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Three dimensional parametrization of electromagnetic shower in Alpha Magnetic Spectrometer Ⅱ ECAL*

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Abstract We develop an empirical formula to parameterize the 3-dimension (3D) distribution of electromagnetic showers in the Alpha Magnetic Spectrometer Ⅱ electromagnetic calorimeter(ECAL). The formula was verified by ECAL test beam data in 2002 and found to perform well. The distribution of electron showers in the ECAL are well described by the formula, which has parameters that allow one to determine the 3D shape of electromagnetic showers in the ECAL. We use this formula to correct for lateral energy leakage and dead channels in the ECAL; good results are obtained.

Key words 3-D parametrization, ECAL, test beam, empirical formula

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1 Introduction

The Alpha Magnetic Spectrometer Ⅱ is a large acceptance particle physics detector designed to be placed and operated on the International Space Station (ISS)[1]. The Electromagnetic Calorimeter (ECAL) of the Alpha Magnetic Spectrometer Ⅱ experiment is a fine grained lead-scintillating fiber sampling calorimeter, which allows precise 3-D imaging of the longitudinal and lateral shower development, providing high e/h discrimination and good energy resolution[2, 3].

The average longitudinal profile of electromagnetic showers can be well described by the Γ-function[4], and the average lateral shape has been examined by many authors[5—9]. But these are all 2 dimensional descriptions. We developed a formula which well describes the 3-dimensional distribution of electromagnetic showers. This formula is verified by the electron test beam data in 2002; the performance of the formula is satisfactory.

In this paper, the ECAL and its 2002 beam test are described in section 2. An empirical 3D-parameterized formula for ECAL electromagnetic showers and the verification of this formula are described in section 3. In section 4, two applications of the parameterized formula including corrections of energy leakage and dead channels are given. The work is summarized in section 5.

2 ECAL and test beam

The ECAL calorimeter is a sampling detector with a lead-scintillating fibers sandwiched structure, it has an active area of 648 mm×648 mm and a thickness of 165 mm, corresponding to ~16 radiation lengths X_0. The calorimeter consists of 9 superlayers with 4 superlayers in x the direction and 5 in the y direction, as shown in Fig. 1(a). Fibers are oriented in the x and y direction, and the signal are read out by photomultipliers (PMT). Each photomultiplier has a cathodic effective area of 18 mm×18 mm subdivided into four square regions of equal area (2×2 pixels), the resulting granularity is, therefore, 9 mm×9 mm; each superlayer is divided into 2 layers. There are a total of 18 depth layers (10 in y and 8 in x direction). As shown in Fig. 1(b), the region covered by one of the four PMT windows is called a cell, the size of one cell corresponds to ~0.5 $R_{\rm m}$ (Molliere radius) both in the

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x and the y directions. A more detailed description of the ECAL can be found in Refs. [2,3] etc.

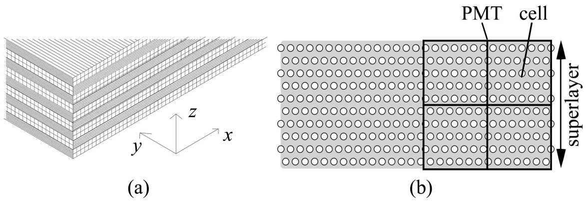

Fig. 1. Structure of Alpha Magnetic Spectrometer II ECAL. (a) The superlayer assembly; (b) the structure of a portion of a superlayer with one PMT.

In July 2002, an engineering model of the ECAL was tested in the CERN SPS H6 beam line using beams of μ, 120 GeV p/$\overline{\mathrm{p}}$, and 3 to 180 GeV e^-. The electron energy values were 3, 6, 10, 15, 20, 30, 35, 50, 80, 120, 150 and 180 GeV. The calorimeter was partially equipped with 7 PMTs per layer, corresponding to 14 cells, in a 126 mm×126 mm corner. As shown in Fig. 2, the dark region in the left of Fig. 2 is the region where the test beam impinged on the calorimeter, the z axis is taken as the beam direction. The right part in Fig. 2 is an enlarged view of the test region with cells numbered from 0 to 13 both in the x and y directions; the 2 dark regions are used for the verification of our empirical formula. A detailed description and the results of this beam test can be found in Refs. [10—14].

