# Five－quark components in $\mathbf{N}^{*}(1535)$ 

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#### Abstract

Here we employ the extended chiral constituent quark model to investigate the five－quark com－ ponents in the $\mathrm{N}^{*}(1535)$ resonance．The axial charge of $\mathrm{N}^{*}(1535)$ and the electromagnetic transition $\gamma^{*} \mathrm{~N} \rightarrow$ $\mathrm{N}^{*}(1535)$ are also analyzed．The results show that there may be sizable strangeness component in $\mathrm{N}^{*}(1535)$ ．


Key words $\mathrm{N}^{*}(1535)$ ，five－quark components，strangeness
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## 1 Introduction

Among the low－lying nucleon excitations，the $S_{11}$ state $\mathrm{N}^{*}(1535)$ plays a special role due to its large $\mathrm{N} \eta$ decay width ${ }^{[1]}$ ，even though its mass is very close to the threshold of the decay．On the other hand， one of the well known puzzles in the classical con－ stituent quark model is the inverse mass order of the $\mathrm{N}^{*}(1440)$ and $\mathrm{N}^{*}(1535)$ resonances ${ }^{[2]}$ ．Therefore，it＇s very important for us to investigate the structure of $\mathrm{N}^{*}$（1535）．

Here we analyze the five－quark components in $\mathrm{N}^{*}(1535)$ ，the results show that there may be sizable strangeness component in this resonance． This＇s consistent with the large $N \eta$ decay rate and strong couplings of $\mathrm{N}^{*}(1535) \mathrm{N} \phi^{[3]}, \mathrm{N}^{*}(1535) \mathrm{K} \Lambda^{[4]}$ and $\mathrm{N}^{*}(1535) \mathrm{N}^{\prime[5]}$ and it＇s also in line with the $\mathrm{K} \Sigma$ quasibound state explanation for $\mathrm{N}^{*}(1535)$ by the mechanism of dynamical resonance formation within the coupled channel approach based on chi－ ral $S U(3)^{[6-9]}$ ．While the five－quark component in the Roper resonance with largest probability may be the one with light quark－antiquark pair ${ }^{[10]}$ ，this may be the origin of the inverse mass order of $\mathrm{N}^{*}(1440)$ and $\mathrm{N}^{*}(1535)$ ．

As applications，we study the axial charge of $\mathrm{N}^{*}(1535)$ employing the extended chiral constituent quark model ${ }^{[11]}$ ，the result is in agreement with that obtained numerically by a recent lattice QCD
calculation ${ }^{[12]}$ ．And we also investigate the role of the five－quark components in the electromagnetic transi－ tion $\gamma^{*} \mathrm{~N} \rightarrow \mathrm{~N}^{*}(1535)^{[13]}$ ，the results fit the data much better than that obtained by the classical constituent quark model．

## 2 The $q q q q \bar{q}$ configurations

First we assume that $\mathrm{N}^{*}(1535)$ is the admixture of three－quark and five－quark components．For the three－quark component in $\mathrm{N}^{*}(1535)$ ，we employ the wave function which is same as that in the classical chiral constituent quark model ${ }^{[14]}$ in the present case． And here we analyze the five－quark components in N＊（1535）explicitly．

As we know，the parity of $\mathrm{N}^{*}(1535)$ is negative， then all the quarks and antiquark in the five－quark component should be in their ground states or higher even orbitally excited states，here we take the one with the lowest energy，i．e．the orbital state of four－ quark subsystem in the five－quark component is taken to be the completely symmetric $[4]_{X}$ ．On the other hand，the color state for the four－quark subsystem is limited to be $[211]_{C}$ in order to combine with the antiquark to form a color singlet．Finally，the flavor－ spin configuration of the four－quark subsystem should be $[31]_{F S}$ ，which is the conjugate state of $[211]_{C}$ ．So we can write the general expression for the five－quark component in $\mathrm{N}^{*}(1535)$ to be the following form ${ }^{[11]}$ ：

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$$
\left.\begin{array}{rl}
\psi_{t, s}^{(i)}= & \sum_{a, b, c} \sum_{Y, y, T_{z}, t_{z}} \sum_{S_{z}, s_{z}} C_{[31]_{a}}^{[144]}[21]_{a} \\
& \left.\left(S, S_{z}, 1 / 2, s_{z} \mid 1 / 2, s\right)_{[F(i)}^{[31]_{b}}\right]_{b}\left[t_{z} \bar{\xi}_{s_{z}} \bar{S}_{z} \varphi_{[5]}\right. \tag{1}
\end{array}\right] .
$$

