)\times \\ &\left\{N\left[\bar{\psi}(k,\delta,x_1)\gamma_\mu\psi(k,\delta,x_1)\underbrace{\bar{\psi}(x_2)\gamma_\nu\psi}(x_2)\right]+\right. \\ &\left.N\left[\bar{\psi}(k,\delta,x_1)\gamma_\mu\underbrace{\psi(x_1)\bar{\psi}}(x_2)\gamma_\nu\psi(k,\delta,x_2)\right]\right\}.\end{aligned}\tag{16}$$

Therefore, the second order self-energies of the nucleon coupling to the vector meson in the nuclear matter corresponding to the first and second terms in Eq. (16) can be calculated as

$$\begin{aligned}\varSigma_1^\omega \;=\; & (-\mathrm{i}g_\omega)^2\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \gamma_\mu\mathrm{i}D^{\mu\nu}(0)\left[\bar{U}(p,\lambda)\gamma_\nu U(p,\lambda)\right]=\gamma_0\frac{g_\omega^2}{m_\omega^2}\rho_\mathrm{V},\end{aligned}\tag{17}$$

with

$$\rho_\mathrm{V}=\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\theta(p_\mathrm{F}-|\boldsymbol{p}|),\tag{18}$$

the number density of protons or neutrons, and

$$\begin{aligned}\varSigma_2^\omega \;=\; & g_\omega^2\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times \\ & \left[\gamma_\mu U(p,\lambda)\mathrm{i}D_0^{\mu\nu}(k-p)\bar{U}(p,\lambda)\gamma_\nu\right]= \\ & g_\omega^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{(\boldsymbol{p}^2+M_\mathrm{N}^2)^{\frac{1}{2}}}\frac{-\gamma_\mu p^\mu+2M_\mathrm{N}}{(k-p)^2-m_\omega^2}\ .\end{aligned}\tag{19}$$

Obviously, the second order self-energies of the nucleon $\varSigma_1^\sigma$, $\varSigma_2^\sigma$, $\varSigma_1^\omega$, $\varSigma_2^\omega$ calculated from Wick's expansion in this section are the same as those results from Walecka model, respectively[3].

2.2 The self-energies of the scalar and vector mesons

In the Wick expansion of the time-ordered product in Eq. (7), only the normal ordering products including one contraction of a pair of nucleon field operator and its conjugate operator should be studied in order to obtain the second order self-energy of the scalar meson in the filled Fermi sea.

When a scalar meson with determined momentum k is studied in the nuclear matter, its field operator $\sigma(k,x)$ can be expressed as

$$\sigma(k,x)=a_k\exp(-\mathrm{i}k\cdot x)+a_k^\dagger\exp(\mathrm{i}k\cdot x).\tag{20}$$

In order to calculate the self-energy of the scalar meson in the nuclear matter, the scalar field operators $\sigma(x_1)$ and $\sigma(x_2)$ in the following normal ordering product should be replaced with Eq. (20),

$$\begin{aligned}&N\left[\bar{\psi}\overbrace{(x_1)\sigma(x_1)\psi(x_1)\bar{\psi}(x_2)\sigma(x_2)\psi}(x_2)\right]+\\&N\left[\bar{\psi}(x_1)\sigma(x_1)\psi\overbrace{(x_1)\bar{\psi}}(x_2)\sigma(x_2)\psi(x_2)\right]\rightarrow\\&2\psi\overbrace{(x_1)\bar{\psi}}(x_2)N\left[\bar{\psi}\underbrace{(x_1)\sigma(k,x_1)\sigma(k,x_2)\psi}(x_2)\right].\end{aligned}\tag{21}$$

