# An overview of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing and $C P$ violation ${ }^{*}$ 

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#### Abstract

I give a brief overview of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing and $C P$ violation in the framework of the standard model． I focus on the theoretical estimate of the $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing parameters and the phenomenological description of several types of $C P$ violation in neutral D －meson decays．


Key words $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing，$C P$ violation
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## 1 Introduction

The phenomenon of meson－antimeson mixing has been of great interest in the long history of parti－ cle physics．Four meson－antimeson mixing systems， together with their characteristic parameters $x$ and $y$ which have experimentally been measured or con－ strained，are listed in Table 1.

Table 1.

| system | $\|x\|$ | $\|y\|$ | year |
| :---: | :---: | :---: | :---: |
| $\mathrm{K}^{0}-\overline{\mathrm{K}}^{0}$ | $\sim 0.47$ | $\sim 1.0$ | 1958 |
| $\mathrm{~B}_{\mathrm{d}}^{0}-\overline{\mathrm{B}}_{\mathrm{d}}^{0}$ | $\sim 0.78$ | $<1 \%$ | 1987 |
| $\mathrm{~B}_{\mathrm{s}}^{0}-\overline{\mathrm{B}}_{\mathrm{s}}^{0}$ | $\sim 27$ | $\sim 0.1$ | 2006 |
| $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ | $\leqslant 1 \%$ | $\sim 1 \%$ | 2007 |

Two lessons were learnt in the development of the standard model（SM）：（1）theorists speculated the ex－ istence of the charm quark and predicted its mass in understanding the observation of $\mathrm{K}^{0}-\overline{\mathrm{K}}^{0}$ mixing；and （2）theorists deduced the correct magnitude of the top quark mass from the observation of $\mathrm{B}_{\mathrm{d}}^{0}-\overline{\mathrm{B}}_{\mathrm{d}}^{0}$ mix－ ing．The measurement of $\mathrm{B}_{\mathrm{s}}^{0}-\overline{\mathrm{B}}_{\mathrm{s}}^{0}$ mixing is consistent with the SM expectation．People feel excited by the preliminary observation of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing，although current data ${ }^{[1]}$ do not hint at any new physics beyond the SM．The charming sleeping beauty is waking up！

In terms of $\mathrm{D}^{0}$ and $\overline{\mathrm{D}}^{0}$ ，the mass states of two neutral D mesons are written as

$$
\begin{align*}
& \left|\mathrm{D}_{1}\right\rangle=p\left|\mathrm{D}^{0}\right\rangle+q\left|\overline{\mathrm{D}}^{0}\right\rangle  \tag{1}\\
& \left|\mathrm{D}_{2}\right\rangle=p\left|\mathrm{D}^{0}\right\rangle-q\left|\overline{\mathrm{D}}^{0}\right\rangle
\end{align*}
$$

where $|p|^{2}+|q|^{2}=1$ holds and $C P \mathrm{~T}$ invariance has been assumed．Two $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing parameters $x$ and $y$ are defined by

$$
\begin{equation*}
x \equiv \frac{M_{2}-M_{1}}{\Gamma}, \quad y \equiv \frac{\Gamma_{2}-\Gamma_{1}}{2 \Gamma} \tag{2}
\end{equation*}
$$

where $M_{1,2}$ and $\Gamma_{1,2}$ are the mass and width of $\mathrm{D}_{1,2}$ ， and $\Gamma \equiv\left(\Gamma_{1}+\Gamma_{2}\right) / 2$ together with $M \equiv\left(M_{1}+M_{2}\right) / 2$ ． One has to take care of the definitions of $x$ and $y$ used in different papers．

Why is the $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ system unique？It is the only meson－antimeson system whose mixing（or oscilla－ tion）takes place via the intermediate states with down－type quarks．The rate of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing is ex－ pected to be very small in the SM，because the third generation（namely，the bottom quark）plays a negli－ gible role in the corresponding box diagrams：on the one hand，$m_{\mathrm{b}}^{2} / m_{\mathrm{w}}^{2} \sim \mathcal{O}\left(10^{-3}\right)$ ；on the other hand， $\left|V_{\mathrm{ub}} V_{\mathrm{cb}}\right|^{2} /\left|V_{\mathrm{us}} V_{\mathrm{cs}}\right|^{2} \sim \mathcal{O}\left(10^{-6}\right)$ ．The $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ system is also the only meson－antimeson system whose mix－ ing parameters $x$ and $y$ are notoriously hard to be calculated in the SM．The reason is simply that the charm quark mass is neither light enough（ $<\Lambda_{\mathrm{QCD}}$ ） nor heavy enough（ $>\Lambda_{\mathrm{QCD}}$ ），and one has no reliable techniques to evaluate $x$ and $y$ in this nonperturbative regime．Therefore，only experimental measurements can reliably tell us how large or how small the rate of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing is．