Table 1. The qqqqq configurations in the $\mathrm{N}^{*}(1535)$ and the corresponding axial charge coefficient $A_{5}^{(i)}$.

| configuration | flavor-spin | $C_{F S}$ | color-spin | $C_{C S}$ | $A_{5}^{(i)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $[31]_{F S}[211]_{F}[22]_{S}$ | -16 | $[31]_{C S}[211]_{C}[22]_{S}$ | -16 | 0 |
| 2 | $[31]_{F S}[211]_{F}[31]_{S}$ | $-40 / 3$ | $[31]_{C S}[211]_{C}[31]_{S}$ | $-40 / 3$ | $+5 / 6$ |
| 3 | $[31]_{F S}[22]_{F}[31]_{S}$ | $-28 / 3$ | $[22]_{C S}[211]_{C}[31]_{S}$ | $-16 / 3$ | $-1 / 9$ |
| 4 | $[31]_{F S}[31]_{F}[22]_{S}$ | -8 | $[211]_{C S}[211]_{C}[22]_{S}$ | 0 | $-4 / 15$ |
| 5 | $[31]_{F S}[31]_{F}[31]_{S}$ | $-16 / 3$ | $[211]_{C S}[211]_{C}[31]_{S}$ | $+8 / 3$ | $+17 / 18$ |

Here $i$ denotes different flavor-spin configurations of the four-quark subsystems in the five-quark components, as shown in Table 1. The numbering of these configurations are in order of increasing energy, if the hyperfine interaction between the quarks is assumed to depend either on flavor and spin or on color and spin. In the table the matrix elements of the schematic hyperfine splitting operator

$$
\begin{equation*}
C_{k S}=-\sum_{i, j} \vec{\lambda}_{i} \cdot \vec{\lambda}_{j} \vec{\sigma}_{i} \cdot \vec{\sigma}_{j} \tag{2}
\end{equation*}
$$

are listed for both the cases where the operators $\vec{\lambda}$ represent either the generators of the color $S U(3)$ $(k=C)$ or the flavor $S U(3)$ group $(k=F)$, respectively. Note that because of their mixed flavor symmetry $[211]_{F}$ both of the configurations (1) and (2) in Table 1 have to contain a strange quark-antiquark pair, that means there may be sizable strangeness components in $\mathrm{N}^{*}(1535)$. This is as expected on the basis of the observed large $\mathrm{N} \eta$ decay branch of the $\mathrm{N}^{*}(1535)^{[1]}$.

## 3 Applications

### 3.1 The axial charge of $\mathrm{N}^{*}(\mathbf{1 5 3 5})$

The axial charge of the $\mathrm{N}^{*}(1535)$ resonance is not accessible experimentally at the present time, it is in this regard that the recent result, obtained numerically by an unquenched QCD lattice calculation, that the axial charge of the $\mathrm{N}^{*}(1535)$ actually may vanish is so interesting ${ }^{[12]}$. As that result appears to be insensitive to the quark mass (the magnitude of the value extrapolated to 0 is less than 0.2 ), it may be taken as a substitute for an experimental value. While the statistical error margins of the calculated values of the axial charge of the $\mathrm{N}^{*}(1535)$ are not yet sufficiently narrow to exclude the small value $-1 / 9$ given by the conventional constituent quark model
with only qqq configurations, it is interesting to explore the phenomenological consequences of a vanishing axial charge.

We calculate the axial charge of $\mathrm{N}^{*}(1535)$ including the contributions from both of the three-quark and five-quark components. The result may be written as a sum of the diagonal matrix elements of all possible configurations:

$$
\begin{equation*}
g_{A}\left(\mathrm{~N}^{*}(1535)\right) \simeq-\frac{P_{3}}{9}+\sum_{n} A_{5}^{(i)} P_{5}^{(i)} . \tag{3}
\end{equation*}
$$

Here $P_{3}$ is the probability of the three-quark component. And $P_{5}^{(i)}$ denotes the probability of the fivequark component with the ith flavor-spin configuration, and $A_{5}^{(i)}$ the corresponding numerical result for the axial charge of $\mathrm{N}^{*}(1535)$ which is shown in Table 1 .

