第 31 卷 第 4 期 高 能 物 理 与 核 物 理 Vol. 31, No. 4
2007 年 4 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Apr., 2007

Isospin Effect on Level Spacing of $N=9$ Isotones*

GUO Yan-Qing[1] REN Zhong-Zhou[1,2;1)]

1 (Department of Physics, Nanjing University, Nanjing 210008, China)
2 (Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou, Lanzhou 730000, China)

Abstract The ground-state properties of N=8 and N=9 isotones are investigated in the framework of the single-particle shell model. The isospin effect on the average nuclear potential is taken into account by introducing an isospin-dependent term in the depth of the Woods-Saxon potential. The theoretical results of RMS radii and spin-parity values are in agreement with the experimental data. Especially, the level inversion between neutron levels $2s_{1/2}$ and $1d_{5/2}$ in N=9 isotones is reproduced. A detailed discussion on numerical results is given and one-neutron halos in the ground states of ^{14}B and ^{15}C are presented.

Key words shell model, isospin effect, level inversion

1 Introduction

Recently, some unexpected phenomena were reported in experiments, such as abnormal spin-parity values of some exotic nuclei[1], the sign of proton halo[2—4], and the disappearance of magic number[5]. All of these new experimental results couldn't be explained by the traditional shell model. More attention has been paid to the microscopic understanding of the change of level spacing and order. In particular, special emphasis is laid on the influence of the nuclear structure and inner interaction on the level inversion. A lot of related studies have been done both on experimental and theoretical sides. Experimental physicists focused on the level inversion between intruder and relevant normal levels through the investigation of β decay or scattering[6—9]. On the other hand, theoreticians tried to describe and explain these abnormal phenomena by using different theoretical methods[10—13]. All of these previous works contribute to our further understanding of the nuclear shell structure.

It was pointed out by Bohr and Mottelson[14] that the nucleons are subject to different average nuclear potentials due to the effect of neutron excess. This effect is proportional to $(N-Z)/A$. For light stable nuclei, the isospin effect could be omitted due to $N\approx Z$. For light nuclei far from stability, the isospin effect will become dominant with the increase of neutron excess. It is interesting to examine the effect of neutron excess on the average nuclear potential. Especially, it is worthy to make a detailed study to see whether the isospin effect could explain the abnormal properties observed in some light nuclei.

N=9 isotones are ideal to investigate this problem for the following reasons: (1) N=9 isotones are in the range of light nuclei where the isospin effect on nuclear potential is significant. (2) Among the N=9 isotones, experiments have showed the existence of level inversion in ^{15}C[15]. (3) From the known experiments[16—18], a halo or a skin has been suggested for ^{14}B and ^{15}C. Based on these considerations,

Received 3 August 2006, Revised 16 November 2006
* Supported by National Natural Science Foundation of China (10125521, 10535010) and Major State Basic Research Development Program (G2000077400)
1) E-mail: zren@nju.edu.cn

in this paper, we will study the isospin effect on the ground-state properties of N=8 and N=9 isotones. The isospin effect is introduced by adding a neutron excess term in the depth of the Woods-Saxon potential. Special attention will be paid to the level spacing between neutron levels $2s_{1/2}$ and $1d_{5/2}$.

2 Theoretical framework

According to the essential assumptions of shell model, an average potential (or called single-particle potential) could be adopted to describe the average potential of single nucleon subjected to others. The motion of a nucleon satisfies the following Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2+U_{\rm ws}(r)+U_{\rm so}(r)+U_{\rm C}(r)\right]\times \phi_{nlj}(\boldsymbol{r})=E_{nlj}\phi_{nlj}(\boldsymbol{r}). \qquad (1)$$

In the above equation, $U_{\rm ws}(r)$ is the Woods-Saxon potential with normal parameters $r_0 = 1.27$fm, $a_0 = 0.67$fm[14, 19]. The standard form of spin-orbit potential can be written as[20, 21]

$$U_{\rm so}(r)=V_{\rm so}\ \frac{2}{r}\ \frac{{\rm d}f(r)}{{\rm d}r}\ (\boldsymbol{l}\cdot\boldsymbol{s}), \qquad (2)$$

where $f(r)$ is the standard Woods-Saxon shape function. The parameter $V_{\rm so}$ takes Thomas form with Thomas form factor λ=25[20, 21]. For protons, the Coulomb potential $U_{\rm C}(r)$ of homogeneously charged sphere with radius $R=r_0(A-1)^{1/3}$[20] is added to the nuclear interaction potential.