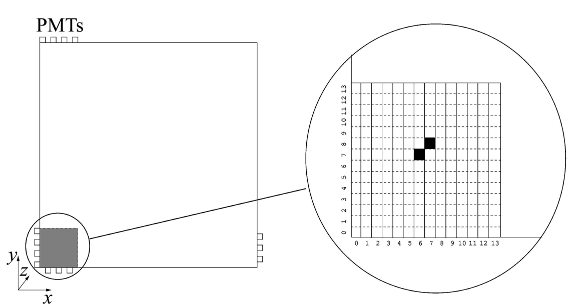

Fig. 2. Schematic view of the test beam setup.

3 3D-parametrization of electromagnetic shower

3.1 Longitudinal shape of electromagnetic shower

The mean longitudinal profile of the energy deposition in an electromagnetic cascade is well described by a Γ-function[15]:

$$\frac{\mathrm{d}E}{\mathrm{d}t}=E_0 b\frac{(bt)^{a-1}\mathrm{e}^{-bt}}{\Gamma(a)}\,, \tag{1}$$

where the shower depth t is measured in units of radiation lengths X_0. E_0, a and b are parameters to be fitted. E_0 equals the incident energy. Parameters a and b are dependent on the atomic number Z of the absorber and the incident beam energy.

3.2 Lateral shape of electromagnetic shower

The lateral profile of an electromagnetic shower can be described by the function[8]:

$$f(r)=\frac{2rR^2}{(r^2+R^2)^2}\,, \tag{2}$$

where R is a phenomenological function of x/X_0 and $\ln E$. Other formulae including the sum of two Gaussians etc.[5—7] have also been studied. But none of them is three dimensional.

3.3 Empirical formula of lateral shower shape in a layer

We propose empirical formula to describe the lateral shower shape in the ECAL :

$$\left.\frac{\mathrm{d}^2E}{\mathrm{d}x\mathrm{d}y}\right|_{\mathrm{layer}}=\frac{3}{\pi}\cdot\frac{E_{\mathrm{layer}}\cdot R^2_{\mathrm{layer}}}{(r+R_{\mathrm{layer}})^4}\,,$$
$$r=\sqrt{(x-x_{\mathrm{c}})^2+(y-y_{\mathrm{c}})^2}\,, \tag{3}$$

where E_{layer}, R_{layer}, x_{c} and y_{c} are parameters to be fitted layer-by-layer. E_{layer} represents the fitted result for the energy deposit in that layer. x_{c} and y_{c} represent the shower's center of gravity in the layer, and are in unit of cell (1 cell = 9.25 mm, corresponding to about 0.5 R_{m}).

3.4 Verification and parameters determination

Electron runs with incident energies ranging from 3 to 180 GeV were used to verify the formula. To avoid lateral leakage, only runs with the beam incident in the center of the test region (the black cells of the right part in Fig. 2) are used.

A χ^2 minimization fit is done using the MINUIT[16] minimization package. The fit minimizes χ^2 given by

$$\chi^2=\sum_{\mathrm{cell}=0}^{13}(E^{\mathrm{fitted}}_{\mathrm{cell}}-E^{\mathrm{deposited}}_{\mathrm{cell}})^2\,, \tag{4}$$

where $E^{\mathrm{fitted}}_{\mathrm{cell}}$ is the expected energy of the ith cell as predicted by formula 3 and $E^{\mathrm{deposited}}_{\mathrm{cell}}$ is the energy deposited in that cell. Simpson's integration formula is used to calculate the deposited energy in each cell.

Some typical results from the fit are shown in Fig. 3. Here E_{average} designates the average energy deposit in each cell over the entire sample of selected events at a given beam energy. The fitted results agree well with the test beam data for the center cells, but small excesses are seen for cells that far from the shower's center of gravity.