Why is the $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ system interesting？It is a sensi－ tive playground to explore possible $C P$－violating new physics，because the SM effects of $C P$ violation in
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neutral D-meson decays are typically of $\mathcal{O}\left(10^{-3}\right)$ or smaller. One may understand this point by considering the charm unitarity triangle of the CKM matrix in the complex plane ${ }^{[2]}: V_{\mathrm{ud}}^{*} V_{\mathrm{cd}}+V_{\mathrm{us}}^{*} V_{\mathrm{cs}}+V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}=0$, in which two sides are comparable in magnitude and much longer than the third one governed by $V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}$. The shape of this triangle is too sharp, implying that the $C P$-violating effects are strongly suppressed in comparison with the $C P$-conserving effects in the charm sector. On the other hand, The $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ system is a nontrivial playground to test the unitarity of the CKM matrix, quantum coherence of the $\mathrm{D}^{0}$ and $\overline{\mathrm{D}}^{0}$ mesons at their production thresholds, $C P \mathrm{~T}$ invariance and $\Delta C=\Delta Q$ rule et al ${ }^{[3]}$. For example, the CKM unitarity together with current data requires

$$
\begin{align*}
\left|V_{\mathrm{tb}}\right|>\left|V_{\mathrm{ud}}\right|>\left|V_{\mathrm{cs}}\right| & \gg\left|V_{\mathrm{us}}\right|>\left|V_{\mathrm{cd}}\right| \\
& \gg\left|V_{\mathrm{cb}}\right|>\left|V_{\mathrm{ts}}\right| \\
& \gg\left|V_{\mathrm{td}}\right|>\left|V_{\mathrm{ub}}\right|>0 \tag{3}
\end{align*}
$$

More accurate measurements of $\left|V_{\mathrm{cd}}\right|$ and $\left|V_{\mathrm{cs}}\right|$ will help test the validity of this hierarchy.

## $2 \quad \mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing

The mixing between $\mathrm{D}^{0}$ and $\overline{\mathrm{D}}^{0}$ mesons arises from the fact that they couple to a subset of virtual or real intermediate states. In this case, the proper time evolution of two flavor states is described by

$$
\begin{equation*}
\mathrm{i} \frac{\mathrm{~d}}{\mathrm{~d} t}\binom{\left|\mathrm{D}^{0}(t)\right\rangle}{\left|\overline{\mathrm{D}}^{0}(t)\right\rangle}=\left(M-\mathrm{i} \frac{\Gamma}{2}\right)\binom{\left|\mathrm{D}^{0}(t)\right\rangle}{\left|\overline{\mathrm{D}}^{0}(t)\right\rangle} \tag{4}
\end{equation*}
$$

where $M$ and $\Gamma$ are $2 \times 2$ Hermitian matrices. The $C P \mathrm{~T}$ invariance implies that $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$ hold. An expansion of the off-diagonal terms $M_{12}$ and $\Gamma_{12}$ to the second order in the SM perturbation theory is

$$
\begin{align*}
\left(M-\mathrm{i} \frac{\Gamma}{2}\right)_{12}= & \frac{1}{2 M}\left[\left\langle\mathrm{D}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=2}\left|\overline{\mathrm{D}}^{0}\right\rangle+\right. \\
& \left.\sum_{n} \frac{\left\langle\mathrm{D}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=1}|\mathrm{n}\rangle\langle\mathrm{n}| \mathcal{H}_{\text {weak }}^{\Delta \mathrm{C}=1}\left|\overline{\mathrm{D}}^{0}\right\rangle}{M-E_{n}+\mathrm{i} \epsilon}\right] \tag{5}
\end{align*}
$$

where $\mathcal{H}_{\text {weak }}^{\Delta C=1}$ and $\mathcal{H}_{\text {weak }}^{\Delta C=2}$ are the effective Hamiltonians of $\Delta C=1$ and $\Delta C=2$ processes, respectively. The first term on the right-hand side of Eq. (5) contributes only to $M_{12}$ and is sensitive to new physics, while the second term contributes both to $M_{12}$ and to $\Gamma_{12}$ and is dominated by the SM contribution.