The nucleon density of nucleus can be written as[22]

$$\rho(r)=\frac{1}{4\pi}\sum_a|u_a(r)|^2n_a\ , \qquad (3)$$

where n_a denotes the nucleon number and $u_a(r)$ is the radial wave function of orbit a.

As proposed by Bohr and Mottelson, the following modification for potential depth V_0 is used[14]

$$V=\begin{cases}V_0\,(1-c\times(N-Z)/A),\ \text{for neutrons,}\\ V_0\,(1+c\times(N-Z)/A),\ \text{for protons.}\end{cases} \qquad (4)$$

The reason for adding this modification is that: with different isospin, neutrons and protons are subject to different average nuclear potentials. When atomic nucleus stays in ground state, inner protons and neutrons prefer to fill in low energy orbits. Because the conjugated neutron and proton in the same orbit form a system with zero isospin, the effect of isospin on total average nuclear potential is contributed entirely by the extra protons or neutrons. So it is reasonable to add the neutron excess term in the potential depth. This modification with the favorable parameters $V_0=52$MeV, $c=0.40$ greatly improves our theoretical results.

Now let's give a brief discussion on the magnitude of the symmetry potential. In Eq. (4), the coefficient of the term $(N-Z)/A$ is approximately 25MeV in the textbook[14]. The value used in our calculation is $V_0\times c = 52\times 0.40\approx 21$MeV. The strength of the symmetry potential in the present paper is close to that in the textbook[14].

It is necessary to point out that the level inversion between $2s_{1/2}$ and $1d_{5/2}$neutron levels could be reproduced when the isospin-dependent term is added in the potential depth. Detailed discussion on the change of level order will be given in the next section.

3 Theoretical results and analysis

By using the method with the parameters mentioned above, we calculate the single-particle energies, RMS radii and density distributions of ground states for N=8 and N=9 isotones with Z=4—8. The numerical results are presented in Tables 1—3 and Figs. 1—3.

In Tables 1 and 2, we list the results of single-particle energies and RMS radii for N=8 and N=9 isotones. $R_{\rm m}$, $R_{\rm n}$ and $R_{\rm c}$ are the RMS radii of matter, neutron and charge distributions, respectively. ϵ(MeV) is the single-particle energy. The corresponding experimental radii are taken from[23—25]. For N=8 isotones, the ground-state properties are studied. For N=9 isotones, calculations are performed for two cases: the outmost neutron occupies either the $2s_{1/2}$ level or $1d_{5/2}$ level. The excited states are denoted by the superscript *. It can be seen from Tables 1 and 2 that: (1) The theoretical RMS radii of N=8 isotones agree with the experimental data well. (2) It is shown that the valence neutrons of the lowest energy states of ^{17}O and ^{16}N occupy $1d_{5/2}$ level. This accords with the spin-parity values of ^{17}O and ^{16}N from the shell model and experiments[1]. (3) For the ground state of ^{15}C, its experimental spin-parity is $(1/2)^{+}$[1] which is in contradiction with the prediction of shell model. Nevertheless, our calculation confirms the experimental result and implies the level inversion between neutron levels $2s_{1/2}$ and $1d_{5/2}$. The ground state of ^{14}B is 2^{-}[1] and we predict its valence neutron occupies the $2s_{1/2}$ level which deserves further experimental test. Our results on ^{13}Be agree with the previous theoretical predictions[10, 26] where the valence neutron of ground state is unbound and occupies the $2s_{1/2}$ level. (4) Basically, the numerical results on RMS radii of ^{17}O, ^{16}N, ^{15}C and ^{14}B agree with the experimental values.

Table 1. The theoretical results of single-particle energies and RMS radii for N=8 isotones (The single-particle energies are in unit of MeV).

