Short range correlations between nucleons in finite nuclei*

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Abstract The short-range correlation between nucleons in finite nuclei is investigated in high energy proton-nucleus and α-nucleus elastic scattering in the framework of Glauber multiple scattering theory without any free parameters. The effects on the p-^{4}He and ^{4}He-^{12}C elastic scattering, and in particular on the proton elastic scattering off hallo-like nuclei, ^{6,8}He, are estimated. Our calculations show that the short-range correlations play an important role in reproducing experimental data and could be also thought of as being possible origin and nature of halo-like phenomena in the nuclear structure. More accurate calculations along this line are needed.

Key words nuclear force, short-range correlations, elastic proton scattering, halo-like nucleus

PACS 25.40.Cm, 28.75.Gz, 21.60.-n

1 Introduction

In many-body physics the word correlation indicates effects beyond Mean-Field Theories. In nuclear physics it is common to distinguish the short range correlation from the long range ones. Nuclear collective phenomena such as vibrations and rotations are ruled by long-range correlations. These effects are well known and studied since the infancy of nuclear physics. On the contrary the study of the short range correlations is a relatively new issue[1] in nuclear physics, in particular, nobody uses the idea to explain the nuclear halo-like phenomena. This correlation is produced by the strong repulsive core of the microscopic nucleon-nucleon interaction at short inter-nucleon distances. In spite of the fact that all the microscopic nuclear theories need the short range correlation, clear signatures of their presence in nuclei have not yet been identified.

In this paper, we discuss the importance of short range nuclear correlations in finite nucleus. To show the importance we make some parameter free calculations of proton-nucleus and nucleus- nucleus elastic scattering at intermediate energies, in particular proton elastic scattering on halo-like nucleus. In Sect. 2, we briefly introduce the description of the short range correlations and some related formulae used in this calculations. In Sect. 3, we present our numerical calculations and theoretical results. Some discussions on the current predictions are also given in this section. We reserve our summary and concluding remarks for Sect. 4.

2 Nuclear wave function with short range correlations

The many-body nuclear state can be described within the framework of the Correlated Basis Function Theory[2] by the wave function

$$|\Psi\rangle = F\,|\Phi_0\rangle\,, \tag{1}$$

where $|\Phi_0\rangle = \Psi_{\rm SD}(\boldsymbol{r}_1,\cdots,\boldsymbol{r}_A)$ indicates a ground state Slater determinant constructed with a set of orthonormal single particle wave functions $|\phi\rangle$, and F

Received 8 January 2007
* Supported by National Natural Science Foundation of China (10565001, 10647002) and Science Foundation of Guangxi (0481030, 0542042, 0575020)
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is a many-body correlation function defined as

$$F = S\prod_{i<j}\{\sum_{p=1}^{n} f^p(r_{ij})\cdot O^p(i,j)\}, \tag{2}$$

where S in Eq. (2) indicates a symmetrizer operator. The two-body correlation functions $f^p(r_{ij})$ have an operatorial dependence of the same type of that of the nucleon - nucleon interaction. In the present calculations we consider correlations up to $p=8$ of the type:

$$O^{p=1,8}(i,j) = [1,\boldsymbol{\sigma}(i)\cdot\boldsymbol{\sigma}(j),\boldsymbol{S}(i,j),\boldsymbol{L}(i,j)\cdot\boldsymbol{S}(i,j)]\otimes [\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)], \tag{3}$$

where $S(i,j)$ is the usual tensor operator. If we only consider the scalar contribution of Eq. (3), the nuclear wave functions of Eq. (1) with two-body correlations between nucleons in a finite nuclei become[3] to

$$|\Psi\rangle \equiv |\Psi\rangle_{JC} = \prod_{i>j=1}^{A} f(r_{ij})\,|\Phi_0\rangle\,. \tag{4}$$

The wave function of Eq. (4) is called Jastrow wave function[3] of nuclear structure and dubbed as $|\Psi\rangle_{JC}$. This wave function gives a good description of the nuclear short range correlation in finite nuclei. The Slater determinant $|\Phi_0\rangle$ in Eqs. (1,4) takes the Pauli effect into consideration. $f(r_{ij})$ represents nuclear two-body short range correlation factor and satisfies

$$f(r_{ij}) = \begin{cases} 0, & \text{if} \quad r_{ij} \leqslant h, \\ 1, & \text{if} \quad r_{ij} > h. \end{cases} \tag{5}$$

where $r_{ij} = |\,\boldsymbol{r}_i - \boldsymbol{r}_j\,|$ is inter-nucleon separation and h is referred to the so-called "healing distance". Clearly, when the inter-nucleon separation r_{ij} is less than the healing distance h the nuclear wave function $|\Psi\rangle_{JC}$ becomes zero. That means there is a strong repulsive core of the microscopic nucleon-nucleon interaction at short inter-nucleon distance which prevents nucleons from getting closer together.