The nucleon field operator $\psi(x_2)$ and the conjugate field operator $\bar{\psi}(x_1)$ in the normal ordering product of Eq. (21) should be expanded in terms of the set of solutions to the Dirac equation, respectively. Therefore, the second order self-energy of the scalar meson can be obtained as[12]

$$\begin{aligned}\varSigma_\sigma\;=\;&(-\mathrm{i}g_\sigma)^2\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times\\&\left[\bar{U}(p,\lambda)\left(\mathrm{i}G(p-k)+\mathrm{i}G(p+k)\right)U(p,\lambda)\right]=\\&g_\sigma^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times\\&\left[\mathrm{Tr}\left(\frac{1}{\not p-\not k-M_\mathrm{N}}\frac{\not p+M_\mathrm{N}}{2M_\mathrm{N}}\right)+\right.\\&\left.\mathrm{Tr}\left(\frac{\not p+M_\mathrm{N}}{2M_\mathrm{N}}\frac{1}{\not p+\not k-M_\mathrm{N}}\right)\right].\end{aligned}\tag{22}$$

Similarly, the normal ordering products relevant to the self-energy of the vector meson in the nuclear matter can be written as

$$\begin{aligned}&N\left[\bar{\psi}\overbrace{(x_1)\gamma_\mu\omega^\mu(x_1)\psi(x_1)\bar{\psi}(x_2)\gamma_\nu\omega^\nu(x_2)\psi}(x_2)\right]+\\&N\left[\bar{\psi}(x_1)\gamma_\mu\omega^\mu(x_1)\psi\overbrace{(x_1)\bar{\psi}}(x_2)\gamma_\nu\omega^\nu(x_2)\psi(x_2)\right]\rightarrow\\&2\psi\overbrace{(x_1)\bar{\psi}}(x_2)\times\\&N\left[\bar{\psi}\underbrace{(x_1)\gamma_\mu\omega^\mu(k,\delta,x_1)\gamma_\nu\omega^\nu(k,\delta,x_2)\psi}(x_2)\right],\end{aligned}\tag{23}$$

with

$$\begin{aligned}\omega_\mu(k,\delta,x)\;=\;&b_{k\delta}\varepsilon_\mu(k,\delta)\exp(-\mathrm{i}k\cdot x)+\\&b_{k\delta}^\dagger\varepsilon_\mu(k,\delta)\exp(\mathrm{i}k\cdot x).\end{aligned}\tag{24}$$

The self-energy of the vector meson in the nuclear matter can be calculated similarly as

$$\begin{aligned}\varSigma_\omega\;=\;&(-\mathrm{i}g_\omega)^2\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times\\&\left[\bar{U}(p,\lambda)\left(\gamma_\nu\mathrm{i}G(p-k)\gamma_\mu+\right.\right.\\&\left.\left.\gamma_\mu\mathrm{i}G(p+k)\gamma_\nu\right)U(p,\lambda)\right]=\\&g_\omega^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)\times\\&\left[\mathrm{Tr}\left(\gamma_\nu\frac{1}{\not p-\not k-M_\mathrm{N}}\gamma_\mu\frac{\not p+M_\mathrm{N}}{2M_\mathrm{N}}\right)+\right.\\&\left.\mathrm{Tr}\left(\gamma_\nu\frac{\not p+M_\mathrm{N}}{2M_\mathrm{N}}\gamma_\mu\frac{1}{\not p+\not k-M_\mathrm{N}}\right)\right].\end{aligned}\tag{25}$$

It is no doubt that the second order self-energies of the scalar and vector mesons calculated from $\hat{S}_2$ matrix directly are the same as those from Walecka model, respectively[3].

The second-order self-energies of the nucleon, the scalar and vector meson in the nuclear matter are calculated from Wick's expansion. It shows the same results as those in Walecka model[3], and then an effective many-body method based on vacuum propagators has been evaluated. Feynman rules on this effective method can be summarized similarly as those in Walecka model[3]. In the new Feynman rules, a factor of

$$\sum_{\lambda=1,2}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{M_\mathrm{N}}{E(p)}\theta(p_\mathrm{F}-|\boldsymbol{p}|)$$

is included for each pair of crosses, which denote the initial and final states of the nucleon in the Fermi sea. Moreover, the momentums and spins of external lines with a cross or without a cross take the same values with each other, respectively. In addition, a factor of (-1) is included in the calculation of exchange diagrams.