	^{16}O	^{15}N	^{14}C	^{13}B	^{12}Be
$R_{\rm m}$/fm	2.73	2.71	2.71	2.73	2.79
$R_{\rm n}$/fm	2.70	2.73	2.77	2.84	2.95
$R_{\rm c}$/fm	2.76	2.70	2.61	2.53	2.42
$R_{\rm m}$(exp.)	2.63 ± 0.06		2.62 ± 0.06	2.75 ± 0.05	2.62 ± 0.07
$R_{\rm n}$(exp.)	2.59 ± 0.11		2.70 ± 0.10	2.93 ± 0.09	2.75 ± 0.11
$R_{\rm c}$(exp.)	2.72 ± 0.02	2.61 ± 0.01	2.56 ± 0.05		
$-\epsilon(1s_{1/2})$(p)	26.44	27.32	28.28	29.33	30.51
$-\epsilon(1p_{3/2})$(p)	13.27	13.69	14.13	14.62	15.15
$-\epsilon(1p_{1/2})$(p)	9.10	9.26			
$-\epsilon(1s_{1/2})$(n)	30.56	28.70	26.65	24.39	21.88
$-\epsilon(1p_{3/2})$(n)	16.93	15.03	12.98	10.79	8.45
$-\epsilon(1p_{1/2})$(n)	12.73	10.84	8.85	6.77	4.65

Table 2. The theoretical results of single-particle energies and RMS radii for N=9 isotones (The single-particle energies are in unit of MeV).

	^{17}O* $2s_{1/2}$	^{17}O $1d_{5/2}$	^{16}N* $2s_{1/2}$	^{16}N $1d_{5/2}$	^{15}C $2s_{1/2}$	^{15}C* $1d_{5/2}$	^{14}B $2s_{1/2}$	^{14}B* $1d_{5/2}$	^{13}Be $2s_{1/2}$	^{13}Be* $1d_{5/2}$
$R_{\rm m}$/fm	2.88	2.79	2.92	2.80	3.04	2.83	3.41			
$R_{\rm n}$/fm	3.00	2.85	3.11	2.89	3.31	2.98	3.82			
$R_{\rm c}$/fm	2.73	2.73	2.67	2.67	2.59	2.59	2.51			
$R_{\rm m}$(exp.)				2.71 (±0.28)	2.78 (±0.09)		3.00 (±0.10)			
$R_{\rm n}$(exp.)				2.79 (±0.50)	2.94 (±0.15)		3.27 (±0.16)			
$R_{\rm c}$(exp.)		2.66 (±0.03)								
$-\epsilon(1s_{1/2})$(p)	28.19	28.19	29.13	29.13	30.14	30.14	31.26	31.26	32.49	32.49
$-\epsilon(1p_{3/2})$(p)	15.07	15.07	15.55	15.55	16.08	16.08	16.64	16.64	17.26	17.26
$-\epsilon(1p_{1/2})$(p)	10.91	10.91	11.14	11.14						
$-\epsilon(1s_{1/2})$(n)	30.24	30.24	28.45	28.45	26.49	26.49	24.34	24.34	21.96	21.96
$-\epsilon(1p_{3/2})$(n)	16.99	16.99	15.17	15.17	13.21	13.21	11.12	11.12	8.90	8.90
$-\epsilon(1p_{1/2})$(n)	13.02	13.02	11.21	11.21	9.30	9.30	7.31	7.31	5.26	5.26
$-\epsilon(2s_{1/2})$(n)	2.95		1.92		1.01		0.32		−0.05	
$-\epsilon(1d_{5/2})$(n)		4.16		2.52		0.89		−0.71		−2.45

In order to describe the level spacing and order more directly, we present the theoretical neutron level scheme of N=9 isotones in Fig. 1. It is clear that the nucleon potential gradually decreases with the increase of neutron excess. The orders and spacing of inner neutron levels are almost unchanged for different nuclei. However, there are great changes in the level spacing and orders between $2s_{1/2}$ and $1d_{5/2}$. The details of the change of this level spacing are given in Table 3 and Fig. 2.

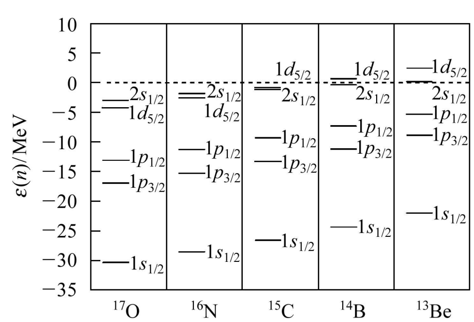

Fig. 1. Neutron levels of N=9 isotones.