It has been proven that Jastrow wave function, $|\Psi\rangle_{JC}$ in Eq. (4), could be rewritten as a new modified Slater determinant[4]

$$|\Psi\rangle_{JC} = \frac{1}{\sqrt{A!}}\,\|\tilde{\phi}_\alpha\|, \tag{6}$$

with

$$\tilde{\phi}_\alpha(r_1) = \phi_\alpha(r_1)\Big\{1-\sum_{\alpha\neq\beta}\langle\beta(2)\,|\,g(r_{12})\,|\,\beta(2)\rangle + \sum_{\alpha\neq\beta\neq\gamma}\langle\beta(2)\gamma(3)\,|\,g(r_{12})g(r_{13})+\cdots\,|\,\beta(2)\gamma(3)\rangle+\cdots\Big\}, \tag{7}$$

where $g(r_{ij})$ is the so-called "auxiliary correlation factor", and $g(r_{ij}) = |\,f(r_{ij})\,|^2 - 1$. $g(r_{ij})$ is zero outside the region in which $f(r_{ij})$ differs from zero. As the usual way, we take $g(r_{ij}) = J_0(q_c r)$ with q_c being correlation parameter and is taken to be $q_c = 300$ MeV/c in this calculation. For light nuclei, calculation of Eq. (7) leads us to the following expressions

$$\tilde{\phi}_{1s}(r) = A_{1s}\phi_{1s}(r) + A_{2s}\phi_{2s}(r) + A_{3s}\phi_{3s}(r) + \cdots\,, \tag{8}$$

$$\tilde{\phi}_{1p}(r) = A_{1p}\phi_{1p}(r) + A_{2p}\phi_{2p}(r) + A_{3p}\phi_{3p}(r) + \cdots\,. \tag{9}$$

Eqs. (8) and (9) show that the short range correlation is just an effect which caused an admixture of different states with identical orbital quantum number (l) but different principal quantum numbers (N). To a good approximation for light nuclei, Eqs. (8,9) can be truncated and then expressed as

$$\tilde{\phi}_{1s}(r) = A_{1s}\phi_{1s}(r) + A_{2s}\phi_{2s}(r)\,, \tag{10}$$

$$\tilde{\phi}_{1p}(r) = A_{1p}\phi_{1p}(r) + A_{2p}\phi_{2p}(r)\,. \tag{11}$$

with $A_{1s} = 0.9624$, $A_{2s} = 0.2719$, $A_{1p} = 0.9431$ and $A_{2p} = 0.3324$ which had been determined by us in our early publications of Refs. [3,4].

3 Numerical calculations and results

We show the importance of nuclear short range correlations in finite nuclei by calculating proton-nucleus and nucleus-nucleus elastic scattering at intermediate energies. In particular, we repay our attentions on the scattering of proton on the halo-like nucleus ^{6,8}He which may reveal the dynamical origin and nature of halo-like phenomena. We start by recalling the formalism of Glauber multiple scattering theory[5], which is the framework of our current investigation. For hadron-nucleus scattering in Glauber theory with Coulomb effect taken into account, the relevant scattering amplitude can be written as

$$F_{\rm fi}(\boldsymbol{q}) = F_{\rm c}(\boldsymbol{q})\delta_{\rm fi} + H_{\rm c.m.}(\boldsymbol{q})\frac{{\rm i}k}{2\pi}\int {\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{b}}\Gamma_{\rm fi}^{\rm c}(\boldsymbol{b}){\rm d}^2\boldsymbol{b}\,, \tag{12}$$

where $F_{\rm c}(\boldsymbol{q})$ is the Coulomb scattering amplitude for a point charge target and is given by Refs. [6,7]

$$F_{\rm c}(\boldsymbol{q}) = -2\xi\frac{k}{q^2}{\rm e}^{{\rm i}\phi_{\rm c}}\,, \tag{13}$$

with

$$\phi_{\rm c} = -2\xi\ln\left(\frac{q}{2k}\right) + 2\eta, \quad \xi = -Z\alpha\frac{m}{k}\,. \tag{14}$$