As the effect of $b$ quark in $D^{0}-\bar{D}^{0}$ mixing is negligibly small, it is an excellent approximation to neglect $C P$ violation in this meson-antimeson system. In this case, the mass states $\left|D_{1}\right\rangle$ and $\left|D_{2}\right\rangle$ are just the $C P$ states $\left|\mathrm{D}_{+}\right\rangle$(even) and $\left|\mathrm{D}_{-}\right\rangle$(odd) under the convention $C P\left|\mathrm{D}^{0}\right\rangle=\left|\overline{\mathrm{D}}^{0}\right\rangle$. One may calculate
$\Delta M \equiv M_{2}-M_{1}$ and $\Delta \Gamma \equiv \Gamma_{2}-\Gamma_{1}$ by using the relations $\Delta M=-2 M_{12}$ and $\Delta \Gamma=-2 \Gamma_{12}$. In other words,

$$
\begin{align*}
x= & \frac{-1}{2 M \Gamma}\left[2\left\langle\mathrm{D}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=2}\left|\overline{\mathrm{D}}^{0}\right\rangle+\right. \\
& \mathcal{P} \sum_{n}\left(\frac{\left\langle\mathrm{D}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=1}|\mathrm{n}\rangle\langle\mathrm{n}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\overline{\mathrm{D}}^{0}\right\rangle}{M-E_{n}}+\right. \\
& \left.\left.\frac{\left\langle\overline{\mathrm{D}}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=1}|\mathrm{n}\rangle\langle\mathrm{n}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\mathrm{D}^{0}\right\rangle}{M-E_{n}}\right)\right] \\
y= & \frac{-1}{4 M \Gamma} \sum_{n}\left[\left\langle\mathrm{D}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=1}|\mathrm{n}\rangle\langle\mathrm{n}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\overline{\mathrm{D}}^{0}\right\rangle+\right. \\
& \left.\left\langle\overline{\mathrm{D}}^{0}\right| \mathcal{H}_{\text {weak }}^{\Delta C=1}|\mathrm{n}\rangle\langle\mathrm{n}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\mathrm{D}^{0}\right\rangle\right](2 \pi) \delta\left(M-E_{n}\right) \tag{6}
\end{align*}
$$

where $\mathcal{P}$ denotes the principle value, and the sum is over all intermediate states n with the implicit phase space $(2 \pi)^{3} \delta^{3}\left(\boldsymbol{p}-\boldsymbol{p}_{n}\right)$.

There are in general two approaches to calculate the values of $x$ and $y$ :

1) to use Eq. (6) at the quark level ( $n=d \bar{d}+s \bar{s}+$ $\mathrm{d} \overline{\mathrm{s}}+\mathrm{s} \overline{\mathrm{d}}) ;$
2) to use Eq. (6) at the hadron level ( $\mathrm{n}=\pi^{+} \pi^{-}+$ $\left.\mathrm{K}^{+} \mathrm{K}^{-}+\pi^{+} \mathrm{K}^{-}+\mathrm{K}^{+} \pi^{-}+\cdots\right)$.
But neither of them is able to give very reliable results, because the charm quark mass lies in an embarrassing (intermediate or non-perturbative) regime where neither the heavy-quark effective theory nor the chiral perturbation theory can work well.

At the quark level, the lowest-order short-distance calculation of the $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing box diagram yields

$$
\begin{equation*}
x_{\mathrm{box}} \propto \frac{m_{\mathrm{s}}^{2}}{m_{\mathrm{W}}^{2}} \times \frac{m_{\mathrm{s}}^{2}}{m_{\mathrm{c}}^{2}}, \quad y_{\mathrm{box}} \propto \frac{m_{\mathrm{s}}^{2}}{m_{\mathrm{c}}^{2}} x_{\mathrm{box}}, \tag{7}
\end{equation*}
$$