The loop diagrams, which relate to the contribution of Dirac sea and cause divergences, are not necessary to be considered in no Dirac sea approximation. Therefore, only the diagrams with crosses should be studied in the calculation of self-energies of particles. Because there do not exist antinucleons in the ground state of nuclear matter, the diagrams with an external line of antinucleons should be excluded, too.

The Feynman diagrams for the second order self-energy of the nucleon in the nuclear matter in Section 2.1 are shown in Fig. 3. The first diagram in Fig. 3 corresponds to the tadpole contribution, and the second corresponds to the exchange term in the relativistic Hartree-Fock approximation[3].

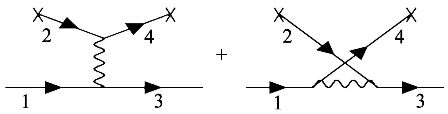

Fig. 3. Feynman diagrams for the second order self-energy of the nucleon in nuclear matter calculated from $\hat{S}_2$ matrix elements. The wave lines denote the scalar meson or vector meson, while 1 and 2 denote particles of the initial state, 3 and 4 denote particles of the final state.

The self-energies of the scalar meson, vector meson or the photon in the nuclear matter can be calculated with the Feynman diagrams in Fig. 4. It shows the same results as the one-fermion-loop approximation in Walecka model[3], i.e., the same effective masses of the photon and mesons in Refs. [12, 14] can be obtained in the one-fermion-loop approximation in Walecka model.

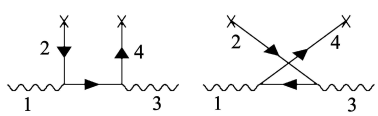

Fig. 4. Feynman diagrams for the second order self-energies of the scalar or vector meson in nuclear matter calculated from $\hat{S}_2$ matrix elements. Same case as in Fig. 3.

In our formalism, the effects of the nuclear medium come from the nucleon condensation, i.e., the scalar density of nucleons. In Walecka's formalism, the propagators of hadrons are defined in the ground state of the nuclear matter, which are different from the propagators defined in vacuum, and the loop diagrams are considered although the meanings are different from those in quantum field theory. Therefore, Feynman rules in our formalism are different from those in Refs. [3, 15], and The condensation of the nucleon is embodied in the integrals of three-momentum space in the new Feynman rules.

In particle physics, people are mostly interested in scattering processes, for which the $\hat{S}$ matrix providing the probability of transition from the initial states to final states, is the most suitable framework. In statistical physics, however, we are mainly concerned the expectation value of physical quantities at finite time. Obviously, these two problems are connected with each other in our formalism. Because vacuum propagators are adopted in our formalism, which are not relevant to the state of the system, it is not difficult to extend this formalism to study the properties of non-equilibrium and finite temperature states. Some works have been done along this direction[16].

Actually, the propagator including the on-shell part of Eq. (5) is not for the nucleon, but for a kind of quasi-nucleon, whose creation and annihilation operators satisfy the same anti-commutation relations as those of the nucleon. There is a Bogoliubov transformation between the creation and annihilation operators of the quasi-nucleon and the nucleon[17].

3 Self-consistent relativistic Hartree approximation

When the isospin $SU(2)$ symmetry is considered in the nuclear matter, the ρ meson interaction should be included in the Lagrangian density,

$$\mathscr{L}_{\rm Int}^{\rho}=-g_{\rho}\bar{\psi}\gamma^{\mu}\frac{\boldsymbol{\tau}}{2}\cdot\boldsymbol{\rho}_{\mu}\psi\ , \tag{26}$$

with $\boldsymbol{\tau}$ being the Pauli matrix. Because the ρ_+ and ρ_- mesons only contribute to the second order self-energy of the nucleon in the exchange terms, only the ρ_0 meson interaction is considered in the relativistic Hartree approximation.