As we can see in Table 1, the five-quark component with the lowest energy does not contribute to the axial charge of $\mathrm{N}^{*}(1535)$. But the numerical result obtained from the next-to-lowest energy five-quark component is positive, so it may naturally cancel the contribution of the three-quark component. If we only take into account the first two configurations in Table 1 and assume that $P_{5}^{(1)}=1-5 P_{5}^{(2)}, g_{A}\left(\mathrm{~N}^{*}(1535)\right)$ would vanish if the total probability of the first two five-quark components falls in the range $20 \%-60 \%$.

### 3.2 The electromagnetic transition $\gamma^{*} \mathbf{N} \rightarrow$ $\mathrm{N}^{*}(\mathbf{1 5 3 5})$

Here we calculate the helicity amplitude $A_{1 / 2}^{p}$ for the electromagnetic transition $\gamma^{*} \mathrm{~N} \rightarrow \mathrm{~N}^{*}(1535)$. The contribution from the three-quark component and the lowest energy five-quark component are considered. We employ the harmonic oscillator model for the orbital wave functions here. The numerical results are shown in Figs. 1 and 2.

As shown in Fig. 1, if we only consider the three-
quark component in $\mathrm{N}^{*}(1535)$, none of the three curves can describe the experimental data satisfactorily. At the photon point, $Q^{2}=0$, the calculated helicity amplitudes are $A_{1 / 2}^{p}=0.147 / \sqrt{\mathrm{GeV}}$ and $A_{1 / 2}^{p}=0.115 / \sqrt{\mathrm{GeV}}$ with the oscillator parameter $\omega_{3}=340 \mathrm{MeV}$ and $\omega_{3}=246 \mathrm{MeV}$, respectively, both of which are larger than the experimental value $A_{1 / 2}^{p}=0.090 \pm 0.030 / \sqrt{\mathrm{GeV}}{ }^{[1]}$. For the curve which is obtained by setting the parameter $\omega_{3}=200 \mathrm{MeV}$, the calculated amplitude describes the data better at the photon point, but it falls too fast in comparison with the data when $Q^{2}$ increases.


Fig. 1. The contribution of the three-quark component to the helicity amplitude $A_{1 / 2}^{p}$.

The numerical result for the helicity amplitude $A_{1 / 2}^{p}$ including the contributions of the three-quark component and the lowest energy five-quark component is shown in Fig. 2. Here we have taken the probability of the qqqq $\bar{q}$ components in the proton as the tentative value $P_{5 q}=20 \%$, and in $\mathrm{N}^{*}(1535) P_{5 q}=$ $45 \%$. The oscillator parameters are $\omega_{3}$ and $\omega_{5}$, which are taken to be $\omega_{3}=340 \mathrm{MeV}$ and $\omega_{5}=600 \mathrm{MeV}$, $\omega_{3}=246$ and $\omega_{5}=600 \mathrm{MeV}$, and $\omega_{3}=340 \mathrm{MeV}$ and $\omega_{5}=1000 \mathrm{MeV}$, respectively. As we can see in Fig. 3, comparing to the results obtained by the traditional qqq constituent quark model, the results here describe the experimental data for $A_{1 / 2}^{p}$ much better when the oscillator parameters are given the values $\omega_{3}=340 \mathrm{MeV}$ and $\omega_{5}=600 \mathrm{MeV}$, both at the photon point and larger $Q^{2}$.

We have also given the contribution of the nondiagonal transition, i.e. the process $\gamma^{*} q q q \rightarrow q q q q \bar{q}$ in Fig. 2, which is represented by the dash-dot line (Here the curve is for $-A_{1 / 2}^{p}$.). Actually, the contribution of the diagonal transition $\gamma^{*} q q q q \bar{q} \rightarrow q q q q \bar{q}$ is very small, which is less than $0.005 / \sqrt{\mathrm{GeV}}$. But as shown in Fig. 2, the non-diagonal transition contributes significantly to the electromagnetic transition $\gamma^{*} \mathrm{p} \rightarrow \mathrm{N}^{*}(1535)$.


Fig. 2. The helicity amplitude $A_{1 / 2}^{p}$ including the contributions of the three-quark component and the lowest energy five-quark component.


Fig. 3. The transverse helicity amplitude $A_{1 / 2}^{p}$ with the phase factor $\delta= \pm 1$.