Here Z is the charge number of the target nucleus, m is the nucleon mass, $\alpha = 1/137$ and k denotes the initial wave number in the center-of-mass system. The quantity η is defined by

$$\eta = \arg\Gamma(1+{\rm i}\xi) = \xi\psi(x=1) + \sum_{n=0}^{+\infty}\left(1+\frac{\xi}{1+n} - \arctan\frac{\xi}{1+n}\right), \tag{15}$$

where $\psi(x)$ is the digamma function and $\psi(x=1)=-0.57721$. The $\Gamma(1+\mathrm{i}\xi)$ in Eq. (15) is Euler's gamma function. The notation $\arg$ stands for the principal argument of a complex number $\Gamma(1+\mathrm{i}\xi)$.

$H_{\mathrm{c.m.}}(q)=\exp(q^2/4A\alpha')$ is the center-of- mass correction factor[7] with α' being harmonic oscillator parameter of the target nucleus.

The Coulomb corrected profile function $\Gamma^c_{\mathrm{fi}}(\boldsymbol{b})$ in Eq. (12) is defined by

$$\Gamma^c_{\mathrm{fi}}(\boldsymbol{b})=\mathrm{e}^{\mathrm{i}x_{\mathrm{c}}(\boldsymbol{b})}\delta_{\mathrm{fi}}-\mathrm{e}^{\mathrm{i}\tilde{x}_{\mathrm{c}}(\boldsymbol{b})}\delta_{\mathrm{fi}}-\Gamma_{\mathrm{fi}}(\boldsymbol{b}), \qquad (16)$$

where x_{c} and $\tilde{x}_{\mathrm{c}}$ are Coulomb phase shift for a point-like charge and an extended charge distribution, respectively. They are given by

$$x_{\mathrm{c}}(b)=2\xi\ln(kb), \qquad \tilde{x}_{\mathrm{c}}(b)=x_{\mathrm{c}}(b)+\bar{x}_{\mathrm{c}}(b), \quad (17)$$

with

$$\bar{x}_{\mathrm{c}}(b) = 2\xi 4\pi b^3\int_0^1 \mathrm{d}x\frac{1}{x^4}\rho_{\mathrm{c}}\left(\frac{b}{x}\right)\times \left[\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)-\sqrt{x-x^2}\right], \quad (18)$$

where $\rho_{\mathrm{c}}(b/x)$ is the charge distribution of the target nucleus. For instance, three parameters Fermi distribution,

$$\rho_{\mathrm{c}}(r)=\rho_0\frac{1+wr^2/c^2}{1+\mathrm{e}^{(r-c)/Z'}}, \qquad (19)$$

is widely used with parameters c, Z', w which vary as the change of nucleus and are given in Table 1[7], i.e.

Table 1. The parameters c, Z', and w for various nucleus: $A=4$, 12, 15, 16, and 40.

nucleus	c	Z'	w
^{4}He	0.964	0.323	0.517
^{12}C	2.355	0.522	−0.149
^{15}N	2.334	0.498	0.139
^{16}O	2.608	0.513	−0.051
^{40}Ca	3.766	0.586	−0.161

The nuclear charge density distribution, $\rho_{\mathrm{c}}(r)$, has been accurately determined by many experiments of electron-nucleus scattering. Given $\rho_{\mathrm{c}}(r)$, the $\bar{x}_{\mathrm{c}}(b)$ can be easily obtained by carrying out numerically the integration over $\boldsymbol{b}$ in Eq. (18).

$\Gamma_{\mathrm{fi}}(\boldsymbol{b})$ in Eq. (16) is the nuclear profile function and defined by the following identity (see Ref. [4]).

$$\Gamma_{\mathrm{fi}}(\boldsymbol{b})=\langle\psi_{\mathrm{f}}\,|\,1-\prod_{j=1}^{A}\left[1-\Gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)\right]\,|\,\psi_{\mathrm{i}}\rangle, \qquad (20)$$

where ψ_{f} and ψ_{i} are the final and initial nuclear state wave functions, respectively. The $\Gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)$ is the two-dimensional Fourier transform of elementary two-body scattering amplitude $f_j(\boldsymbol{q})$,

$$\Gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)=\frac{1}{2\pi\mathrm{i}k}\int\mathrm{e}^{-\mathrm{i}\boldsymbol{q}(\boldsymbol{b}-\boldsymbol{s}_j)}f_j(\boldsymbol{q})\mathrm{d}^2\boldsymbol{q}. \qquad (21)$$