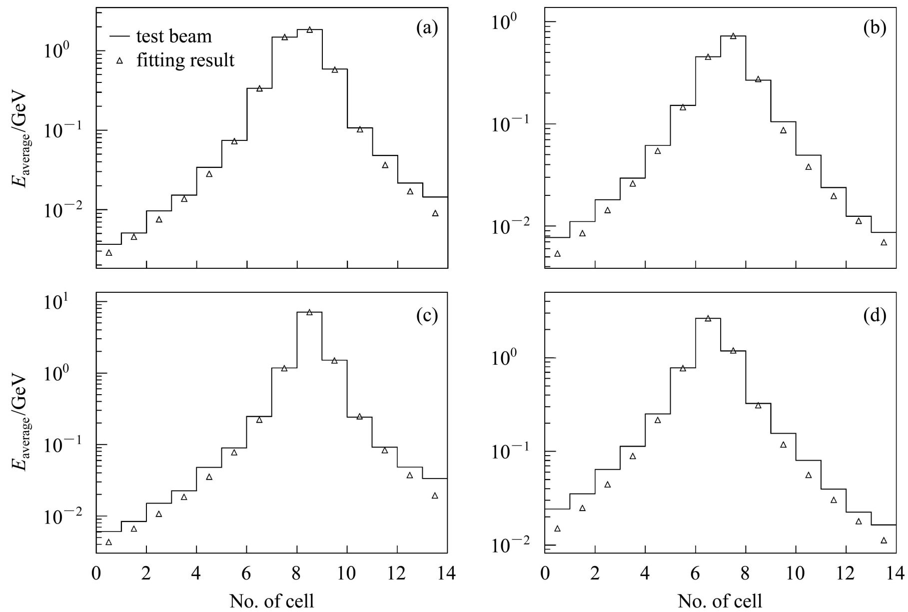

Fig. 3. Fit results for the energy in the (a) the 9th layer of 50 GeV; (b) the 15th layer of 50 GeV; (c) the 9th layer 120 GeV; (d) the 15th layer of 120 GeV.

Figure 4(a) shows the comparison of the fitted and original deposited energy in the 9th layer of 120 GeV electron beams. When the energy deposited in the cells far away the center of gravity is small, the excesses in Fig. 3 have little effect. The fitted energies for all layers are consistent with the deposited energy as shown in Fig. 4(b); the solid line in Fig. 4(b) is the result of a Γ-function fit.

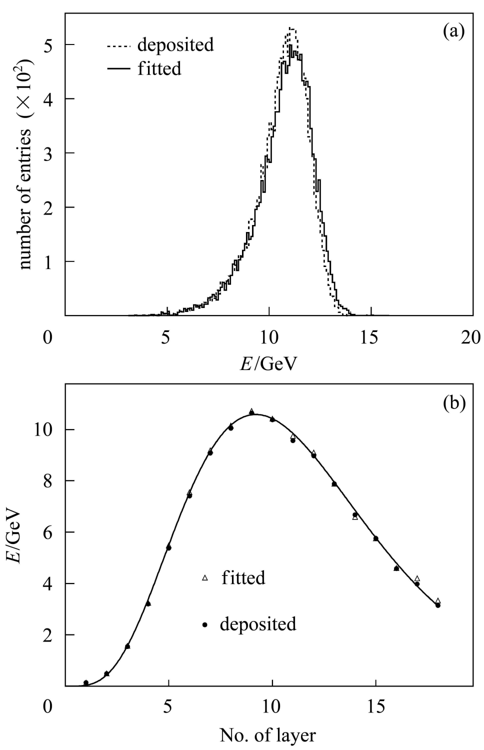

Fig. 4. (a) comparison of the fitted and original deposited energy in the 9th layer; (b) fitting energy and deposited energy in all 18 layers.

The distribution of the fitted results for of R_{layer} for the 8th layer in the case of 120 GeV electrons is shown in Fig. 5(a); it is well described by a Gaussian function.