In Table 3, we list theoretical values of neutron level spacing $[\epsilon(2s_{1/2})-\epsilon(1d_{5/2})]$ for N=9 isotones and the corresponding experimental data[15, 27, 28]. In order to see these more clearly, the trend of the variation is plotted in Fig. 2. It is seen that the level spacing is positive when neutron excess is small. With the increase of neutron excess, the spacing gradually diminishes and becomes negative when $N-Z$=3. In other words, there is the inversion of the neutron levels $2s_{1/2}$ and $1d_{5/2}$ in ^{15}C. Our prediction on ^{15}C spin-parity agrees with the experimental value $(1/2)^{+}$[1]. Based on the theoretical spacing which is negative when $N-Z$=4 and $N-Z$=5, we predict that there also exists level inversion in ^{14}B and ^{13}Be. Furthermore, it is evident that the global trend of theoretical variation of level spacing is consistent with the experimental fact.

Table 3. The difference of neutron single-particle energies between $2s_{1/2}$ and $1d_{5/2}$ for N=9 isotones.

	$N-Z$				
	1	2	3	4	5
Exp./MeV	0.87	0.15	−0.74		
Cal./MeV	1.20	0.60	−0.13	−1.03	−2.40

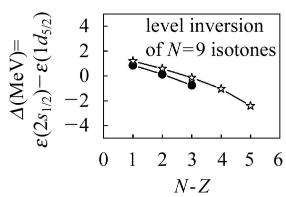

Fig. 2. The variation of neutron level spacing between $2s_{1/2}$ and $1d_{5/2}$ with neutron excess for N=9 isotones. The black circle represents the experimental value and the hollow star denotes the theoretical one.

From the known experiments[16—18], a halo or a skin has been suggested for ^{14}B and ^{15}C. It is noted from Table 2 that the RMS radii of neutron distribution for the ground states of ^{15}C and ^{14}B are much larger than that of ^{17}O and ^{16}N. So we plot the density distributions for the ground states of ^{17}O, ^{16}N, ^{15}C and ^{14}B in Fig. 3. The dash-dotted, dashed, solid and dotted curves represent the density distributions of matter, neutron, proton and the last neutron, respectively. It is evident that proton density distributions are similar for different nuclei. However, the spatial extensions of matter, neutron and the last neutron density distributions gradually extend with the increase of neutron excess. Long tails of matter density distributions emerge in ^{15}C and ^{14}B. All of these analyses show that the large spatial extension of matter density distribution comes from the contribution of the last neutron and there exist one-neutron halos in the ground states of ^{15}C and ^{14}B.

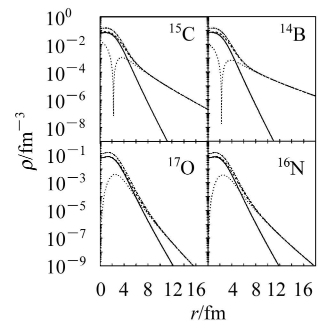

Fig. 3. The density distributions of matter (dash-dotted lines), neutron (dashed lines), proton (solid lines) and the last neutron (dotted lines) for the ground states of ^{17}O, ^{16}N, ^{15}C and ^{14}B.

4 Conclusion

With the single-particle shell model, we studied the ground-state properties of N=8 and N=9 isotones with Z=4—8. An isospin-dependent term is introduced in the depth of the Woods-Saxon potential. The theoretical results of RMS radii and spin-parity values are in agreement with the experimental data. In particular, the level inversion between neutron levels $2s_{1/2}$ and $1d_{5/2}$ in ^{15}C, ^{14}B and ^{13}Be is reproduced. The global tendency of the level spacing agrees with the observed fact. In addition, our study implies that the ^{15}C and ^{14}B are one-neutron halo nuclei which deserve further experimental investigation in China.

It is noted that the reasonable results of level inversion can be obtained when the isospin-dependent term is added in the depth of the Woods-Saxon potential. This may imply that the isospin effect is an important factor for the change of average nuclear potential of light nuclei far from stability. In order to obtain comprehensive understanding of the isospin effect, it is interesting to study other light nuclei by using the above method and this work is in progress.