In the case of proton - nucleus scattering, the j denotes projectile scattering off the j-th nucleon in target. Of course, $f_j(\boldsymbol{q})$ is of spin-isospin dependence two-body amplitude. For our present purpose we neglect the spin-flip parts of the two-body amplitude $f_j(\boldsymbol{q})$, and parameterize the central part of $f_j(\boldsymbol{q})$ as

$$f_j(\boldsymbol{q})=\frac{\mathrm{i}k\sigma_j}{4\pi}(1-\mathrm{i}\rho_j)\mathrm{e}^{-\beta_j^2q^2/2}, \qquad (22)$$

where σ_j, ρ_j and β_j are respectively total cross section, ratio of the real-to-imaginary part of forward scattering amplitude, and slope parameter of the amplitude for two body scattering. They are of energy-dependence, and determined by experiment. To perform the calculation of the scattering amplitude, sufficiently accurate values of these parameters are needed since they are the fundamental input data. In this work, the values of σ_{pp} and ρ_{pp} have been deduced by interpolation of the results from a free pp scattering phase shift analysis[8]. For the pn scattering, the available data on the elementary cross sections are more scarce and even partly inconsistent. We choose the values for σ_{pn} and ρ_{pn} from Ref. [9]. The slope parameters β_{pn} are obtained from Ref. [8]. No difference has been made between β_{pp} and β_{pn}, and they are taken to be $\beta_{\mathrm{pp}}=\beta_{\mathrm{pn}}=0.17\ \mathrm{fm}^2$. All the parameters used in this calculation are listed in Table 2.

Table 2. The parameters of the pn scattering amplitude used in the present calculations. E is the incident particle energy.

target nucleus	E/GeV	σ_{pp}/mb	ρ_{pp}	σ_{pn}/mb	ρ_{pn}	$\beta_{\mathrm{pp}}=\beta_{\mathrm{pn}}$/fm^2
^{4}He	0.702	43.5	0.095	37.6	−0.297	0.17
^{6}He	0.721	44.6	0.069	37.7	−0.307	0.17
^{8}He	0.678	41.9	0.129	37.4	−0.283	0.17
^{12}C	1.370	43.2	0.230	37.5	−0.290	0.17

Using the Jastrow wave function given by Eq. (6), the nuclear matrix elements in the Glauber amplitude, Eq. (20), can be expressed as

$$\left\langle\psi_{JC}\,|\,1-\prod_{j\neq1}^{A}[1-\Gamma_j(\boldsymbol{b}-\boldsymbol{s}_j)]\,|\,\psi_{JC}\right\rangle=1-\|\tilde{O}_{nm}\|, \qquad (23)$$

where

$$\tilde{O}_{nm}=\delta_{nm}-\int\tilde{\phi}^*_m(\boldsymbol{r})\Gamma(\boldsymbol{b}-\boldsymbol{s})\tilde{\phi}_n(\boldsymbol{r})\mathrm{d}^3\boldsymbol{r}. \qquad (24)$$

where $\tilde{\phi}$ are given by Eqs. (10,11). Using all ingredients discussed above the total elastic scattering amplitude $F_{ii}(\boldsymbol{q})$ for proton-nucleus elastic scattering ($f=i$), and consequently the differential cross section can be obtained from Eq. (12) and definition of $\mathrm{d}\sigma/\mathrm{d}t=|F_{ii}(q)|^2$.

For ^{4}He-^{12}C elastic scattering, the scattering amplitude given by Glauber theory can be expressed as

$$F_\alpha(q) = \frac{\mathrm{i}k}{2\pi}\int \mathrm{d}^2 b \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{b}}\{1-\langle\Psi(^{12}\mathrm{C})\,|\times \prod_{j\in{}^{12}\mathrm{C}}\langle\Psi(^4\mathrm{He})\,|\prod_{k\in\alpha}[1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_j+\boldsymbol{s}_k)]\times |\,\Psi(^4\mathrm{He})\rangle\,|\,\Psi(^{12}\mathrm{C})\rangle\}. \tag{25}$$

where $\boldsymbol{s}_j$ is the projected vector of the j-th nucleon space coordinate vector $\boldsymbol{r}_j$ which is perpendicular to the incident plane. Using this formulism the differential cross section for ^{4}He-^{12}C can be predicted numerically.

Within the framework of Glauber multiple scattering theory and neglecting the spin-flip effect, we made parameter free calculations of p-^{4}He and ^{4}He-^{12}C elastic scattering at intermediate energy region. The theoretical predictions are shown in Figs. 1—2.