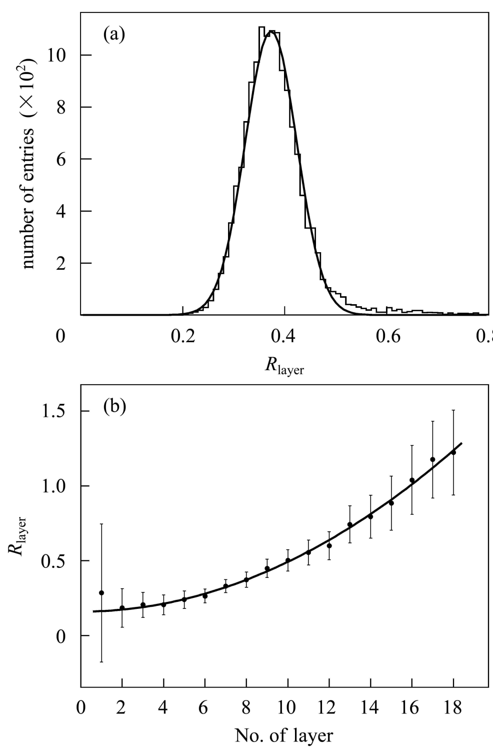

Fig. 5. (a) Distribution of R_{layer} on the 8th layer for 120 GeV electrons; (b) R_{layer} variation with the layer number for 120 GeV electrons.

The mean value of R_{layer} can be written as a function of layer number as following:

$$R_{\text{layer}} = A \times l^2 + B\,, \tag{5}$$

for all beam energies,where l represents the layer number, and A & B are the parameters determined from the fit. Fig. 5(b) shows the variation with layer number of the mean value of R_{layer} for 120 GeV electrons. The error bar represents the sigma from Gaussian fitting of R_{layer} on each layer. The sigmas of the fitted Gaussians for the first two layers are a little larger because of fluctuations in the shower's starting point.

Parameter A can be described as a function of the beam energy Fig. 6(a):

$$A = p0 \times \log(E_{\text{beam}}) + p1\,, \tag{6}$$

where E_{beam} represents the energy of the incident electron beams, and $p0$ & $p1$ are the parameters that are determined from the fit. The fitting value of $p0$ and $p1$ is $(-6.90\pm1.66)\times10^{-4}$ and $(6.60\pm0.74)\times10^{-3}$, respectively. Parameter B is a constant Fig. 6(b):

$$B = 0.176\pm0.012\,. \tag{7}$$

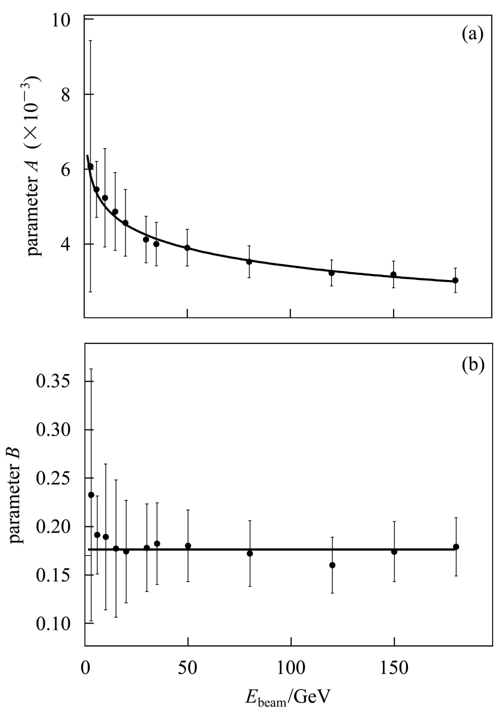

Fig. 6. Variation of parameters A (a) and B (b) with different beam energies.

4 Application

The energy deposited by electrons and photons in each layer can be predicted by Formula 1 and the energy deposited in all cells of a layer by Formula 3. With these Formulae, corrections for leakage and dead channels correction can be accomplished as described below.