With the similar method, the second order self-energy corrections of the proton and neutron coupling to the ρ_0 meson can be written as

$$\Sigma_1^{\rho}=\gamma_0\frac{g_{\rho}^2}{\pm 4m_{\rho}^2}\left(\rho_{\rm p}-\rho_{\rm n}\right), \tag{27}$$

with the plus for the proton and the minus for the neutron, where $\rho_{\rm p}$ and $\rho_{\rm n}$ are the number density of protons and neutrons, respectively. Obviously, the results in Eq. (27) are the same as those in the relativistic mean-field approximation[3].

In the calculation of the second order self-energies of hadrons in the nuclear matter in Sect. 2, the non-interacting hadron propagators are used. Although the second order results can be summed to all orders with Dyson equation, this procedure is not self-consistent. Self-consistency can be achieved by using the interacting propagators to also determine the self-energy[3]. In the relativistic Hartree approximation, the self-energy of the nucleon in the nuclear matter can be calculated self-consistently with the interacting propagator of the nucleon

$$\mathrm{i}G_{\mathrm{H}}(p)=\frac{-1}{\gamma_\mu\bar{p}^\mu-M_{\mathrm{N}}^*+\mathrm{i}\varepsilon}, \tag{28}$$

where

$$M_{\mathrm{N}}^*=M_{\mathrm{N}}-\frac{g_\sigma^2}{m_\sigma^2}(\rho_{\mathrm{p}}^S+\rho_{\mathrm{n}}^S), \tag{29}$$

$$\bar{p}^0=p^0-\frac{g_\omega^2}{m_\omega^2}(\rho_{\mathrm{p}}+\rho_{\mathrm{n}})-\frac{g_\rho^2}{\pm 4m_\rho^2}(\rho_{\mathrm{p}}-\rho_{\mathrm{n}}), \tag{30}$$

with the plus for the proton and the minus for the neutron, and

$$\bar{\boldsymbol{p}}=\boldsymbol{p}\ . \tag{31}$$

In Eq. (29), ρ_{p}^S and ρ_{n}^S are the scalar densities of protons and neutrons, respectively. It corresponds to the transformation

$$M_{\mathrm{N}}\to M_{\mathrm{N}}^*,\qquad E(p)\to E^*(p)\ , \tag{32}$$

in the self-energy of the nucleon in Eqs. (13) and (17), where $E^*(p)=(\boldsymbol{p}^2+M_{\mathrm{N}}^{*2})^{1/2}$.

The effective nucleon mass can be defined as the pole of the nucleon propagator in the limit of the space-momentum of the nucleon $\boldsymbol{p}\to 0$, which corresponds to the mass spectra of the collective excitations in the nuclear matter[18, 19]. According to Eq. (13), the effective nucleon mass in the nuclear matter is defined in Eq. (29) in the relativistic Hartree approximation. In Walecka-2 model, the nonlinear self-coupling terms of the scalar meson are introduced to replace the mass term $\frac{1}{2}m_\sigma^2\sigma^2$[8],

$$U(\sigma)=\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4\ . \tag{33}$$

Because the boson distribution functions of the mesons are zero in the nuclear matter at zero temperature, the self-coupling terms of the scalar meson have no contribution to the self-energy corrections of the nucleon and the meson when the loop diagrams are ignored. Therefore, the effective nucleon mass still takes the form in Eq. (29) in Walecka-2 model, which is important to conserve the self-consistency in the calculation of relativistic Hartree approximation.

In the relativistic mean-field approximation of Walecka-2 model, the effective nucleon mass in the nuclear matter can be obtained from the Dirac equation of the nucleon,

$$M_{\mathrm{N}}^*=M_{\mathrm{N}}+g_\sigma\sigma_0\ , \tag{34}$$

and the expectation value of the scalar field σ_0 is calculated repeatedly with the equation

$$m_\sigma^2\sigma_0+g_2\sigma_0^2+g_3\sigma_0^3=-g_\sigma(\rho_{\mathrm{p}}^S+\rho_{\mathrm{n}}^S). \tag{35}$$

If the nonlinear self-coupling terms of the scalar meson are considered, the effective nucleon mass in Eq. (34) is different from that in Eq. (29) although they are same as one another in Walecka-1 model, where the nonlinear self-coupling terms of the scalar meson are ignored. Therefore, the self-consistent calculation is realized differently in the relativistic mean-field approximation and relativistic Hartree approximation of Walecka-2 model.