which are of $\mathcal{O}\left(10^{-5}\right)$ and $\mathcal{O}\left(10^{-7}\right)$, respectively. The small factor of $y_{\text {box }} / x_{\text {box }}$ can simply be understood as the helicity suppression. It was first pointed out by Georgi that higher-order contributions to $x$ and $y$ in the operator product expansion have fewer powers of $m_{\mathrm{s}}$ suppression, because the chiral suppression can be lifted by quark condensates instead of mass insertions ${ }^{[4]}$. The 8 -quark operator contributions to $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing are only suppressed by $m_{\mathrm{s}}^{2}$, and thus they are the dominant short-distance effects. More explicit estimates ${ }^{[5]}$, which depend on some assumptions and involve large uncertainties in dealing with the hadronic matrix elements, give $x \sim y \sim \mathcal{O}\left(10^{-3}\right)$ or smaller values.

At the hadron level, one may take the intermediate states n to be the exclusive hadronic states. This long-distance approach is reasonable because $m_{\mathrm{c}}$ (or $M)$ lies in a region populated by the excited lightquark states. It is impossible to sum over all the
possible intermediate hadronic multiplets in practice, however. Once the b-quark contribution is neglected, $x$ and $y$ will vanish in the limit of flavor $S U(3)$ symmetry. This point can be illustrated by assuming $n$ to be the two-body charged-pseudoscalar meson states $\mathrm{n}=\left\{\pi^{+} \pi^{-}, \mathrm{K}^{+} \mathrm{K}^{-}, \pi^{+} \mathrm{K}^{-}, \mathrm{K}^{+} \pi^{-}\right\}$. Their relative contributions to $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing are proportional to $\{+1,+1,-1,-1\} \cos ^{2} \theta_{\mathrm{C}} \sin ^{2} \theta_{\mathrm{C}}$, as a consequence of flavor $S U(3)$ symmetry. Hence the sum of these dispersive contributions vanishes, implying a perfect realization of the GIM mechanism. But we know that the flavor $S U(3)$ symmetry is badly broken in neutral D-meson decays. Non-vanishing $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing can actually arise as the second-order effect of $S U(3)$ symmetry breaking ${ }^{[6]}$,

$$
\begin{equation*}
x \sim y \sim \sin ^{2} \theta_{\mathrm{C}} \times[S U(3) \text { breaking }]^{2} \tag{8}
\end{equation*}
$$

How to estimate the size of $S U(3)$ breaking effects is a big challenge. One finds that a calculation of $y$ in this exclusive approach is less model-dependent, while the estimate of $x$ involves off-shell hadronic states and thus is less reliable. In this case, one may choose to use the dispersion relation

$$
\begin{equation*}
\Delta M=-\frac{1}{2 \pi} \mathcal{P} \int_{2 m_{\pi}}^{\infty} \mathrm{d} E\left[\frac{\Delta \Gamma(E)}{E-M}+\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{E}\right)\right] \tag{9}
\end{equation*}
$$

which has been proved in the heavy-quark effective theory ${ }^{[7]}$, to get $x$ from $y$.

As emphasized by Ligeti ${ }^{[8]}$, the most important long-distance effect is expected to be due to the $S U(3)$ symmetry breaking in phase space. Contrary to the breaking of $S U(3)$ symmetry in hadronic matrix elements, the breaking of $S U(3)$ symmetry in phase space is calculable in a less model-dependent way. A detailed analysis shows that there do exist some exclusive states which can induce large $S U(3)$ symmetry breaking and contribute to $y$ near the $1 \%$ level ${ }^{[7]}$. The dispersion relation implies that the magnitude of $x$ is similar to that of $y$. With the help of some fair assumptions, one typically gets $x \lesssim y$ and $10^{-3}<|x|<10^{-2}$. We can therefore draw a preliminary conclusion: the SM predictions for $x$ and $y$ remain quite uncertain, but the above order-ofmagnitude estimates seem reasonable.