There is another important parameter for the nondiagonal transition, the phase factor $\delta$ between the qqq and qqqq $\bar{q}$ components of the $\mathrm{N}^{*}(1535)$ resonance, which has been taken to be +1 . But in principle, this factor could be an arbitrary complex one $\exp \{\mathrm{i} \phi\}$. As we know, the helicity amplitude $A_{1 / 2}^{p}$ is real, so here we may choose $\delta$ to be $\pm 1$. The numerical results for $\delta=-1$ is given by the bold solid line in Fig. 3. As we can see in Fig. 3, the transverse helicity amplitude $A_{1 / 2}^{p}$ seems to favor a negative value for the mixing phase factor $\delta$. Note that here we have taken the probability of the strangeness five-quark component to be $85 \%$, which may be too large. But there is no evidence from both of the theoretical and experimental analysis has indicated the strangeness in $\mathrm{N}^{*}(1535)$ cannot be so large.

And if we take the mixing phase factor to be $\delta=-1$, the physical picture for the model is much clearer. In our extended quark model with each
baryon as a mixture of the three-quark and five-quark components, the two components represent two different states of the baryon. For the qqqqq $\bar{q}$ state, there are more color sources than the qqq state, and may make the effective phenomenological confinement potential stronger. This is consistent with other empirical evidence favoring larger value of the oscillator parameter of the five-quark component ${ }^{[10,15-17]}$, and it leads to better description for the $A_{1 / 2}^{p}$ than the three-quark model. An intuitive picture for our extended quark model is like this: the qqq state has weaker potential; when quarks expand, a $q \bar{q}$ pair is pulled out and results in a $q q q q \bar{q}$ state with stronger potential; the stronger potential leads to a more compact state which then makes the $\bar{q}$ to annihilate with a quark easily and transits to the qqq state; this leads to constantly transitions between these two states.

On the other hand, as we know, the numerical results for the longitudinal helicity amplitude $S_{1 / 2}^{p}$ obtained by the traditional qqq constituent quark model are always positive (In Ref. [13], we give a wrong sign for $S_{1 / 2}^{p}$ obtained by the three-quark component), it's just opposite to the sign of the data. If we take into account the contributions of the five quark component and set the phase factor between the three- and five-quark components to be $\delta=-1$, then the nondiagonal transition would contribute a negative value to $S_{1 / 2}^{p}$. Although the longitudinal helicity amplitude cannot be described very well with the same parameters which can fit the $A_{1 / 2}^{p}$ data best, the non-diagonal transition may be the origin of the negative $S_{1 / 2}^{p}$.

## 4 Conclusion

Here we have analyzed the five-quark components
in $\mathrm{N}^{*}(1535)$, the results show that there may be sizable ( $40 \%$ or more) strangeness component in this resonance. As applications, we calculate the axial charge of $\mathrm{N}^{*}(1535)$ and the helicity amplitude for the electromagnetic transition $\gamma^{*} \mathrm{p} \rightarrow \mathrm{N}^{*}(1535)$ employing this model, the results fit the data better than that obtained from the traditional qqq constituent quark model.

The conclusion that there is sizable strangeness component is not only in line with the large $\mathrm{N} \eta$ decay width of $\mathrm{N}^{*}(1535)^{[1]}$, but is also consistent with most of the recent phenomenological analysis ${ }^{[3-9]}$. And what is most important is that this may be the origin of the inverse mass ordering of $\mathrm{N}^{*}(1440)$ and $\mathrm{N}^{*}(1535)$.

On the other hand, we find that the non-diagonal transition, i.e. the transition between the three-quark component in proton and the five-quark component in $\mathrm{N}^{*}(1535)$ contributes significantly to the electromagnetic transition $\gamma^{*} \mathrm{p} \rightarrow \mathrm{N}^{*}(1535)$. If we take the mixing phase factor between the three- and five-quark components in $\mathrm{N}^{*}(1535)$ to be $\delta=-1$, it may lead to negative longitudinal helicity amplitude $S_{1 / 2}^{p}$. And the non-diagonal transition leads to good description for the transverse helicity amplitude $A_{1 / 2}^{p}$ whatever the sign for the phase factor $\delta$ is.

Based on the above statement, we conclude that there should be sizable strangeness five-quark component in $\mathrm{N}^{*}(1535)$, and it plays significant role in the electromagnetic and other properties of this resonance.

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