Because of the strong coupling between hadrons, the nuclear systems can only be studied approximately and effectively in the framework of quantum field theory, and the renormalization is even meaningless on nuclear systems. In nuclear many-body theories, the self-consistency should be considered, and the different interacting propagators should be adopted in the calculation of different order approaches.

4 Density-dependent relativistic mean-field method

According to the self-energies of the scalar meson and vector meson in Eqs. (22) and (25), the screening masses of the mesons in the nuclear matter can be extracted directly at the limit of zero momentum $\boldsymbol{k}\to 0$. A more detailed derivation can be found in our previous article[12]. The screening masses of the scalar meson, the vector meson and ρ meson can be written as

$$m_\sigma^*=\sqrt{m_\sigma^2+\frac{g_\sigma^2(\rho_S^{\mathrm{p}}+\rho_S^{\mathrm{n}})}{M_{\mathrm{N}}^*}}, \tag{36}$$

$$m_\omega^* = \sqrt{m_\omega^2 + \frac{g_\omega^2(\rho_S^p + \rho_S^n)}{2M_N^*}}, \tag{37}$$

and

$$m_\rho^* = \sqrt{m_\rho^2 + \frac{g_\rho^2(\rho_S^p + \rho_S^n)}{8M_N^*}}, \tag{38}$$

where ρ_S^p and ρ_S^n denote the scalar densities of protons and neutrons, respectively. It can be seen from Eqs. (36), (37) and (38) that the screening masses of mesons increase with the nucleon number density in the nuclear matter.

By replacing the meson masses in the Lagrangian of Eq. (1) with the corresponding screening masses of mesons, respectively, a density-dependent relativistic mean-field method is obtained[12]. With the parameters

$$\frac{g_\sigma^2}{m_\sigma^2} = 8.297\text{fm}^2, \quad \frac{g_\omega^2}{m_\omega^2} = 3.683\text{fm}^2, \quad \frac{g_\rho^2}{m_\rho^2} = 5.187\text{fm}^2, \tag{39}$$

we obtain a saturation density of 0.149fm^{-3}, a binding energy of 16.669MeV, a compression modulus of 280.1MeV, a symmetry energy coefficient of 32.8MeV and an effective nucleon mass of $0.808M_N$ for the symmetric nuclear matter.

Recently, we noticed the density-dependent relativistic Hartree-Fock method developed by W. H. Long et al[20], where the meson-nucleon coupling constants are taken as functions of the nucleon number density. Considering the ratio of the meson-nucleon coupling constant and the corresponding meson mass can be treated as one parameter in the conventional relativistic mean-field calculations on the properties of nuclear matter, either changing the coupling constants as functions of the nucleon number density or replacing the meson masses with their corresponding screening masses would be a choice to achieve a density-dependent relativistic many body method. The ratios of the density-dependent coupling constants and the corresponding meson masses with the parameters PKO1 in Ref. [20] and the ratios of the fixed coupling constants and the screening meson masses with the parameters in Eq. (39) as functions of the nucleon number density are shown in Fig. 5. Although the values of $g_\sigma(\rho)/m_\sigma$, $g_\omega(\rho)/m_\omega$, $g_\rho(\rho)/m_\rho$ in the model of Ref. [20] are far different from the corresponding values of g_σ/m_σ^*, g_ω/m_ω^*, g_ρ/m_ρ^* in our model, they all decrease with the increasing nucleon number density. It implies that these two kinds of density-dependent models are consistent with each other. Actually, the interaction between two fixed nucleons in the nuclear matter becomes weaker if the screening effect of the nuclear medium is considered.