The BaBar collaboration has obtained the experimental evidence for $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing from a measurement of the time dependence of the doubly-Cabibbosuppressed decay $\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \pi^{-}$and its CP-conjugate mode ${ }^{[1]}$. The decay rate of $\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \pi^{-}$can be expressed as

$$
\begin{align*}
& \Gamma\left[\mathrm{D}^{0}(t) \rightarrow \mathrm{K}^{+} \pi^{-}\right] \propto \\
& \quad \mathrm{e}^{-\Gamma t}\left[R+\sqrt{R} y^{\prime}(\Gamma t)+\frac{\left.{x^{\prime 2}+y^{\prime 2}}_{4}^{4}(\Gamma t)^{2}\right]}{}\right. \tag{10}
\end{align*}
$$

with

$$
\begin{equation*}
x^{\prime}=x \cos \delta+y \sin \delta, \quad y^{\prime}=y \cos \delta-x \sin \delta \tag{11}
\end{equation*}
$$

where $A\left(\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \pi^{-}\right) / A\left(\overline{\mathrm{D}}^{0} \rightarrow \mathrm{~K}^{+} \pi^{-}\right) \approx \mathrm{Re}^{\mathrm{i} \delta}$ and $|q / p| \approx 1$ have been used in the neglect of tiny $C P$ violation. The BaBar measurement yields $y^{\prime}=$ $(0.97 \pm 0.54) \times 10^{-2}$ and $x^{\prime 2}=(-2.2 \pm 3.7) \times 10^{-4}$, which at least indicates $y \sim 1 \%$.

The Belle collaboration has used the decay mode $\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}$to extract the information on $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing ${ }^{[1]}$. To a good degree of accuracy, the decay rates of $\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}$and $\mathrm{D}^{0} \rightarrow \pi^{+} \mathrm{K}^{-}$can approximate respectively to $\Gamma\left(\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}\right) \approx \mathrm{e}^{-\Gamma(1+y \cos \phi) t}$ and $\Gamma\left(\mathrm{D}^{0} \rightarrow \pi^{+} \mathrm{K}^{-}\right) \approx \mathrm{e}^{-\Gamma t}$, where $\phi$ is the weak phase of $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing. Hence we have the lifetime ratio

$$
\begin{equation*}
\frac{\tau\left(\mathrm{D}^{0} \rightarrow \pi^{+} \mathrm{K}^{-}\right)}{\tau\left(\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}\right)} \approx 1+y \cos \phi \tag{12}
\end{equation*}
$$

from which the effective mixing parameter

$$
\begin{equation*}
y_{C P} \equiv y \cos \phi \approx \frac{\tau\left(\mathrm{D}^{0} \rightarrow \pi^{+} \mathrm{K}^{-}\right)}{\tau\left(\mathrm{D}^{0} \rightarrow \mathrm{~K}^{+} \mathrm{K}^{-}\right)}-1 \tag{13}
\end{equation*}
$$

can be extracted. Current experimental data yield $y_{C P}=(1.31 \pm 0.32 \pm 0.25) \times 10^{-2}$, consistent with $y \sim 1 \%$.

## $3 \quad C P$ violation

In principle, there may be four different types of $C P$-violating signals in neutral $\mathrm{D}-$ meson decays.

1) $C P$ violation in $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing. This implies $|q / p| \neq 1$. In practice, we have the following $C P$ violating observable:

$$
\begin{equation*}
\Delta_{\mathrm{D}} \equiv \frac{|p|^{4}-|q|^{4}}{|p|^{4}+|q|^{4}} \tag{14}
\end{equation*}
$$

It is expected that the magnitude of $\Delta_{\mathrm{D}}$ should be at most of the order $10^{-3}$ in the $\mathrm{SM}^{[9]}$. However, a reliable estimation of $\Delta_{\mathrm{D}}$ suffers from large long-distance uncertainties.
2) $C P$ violation in the direct decay. For a decay mode $\mathrm{D}^{0} \rightarrow \mathrm{f}$ and its $C P$-conjugate process $\overline{\mathrm{D}}^{0} \rightarrow \overline{\mathrm{f}}$, this implies

$$
\begin{align*}
& \left.\left|\langle\overline{\mathrm{f}}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\right| \overline{\mathrm{D}}^{0}\right\rangle|\equiv| \sum_{n}\left[A_{n} \mathrm{e}^{\mathrm{i}\left(\delta_{n}-\phi_{n}\right)}\right] \mid \neq \\
& \left.\left|\langle\mathrm{f}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\right| \mathrm{D}^{0}\right\rangle|\equiv| \sum_{n}\left[A_{n} \mathrm{e}^{\mathrm{i}\left(\delta_{n}+\phi_{n}\right)}\right] \mid \tag{15}
\end{align*}
$$