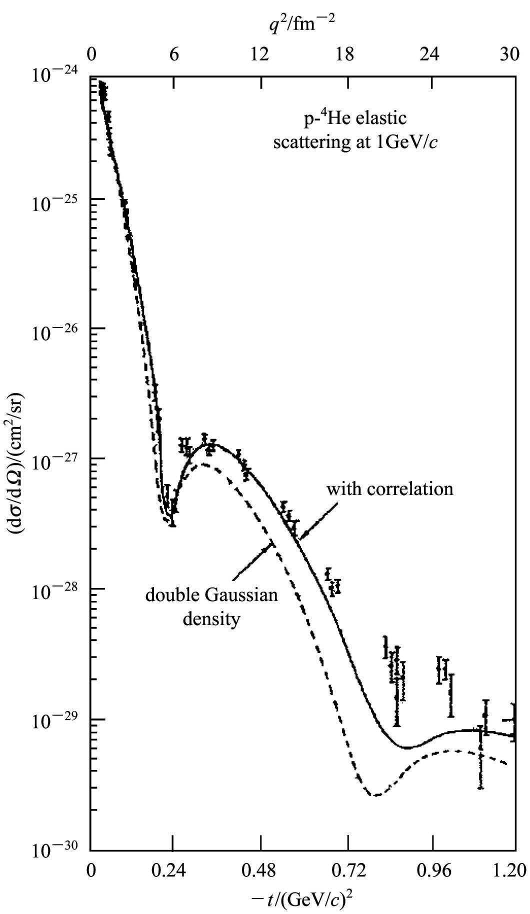

Fig. 1. Effect of nuclear short range correlation on the differential cross section of p-^{4}He elastic scattering at the incident proton energy of 1.0 GeV. The solid curve is our prediction of the short range correlation wave function, Eq. (6), while the dashed line denotes the prediction by use of double Gaussian nuclear wave function of ^{4}He. The data come from Ref. [10].

As we can see from Figs. 1—2, the short range correlation clearly plays an essential role in fitting experimental data. Particularly, our theoretical predictions on ^{4}He-^{12}C elastic scattering at the energy of 1.37 GeV in Fig. 2 evidently show the importance of the nucleon short range correlation. The Jastrow wave functions with the short range correlations between nucleons taken into consideration reproduce the data very well. We got an excellent agreement with data until the 3-rd maximum of the differential cross section of ^{4}He-^{12}C elastic scattering.

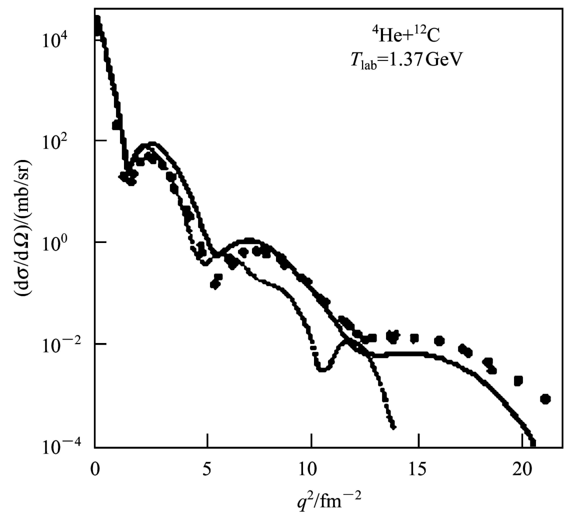

Fig. 2. Differential cross section of ^{4}He-^{12}C heavy ion elastic scattering at the incident energy of 1.37 GeV. The solid curve stands for the prediction of Jastrow wave function, Eq. (6), while the dotted curve denotes the prediction without short range correlation taken into consideration. The data come from Ref. [11].

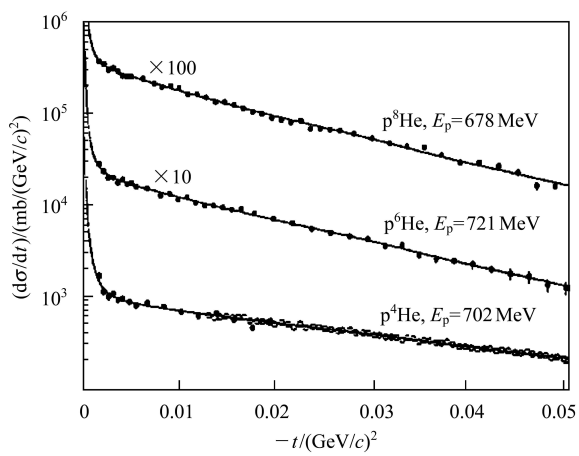

Fig. 3. Differential cross section $\mathrm{d}\sigma/\mathrm{d}t$ versus the four momentum transfer squared t for p-^{4}He, p-^{6,8}He elastic scattering at the proton Laboratory energies of $E_\mathrm{p}=0.702$ GeV, 0.721 GeV and 0.678 GeV, respectively. The solid curves are the results predicted by Jastraw wave function, Eq. (6), while the solid points are experimental data[10].