References

1 Audi G, Bersillon O, Blachot J et al. Nucl. Phys., 2003, **A729**: 3—128; Audi G, Wapstra A H, Thibault C. Nucl. Phys., 2003, **A729**: 337—676

2 CAI X Z, ZHANG H Y, SHEN W Q et al. Phys. Rev., 2002, **C65**: 024610

3 LIU Z H, RUAN M, ZHAO Y L et al. Phys. Rev., 2004, **C69**: 034326

4 ZHANG H Y, SHEN W Q, REN Z Z et al. Nucl. Phys., 2002, **A707**: 303—324

5 Utsuno Y, Otsuka T, Glasmacher T et al. Phys. Rev., 2004, **C70**: 044307

6 Nummela S, Nowacki F, Baumann P et al. Phys. Rev., 2001, **C64**: 054313

7 Maréchal F, Balabanski D L, Borremans D et al. Phys. Rev., 2005, **C72**: 044314

8 Tripathi V, Tabor S L, Mantica P F et al. Phys. Rev. Lett., 2005, **94**: 162501

9 Markenroth K, Axelsson L, Baxter S et al. Phys. Rev., 2000, **C62**: 034308

10 REN Z Z, CHEN B Q, MA Z Y et al. Z. Phys., 1997, **A357**: 137—141

11 ZHANG H Y, SHEN W Q, REN Z Z et al. Nucl. Phys., 2003, **A722**: 518c—522c

12 PEI J C, XU F R, Stevenson P D. Nucl. Phys., 2006, **A765**: 29—38

13 LIU Jian-Ye, GUO Wen-Jun, REN Zhong-Zhou et al. HEP & NP, 2006, **30**(4): 317—321 (in Chinese)
(刘建业, 郭文军, 任中洲等. 高能物理与核物理, 2006, **30**(4): 317—321)

14 Bohr A, Mottelson B R. Nuclear Structure (Vol. I). Singapore: World Scientific Publishing Co. Pte. Ltd., 1998. 144—149, 236—240

15 Fortune H T. Phys. Rev., 1995, **C52**: 2261—2263; Fortune H T, Sherr R. Phys. Rev., 2005, **C72**: 024319

16 Ozawa A, Tanihata I, Kobayashi T et al. Nucl. Phys., 1996, **A608**: 63—76

17 FANG D Q, SHEN W Q, FENG J et al. Phys. Rev., 2000, **C61**: 064311

18 WU Cui-E, YE Yan-Lin, JIANG Dong-Xing et al. HEP & NP, 2005, **29**(11): 1052—1056 (in Chinese)
(吴翠娥, 叶沿林, 江栋兴等. 高能物理与核物理, 2005, **29**(11): 1052—1056)

19 LIU Z H, ZHANG X Z, ZHANG H Q. Phys. Rev., 2003, **C68**: 024305

20 Satchler G R. Direct Nuclear Reactions. New York: Oxford University Press, 1983. 455—460, 713

21 LIN C J, ZHANG H Q, LIU Z H et al. Phys. Rev., 2002, **C66**: 067302

22 WANG Zai-Jun, REN Zhong-Zhou. Science in China, 2003, **G33**: 385—392 (in Chinese)
(王再军, 任中洲. 中国科学, 2003, **G33**: 385—392)

23 de Vries H, de Jager C W, de Vries C. At. Data Tables Nucl. Data Tables, 1987, **36**: 495—536

24 Liatard E, Bruandet J F, Glasser F et al. Europhys. Lett., 1990, **13**: 401—404

25 Saint-Laurent M G, Anne R, Bazin D et al. Z. Phys., 1989, **A332**: 457—465

26 REN Z Z, XU G O, CHEN B Q et al. Phys. Lett., 1995, **B351**: 11—17

27 Tilley D R, Weller H R, Cheves C M. Nucl. Phys., 1993, **A564**: 1—183

28 Ajzenberg-Selove F. Nucl. Phys., 1991, **A523**: 1—196

同位旋效应对 $N=9$ 同中子素能级间隙的影响*

郭艳青[1]　任中洲[1,2;1)]

1 (南京大学物理系　南京　210008)
2 (兰州重离子加速器国家实验室原子核理论中心　兰州　730000)

摘要　在单粒子壳模型下, 研究了 N=8 及 N=9 同中子素的基态性质. 通过在伍兹–萨克逊势深中引入同位旋依赖项, 考虑了同位旋效应对核平均势的影响. 理论计算的均方根半径和自旋宇称值与实验结果整体上符合较好, 尤其是 N=9 同中子素中的 $2s_{1/2}$ 与 $1d_{5/2}$ 中子能级间的能级反转得到了较好的解释. 计算结果还显示在 ^{14}B 和 ^{15}C 基态中存在单中子晕.

关键词　壳模型　同位旋效应　能级反转

2006 – 08 – 03 收稿,　2006 – 11 – 16 收修改稿

∗ 国家自然科学基金(10125521, 10535010)和国家重点基础研究发展规划项目(G2000077400)资助

1) E-mail: zren@nju.edu.cn