where a parametrization of the decay amplitudes with the weak $\left(\phi_{n}\right)$ and strong $\left(\delta_{n}\right)$ phases is also given. We see that $n \geqslant 2, \phi_{m}-\phi_{n} \neq 0$ or $\pi$ and $\delta_{m}-\delta_{n} \neq 0$ or $\pi$ are necessary conditions for the above direct $C P$ violation.
3) $C P$ violation from the interplay of decay and mixing. Let us define two rephasing-invariant quan-
tities

$$
\begin{align*}
& \lambda_{\mathrm{f}} \equiv \frac{q}{p} \cdot \frac{\langle\mathrm{f}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\overline{\mathrm{D}}^{0}\right\rangle}{\langle\mathrm{f}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\mathrm{D}^{0}\right\rangle}, \\
& \bar{\lambda}_{\overline{\mathrm{f}}} \equiv \frac{p}{q} \cdot \frac{\langle\bar{f}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\mathrm{D}^{0}\right\rangle}{\langle\overline{\mathrm{F}}| \mathcal{H}_{\text {weak }}^{\Delta C=1}\left|\overline{\mathrm{D}}^{0}\right\rangle}, \tag{16}
\end{align*}
$$

where the hadronic states f and $\overline{\mathrm{f}}$ are common to the decay of $\mathrm{D}^{0}$ (or $\overline{\mathrm{D}}^{0}$ ). Even in the assumption of $|q / p|=1$, indirect $C P$ violation can appear if

$$
\begin{equation*}
\operatorname{Im} \lambda_{\mathrm{f}}-\operatorname{Im} \bar{\lambda}_{\overline{\mathrm{f}}} \neq 0 \tag{17}
\end{equation*}
$$

Provided f is a $C P$ eigenstate (i.e., $|\overline{\mathrm{f}}\rangle= \pm|\mathrm{f}\rangle$ ) and the decay is dominated by a single weak phase, then we have $\bar{\lambda}_{\bar{f}}=\lambda_{\mathrm{f}}^{*}$.
4) $C P$ violation in the $C P$-forbidden decay. On the $\psi(3770)$ (or $\psi(4140)$ ) resonance, a $C P$-odd (or $C P$-even) $\mathrm{D}^{0} \overline{\mathrm{D}}^{0}$ pair can be coherently produced:

$$
\begin{align*}
& \psi(3770) \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{0} \\
& \psi(4140) \rightarrow \mathrm{D}^{0} \bar{D}^{* 0} \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{0} \pi^{0}, \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{0} \gamma \tag{18}
\end{align*}
$$

The $\mathrm{D}^{0} \overline{\mathrm{D}}^{0}$ pairs produced in the above three processes are $C P$-odd, $C P$-odd and $C P$-even, respectively. Then the decay

$$
\begin{equation*}
\left(\mathrm{D}^{0} \overline{\mathrm{D}}^{0}\right)_{C P= \pm 1} \longrightarrow\left(\mathrm{f}_{1} \mathrm{f}_{2}\right)_{C P=\mp 1} \tag{19}
\end{equation*}
$$

where $\mathrm{f}_{1}$ and $\mathrm{f}_{2}$ are proper $C P$ eigenstates (e.g., $\pi^{+} \pi^{-}$, $\mathrm{K}^{+} \mathrm{K}^{-}$and $\mathrm{K}_{\mathrm{S}} \pi^{0}$ ), is a $C P$-forbidden process and can only occur due to $C P$ violation.

Besides these four types of $C P$-violating effects in neutral D-meson decays, one may expect the effect of $C P$ violation induced by $\mathrm{K}^{0}-\overline{\mathrm{K}}^{0}$ mixing in those decay modes with $\mathrm{K}_{\mathrm{S}}$ or $\mathrm{K}_{\mathrm{L}}$ in their final states ${ }^{[10]}$. Its magnitude is typically ${ }^{[11]}$

$$
\begin{equation*}
2 \operatorname{Re}\left(\epsilon_{\mathrm{K}}\right) \approx 3.3 \times 10^{-3} \tag{20}
\end{equation*}
$$

which may be comparable in magnitude with the charmed $C P$-violating effects listed in Eqs. (14), (15) and (17). Kaplan emphasized that this kind of known $C P$-violating asymmetry should be measured in the charm factory as a calibration for the experimental systematics of asymmetries at the $0.1 \%$ level ${ }^{[12]}$.