4.1 Leakage correction

Longitudinal leakage can be corrected using Formula 1 and the lateral leakage using Formula 3. It is too complex to correct when there is lateral leakage in both the x and the y directions, so only data samples with lateral leakage in one direction are considered in this analysis. In order to show the significance of the lateral leakage correction, we choose events with the center of gravity of the deposited energy close to the edge, but not right at edge to ensure the main part of the electromagnetic shower is still contained in ECAL. Fig. 7 shows the fitted results for energy deposited in the 8th layer and the 16th layer for 120 GeV electrons, when there is lateral leakage only in x direction .

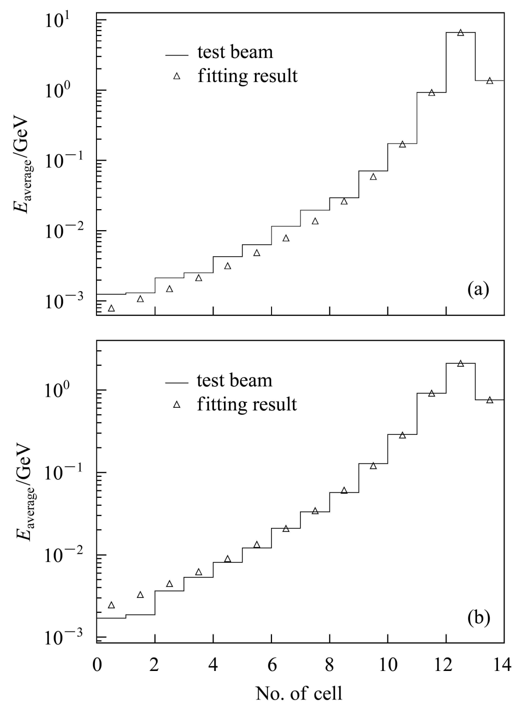

Fig. 7. Fitted results for energy in (a) the 8th layer and (b) the 16th layer for 120 GeV electrons.

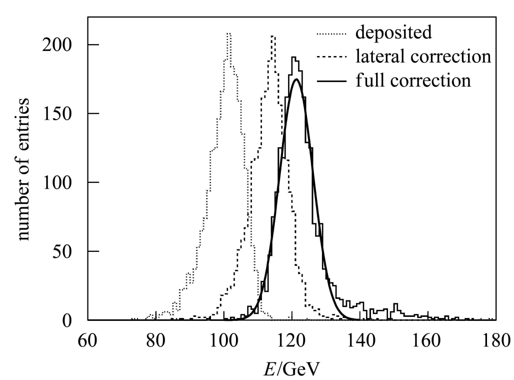

Fig. 8. Reconstructed electron energies from the Γ-function fit after the correction for lateral leakage using the empirical formula.

After the correction for lateral leakage, Formula 1 is used to correct for the longitudinal leakage. The fitted result is shown in Fig. 8, where the dot-dash line represents the deposited energy, the dashed line represents the total energy after the correction for the lateral leakage, and the solid line represents the reconstructed energy after the longitudinal energy leakage correction.

4.2 Dead channel correction

There were no dead channels during the beam test, so the outputs of some channels are artificially set to 0 in order to check whether it's possible to correct for dead channels using the empirical formula.

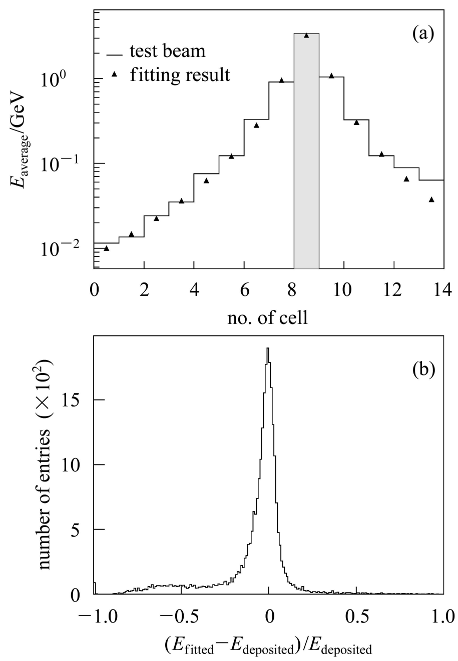

Fig. 9. Dead channel comppensation. (a) The output of cell 8 is set to zero and the lost energy is regained after the fit; (b) The performance of the energy correction based on the empirical formula.