In order to emphasize once again the importance of the nuclear short range correlations, in Fig. 3 we re-show our previous calculations of differential cross sections of the proton elastic scattering at the energy about 1 GeV on the halo-like nuclei ^{6,8}He[3b]. As is seen from Fig. 3, we obtained perfect fits to experimental data available without using any free parameters. Recently, it has been claimed by many that proton-nucleus elastic scattering at the energy region of about 1 GeV is a perfect tool[7] to study halo-like phenomena. Therefore, our excellent fit to the data may indicate that the dynamical origin of halo-like phenomena could be the short range correlations between nucleons in finite halo-like nuclei.

4 Summary and concluding remarks

We study the importance of nucleonic short range correlations in finite nucleus in proton - nucleus and α-nucleus elastic scattering at intermediate energies within the framework of Glauber theory without any free parameters. Our parameter free calculations show that the short range correlations between nucleons in target nucleus play an important role in reproducing experimental data, in particular at high momentum transfers. The data on all He isotopes are well described in terms of the Jastrow wave function which takes the nuclear short range correlations into account. All the results evidently show that this effect must be included in an accurate calculation of any observable, and that the possible origin of an extended neutron halos in ^{6,8}He. For the later, the reason is due to the fact that the short range nuclear force causes an mixture of different states with identical orbital quantum number (l) but different principal quantum numbers (N) which makes nucleus extended largely so that it has a larger size comparing with that predicted by the theory of nuclear structure without considering the short range correlation as done so far. Therefore, it might be possible to claim that nuclear short range correlation could be the origin and nature of the halo-like phenomena of nuclear structure. This may be a very important result for halo-like nucleus study.

References

1 Bertulani C A. Short -Range Correlations in Two Nucleon Knockout Reactions. arXiv: nucl-th / 0504048; Barbieri C, Dickhoff W H. Phys. Rev., 2001, **C63**: 034313; Barbieri C, Dickhoff W H. Phys. Rev., 2002, **C65**: 064313; Guardiola R, Oset E. Nucl. Phys., 1974, **A234**: 458—468

2 Preston M A. Physics of the Nucleus. Reading, Massachusetts, PALO ALTO, London: Addison-Wesley Publishing Company INC., 1962, 215; Clark J W. Prog. Part. and Nucl. Phys., 1979, **2**: 89

3 a) Jastrow R. Phys. Rev., 1951, **81**: 165; Jastrow R. Phys. Rev., 1955, **98**: 1479; b) ZHOU Li-Juan et al. Commun. Theor. Phys., 2004, **42**(1): 90—94

4 LU Shi-Qi, WANG Ru-Lin, MA Wei-Xing. HEP & NP, 1981, **5**(3): 374—377 (in Chinese)

5 Glauber R J. Lecture in Theoretical Physics (Vol. 1). Eds. Britten W E, Danham L G. New York: Interscience, 1959. 315

6 MA Wei-Xing, Strottman D D. Phys. Rev., 1991, **C44**: 615

7 ZHOU Li-Juan et al. Commun. Theor. Phys., 2005, **43**: 699—703

8 Arndt R A, Ropa L D, Bryan R A et al. Phys. Rev., 1983, **D28**: 97

9 Devlin T J, Johnson W, Noren J et al. Phys. Rev., 1973, **D8**: 136; Grein W, Kroll P. Nucl. Phys., 1982, **A377**: 505; Silverman B H, Lugol J C, Saudinos J et al. Nucl. Phys., 1989, **A499**: 763

10 Alkhazov G D, Andronenko M N, Dobrovolsky A D et al. Phys. Rev. Letts., 1997, **78**(12): 2313—2316

11 Alkhazov G D et al. Phys. Letts., 1972, **B42**: 121; Bertini R et al. Phys. Letts., 1973, **B45**: 119; Alkhazov G D et al. JETP Lett., 1973, **10**: 101; Starodubsky V E. Leningrad Nucleaar Physics Inst., 1974, **2**: 229