We have pointed out that the SM expectation for $C P$ violation in the charm sector is of $\mathcal{O}\left(10^{-3}\right)$ or smaller. The reason is simply that the charm unitarity triangle of the CKM matrix, formed by $V_{\mathrm{ud}}^{*} V_{\mathrm{cd}}+V_{\mathrm{us}}^{*} V_{\mathrm{cs}}+V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}=0$ in the complex plane, is too sharp. In the Wolfenstein phase convention recommended by the $\mathrm{PDG}^{[11]}$, we have

$$
\begin{equation*}
\operatorname{Im}\left(V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}\right) \approx-\lambda^{6} \sin \gamma \tag{21}
\end{equation*}
$$

where $\lambda \equiv \sin \theta_{\mathrm{C}} \approx 0.22$, and $\gamma \approx 65^{\circ}$ is one of the inner angles of the well-known beauty unitarity triangle of the CKM matrix. Hence the imaginary part of $V_{\mathrm{ud}}^{*} V_{\mathrm{cd}}+V_{\mathrm{us}}^{*} V_{\mathrm{cs}}$ must be $+\lambda^{6} \sin \gamma$, and the ratio of the $C P$-violating part to the $C P$-conserving part in
many D-meson decay channels is characterized by

$$
\begin{align*}
\frac{\operatorname{Im}\left(V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}\right)}{\left|V_{\mathrm{ud}}^{*} V_{\mathrm{cd}}\right|} \approx & \frac{\operatorname{Im}\left(V_{\mathrm{ub}}^{*} V_{\mathrm{cb}}\right)}{\left|V_{\mathrm{us}}^{*} V_{\mathrm{cs}}\right|} \approx \\
& A^{2} \lambda^{4} \sqrt{\rho^{2}+\eta^{2}} \mathrm{e}^{-\mathrm{i} \gamma} \approx \lambda^{6} \mathrm{e}^{-\mathrm{i} \gamma} \tag{22}
\end{align*}
$$

which is about $5 \times 10^{-4}$ in magnitude. This naive but reasonable estimate implies that the magnitudes of $C P$-violating asymmetries in neutral (and charged) D-meson decays are at most of $\mathcal{O}\left(10^{-3}\right)$ in the SM, even if there are large final-state interactions. In general, the singly Cabibbo-suppressed Dmeson decays may have larger $C P$-violating effects than those Cabibbo-favored and doubly Cabibbosuppressed decays ${ }^{[13]}$.
$C P$ violation at the percent level has not been observed in any experiments ${ }^{[13]}$. But signals of $\mathcal{O}\left(10^{-3}\right)$ are expected to show up in some neutral D-meson decays within the $\mathrm{SM}^{[14]}$, although such theoretical estimates involve large uncertainties. If a $C P$-violating asymmetry of $\mathcal{O}\left(10^{-2}\right)$ is observed in the (near) future, it will clearly signify the existence of new physics in the charm sector.

## 4 Concluding remarks

Now let me make some concluding remarks.

1) The SM predictions for $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing and $C P$ violation have very large uncertainties, and they are very hard to get improved in the foreseeable future.
2) However, $D^{0}-\bar{D}^{0}$ mixing up to the $1 \%$ level is very likely in the SM, consistent with current experimental evidence. The fact $x \lesssim y$ implies that this meson-antimeson mixing system might not be sensitive to new physics.
3) $C P$ violation up to the $0.1 \%$ level is expected in the SM, and thus a signal of $C P$ violation at the $1 \%$ level would serve as robust evidence of new physics in the charm sector.
4) But personally, I believe that new physics might essentially be decoupled to the standard flavor physics at low energy scales, just like the physics of massive neutrinos. I hope that I would be wrong.
5) No matter whether there is new physics or not in the charm sector, it is interesting and important to study $\mathrm{D}^{0}-\overline{\mathrm{D}}^{0}$ mixing and $C P$ violation at the BEPCII collider and other facilities.

Let us see what will happen in the coming years.
Many people in the audience believe that $(C P)$ symmetry is beautiful. My friend, Peter Minkowski, believes that ( $C P$-violating) asymmetry is a sister of $(C P)$ symmetry. So, I have a very good reason to believe that ( $C P$-violating) asymmetry is also beautiful! Then let us look for this sleeping beauty in the charm sector.

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