The cells number with the largest energy deposit in the sample of 120 GeV electron events used in the analysis is 8 in the y direction and 6 in the x direction. The readout at the cell of the center of gravity is set to zero and the empirical formula is used to fit the distribution of energy layer by layer. The lost energy caused by the mimicked dead channels is corrected as shown in Fig. 9(a), which shows the case for layer 14 after fitting; the lost energy is regained (dark region). The ratio of the difference between the fitted energy and the deposited energy to the original deposited energy $((E_{\rm fitted} - E_{\rm deposited})/E_{\rm deposited})$ is shown in Fig. 9(b); the fitted energy agrees well with the measured deposited energy in this cell, but a long tail appears on the left side where the MINUIT fit gives a poor result.

5 Summary

An empirical formula is developed using Alpha Magnetic Spectrometer Ⅱ ECAL test beam data. This formula can be described as follows:

$$\left.\frac{{\rm d}^2E}{{\rm d}x{\rm d}y}\right|_{\rm layer} = \frac{3}{\pi}\cdot\frac{E_{\rm layer}\cdot R^2_{\rm layer}}{(r+R_{\rm layer})^4},$$

$$r=\sqrt{(x-x_{\rm c})^2+(y-y_{\rm c})^2}\,.$$

Here $E_{\rm layer}$ is the energy deposit in a layer, which is described by the Γ-function. The parameter $R_{\rm layer}$ can be written as the function of layer number $R_{\rm layer}=A\times l^2+B$. Parameter A is a function of the incident beam energy: $A=p0\times\log(E_{\rm beam})+p1$, where $p0=(-6.90\pm1.66)\times10^{-4}$, $p1=(6.60\pm0.74)\times10^{-3}$. Parameter B is a constant $B=0.176\pm0.012$.

Using these formulae, the longitudinal and lateral energy leakage as well as energy leakage from dead channels can be corrected for quite well.

The 3D-parameterized formula for the electromagnetic shower is the combination of formula 1 and 3. The 3D parametrization presented here might be used in fast Monte Carlo simulations in order to save the CPU time spent in the full simulation with GEANT4. Further studies are still needed to complete the implementation of this type of fast simulation.

References

1 Battiston R et al. Astropart. Phys., 2000, **13**: 51—74
2 Antonelli M et al. Nuclear Physics B, 1977, **54**: 14—19
3 Cadoux F et al. Nuclear Physics B, 2002, **113**(Proc.Suppl.): 159—165
4 Montanet L et al. Physics Review D, 1994, **50**: 1173—1814
5 Acosta D et al. Nuclear Instruments and Methods in Physics Research A, 1992, **316**: 184—201
6 Nelson W R et al. Phys. Rev., 1966, **149**: 201
7 Bathow G et al. Nucl. Phys. B, 1970, **20**: 592
8 Gridhammer G et al. In: Donalson R, Gilchriese M G D ed. Proceeding of the Workshop on Calorimetry for the Supercollider. New Jersey: World Scientific, 1989. 151
9 YAO W M et al. J. Phys. G, 2006, **33**: 1
10 Girard L et al. Performance of the Alpha Magnetic Spectrometer II electromagnetic calorimeter. In: Proceedings for 8th International Conf on Advanced Technology and Particle Physics (ICATPP 2003), October 2003
11 LI Zu-Hao et al. HEP & NP, 2004, **28**: 521 (in Chinese)
12 LI Zu-Hao et al. HEP & NP, 2004, **28**: 1188 (in Chinese)
13 WANG Xiao-Bin et al. HEP & NP, 2005, **29**: 1071 (in Chinese)
14 LI Xin-Qiao et al. HEP & NP, 2006, **30**: 996 (in Chinese)
15 Longo E, Sestili I. Nucl. Instrum. Methods, 1975, **128**: 283
16 James F, Roos M. Comput. Phys. Commun., 1975, **10**: 343