In fact, either density-dependent coupling constants or density-dependent meson masses mean including the nonlinear density-dependent potential in the relativistic many body method. This point can be seen easily from the equations on the expectation values of mesons in the relativistic mean-field approximation. In Fig. 5, the values of $g_\sigma(\rho)/m_\sigma$ and g_σ/m_σ^* decrease rapidly with the increasing nucleon number density, however, the corresponding values for ρ meson change slightly with the nucleon number density, it means the nonlinear attractive interaction between nucleons by exchanging scalar mesons is more important to construct the density-dependent nuclear models.

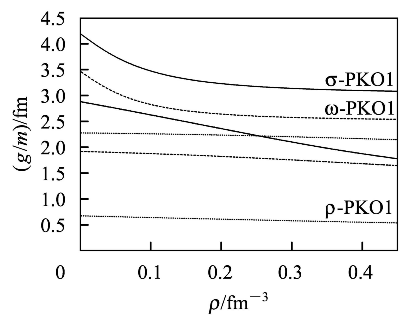

Fig. 5. Ratios of the density-dependent coupling constants and the meson masses with the parameters PKO1[20] and ratios of the coupling constants and the screening meson masses with the parameters in Eq. (39) as functions of the nucleon number density ρ. The solid lines are for cases of σ meson, and the dashed lines for ω meson, and the dot lines for the ρ meson. Those lines for the model in Ref. [20] are labeled in the figure, and the lines without special labels are for the model in Ref. [12].

5 Summary

In conclusion, an effective formalism to solve nuclear many-body problems is evaluated, and we find this formalism with off-shell propagators gives the same results as those in Walecka model in the calculation of self-energies of particles in nuclear matter. The self-consistency of Walecka-2 model is discussed in the relativistic mean-field approximation. In addition, the relativistic many body method with density-dependent coupling constants is compared with the method with density-dependent meson masses.

References

1 Walecka J D. Ann. Phys.(N.Y.), 1974, **83**: 491
2 Chin S A. Ann. Phys.(N.Y.), 1977, **108**: 301
3 Serot B D, Walecka J D. Adv. Nucl. Phys., 1986, **16**: 1
4 Reinhard P G. Rept. Prog. Phys., 1989, **52**: 439
5 Ring P. Prog. Part. Nucl. Phys., 1996, **37**: 193
6 Serot B D, Walecka J D. Int. J. Mod. Phys., 1997, **E6**: 515
7 MENG J, Toki H, ZHOU S G et al. Prog. Part. Nucl. Phys., 2006, **57**: 470
8 Boguta J, Bodmer A R. Nucl. Phys., 1977, **A292**: 413
9 Zimanyi J, Moszkowski S A. Phys. Rev., 1990, **C42**: 1416
10 Brockmann R, Toki H. Phys. Rev. Lett., 1992, **68**: 3408
11 SHEN H, Sugahara Y, Toki H. Phys. Rev., 1997, **C55**: 1211
12 SUN B X et al. Int. J. Mod. Phys., 2003, **E12**: 543. nucl-th/0206029
13 Itzykson C, Zuber J B. Quantum Field Theory. Mc Graw-Hill Inc, 1980
14 SUN B X et al. Mod. Phys. Lett., 2003, **A18**: 1485. nucl-th/0204013
15 Furnstahl R J, Serot B D. Phys. Rev., 1991, **C44**: 2141
16 SUN B X et al. Commun. Theor. Phys., 2006, **45**: 527. nucl-th/0209041
17 Umezawa H, Matsumoto H, Tachiki M. Thermofield Dynamics and Condensed States. North-Holland, 1982
18 SONG C. Phys. Rev., 1993, **D48**: 1375
19 GAO S, ZHANG Y J, SU R K. Phys. Rev., 1995, **C52**: 380
20 LONG W H, Giai N V, MENG J. Phys. Lett., 2006, **B640**: 150

一种处理原子核多体问题的有效方法*

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摘要 由二级散射矩阵元出发计算了核物质内强子的二级自能修正, 并且归纳了一种研究原子核多体问题的有效方法. 另外, 还讨论了各种密度相关的相对论多体方法之间的关系.

关键词 核物质 自能 散射矩阵

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