Calculation of α decay half-lives for superheavy elements using the double folding model*

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Abstract α decay half-lives of some new synthesized superheavy elements, possibly synthesized superheavy elements and decay products are calculated theoretically within the WKB approximation by using microscopic α-nucleus interaction potentials. These nuclear potentials between the α particle and daughter nuclei are obtained by using the double folding integral of the matter density distribution of the α particle and daughter nuclei with a density-dependent effective nucleon-nucleon interaction, in which the zero-range exchange term is supplemented. The calculated α decay half-lives are compared with those of the different models and experimental data. It is shown that the present calculation successfully provides the half-lives of the observed α decays for some new superheavy elements and therefore gives reliable predictions for α decay of possibly synthesized superheavy elements in future experiments.

Key words double folding model, superheavy element, half-life, α decay

PACS 27.90.+b, 23.60.+e, 21.60.-n

1 Introduction

In 1896, Becquerel first observed some unknown radioactivity; in 1908 Rutherford found the unknown radioactivities and called it α decay in the first experimental observation of α radioactivity[1]. In the 1920s, Gamow et al. successfully provided a theoretical explanation of α radioactivity in terms of a quantum tunnelling effect[2, 3]. From then on the ground state of unstable nuclei has been observed to have different kinds of decay modes: α decay, β decay, one proton emission, two proton emission, ^{14}C radioactivity, spontaneous fission etc. α decay is one of the most important decay modes because it can provide information on nuclear structure such as ground state lifetime, nuclear spin, parity etc. Experimentally, α decay of nuclei is used to identify new nuclides and new elements through an α decay chain from an unknown parent nucleus to a known nuclide. According to this method, some new synthesized superheavy elements have been identified at GSI, Dubna, Berkeley, RIKEN and GANIL' such as $Z = 110$, 111, 112, especially for $Z = 114$, 116, and $^{294}118$ elements have recently been produced in fusion-evaporation reactions[4, 5]. Theoretically different approaches have been proposed to describe α radioactivity, such as the shell, cluster and fission-like model; primarily motivated by an increase in the role of α decay in the spectroscopy of unstable nuclei and the synthesis of new elements in the study of superheavy elements. Varga et al.[6] calculated the α decay half-life for the ground state of ^{212}Po by using both shell-model and cluster-model configurations. Buck has shown that the large amount of data on lifetimes for favored α decay of heavy nuclei can be reproduced satisfactorily using a simple α-cluster model[7]. It is very interesting to study α radioactivity on the basis of an α-cluster model. In this work, we use the double folding model to calculate the microscopic nuclear potentials between an α particle and daughter nuclei, which are composed of parent nuclei. The nuclear potentials are obtained by folding the matter density distributions of both the α particle and daughter nuclei with realistic M3Y effective nucleon-nucleon (NN) interaction. The Coulomb

Received 5 June 2008

* Supported by National Natural Science Foundation of China (60572177)

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potential is calculated simply by the point-like model where a point α particle is assumed to interact with the daughter nucleus. α decay half-lives of new superheavy elements are calculated with the above approach for nuclear and Coulomb potentials. The results are compared with existing theoretical and experimental data to test the validity of this method. When the calculated results are in agreement with the available experimental data, α decay half-lives of possibly synthesized new elements in future experiments are predicted.

2 The framework of the double folding model

In the double folding model the nuclear potential is calculated as

$$V_{\rm N}(R)=\iint\rho_1(r_1)v(s=|\boldsymbol{R}+\boldsymbol{r}_2-\boldsymbol{r}_1|)\rho_2(r_2){\rm d}\boldsymbol{r}_1{\rm d}\boldsymbol{r}_2\,, \tag{1}$$

where $\rho_1(r_1)$ and $\rho_2(r_2)$ are the matter density distribution functions of the two nuclei. The density distribution function has a Gaussian form for an α particle

$$\rho(r)=0.4229\exp(-0.7024r^2), \tag{2}$$

whose volume integral is equal to the mass number 4 of an α particle. The density distribution function of the daughter nucleus can be described by the spherically symmetric Fermi function,

$$\rho(r)=\rho_0/[1+\exp((r-c)/a)], \tag{3}$$

where the half-density radius c is followed as[8]

$$c=r_\rho(1-\pi^2a^2/3r_\rho^2), \tag{4}$$

$$r_\rho=1.13A_{\rm d}^{1/3}\,, \tag{5}$$

where the diffuseness $a=0.54$ fm and $A_{\rm d}$ is the mass number of the daughter nucleus. The value of ρ_0 is determined by normalization so that

$$\int\rho_i(r_i){\rm d}\boldsymbol{r}_i=A_{\rm d}\,. \tag{6}$$

The NN interaction v between two nucleons in Eq. (1) is frequently given by the M3Y interaction, which is designed to reproduce the G-matrix elements on an oscillatory basis[9]. We refer to this as the density-independent NN interaction. It is composed of the direct part and zero-range pseudo-potential representing knock-on exchange. The density-independent NN interaction is given by

$$v^{\rm (M3Y)}(s,E)=7999\frac{{\rm e}^{-4s}}{4s}-2134\frac{{\rm e}^{-2.5s}}{2.5s}+\hat{J}_{00}(E)\delta(s)\,. \tag{7}$$

where the first and the second terms denote the direct part, and the zero-range pseudo term $\hat{J}_{00}(E)$ follows as

$$\hat{J}_{00}(E)\approx-276[1-0.005(E/A)]\ {\rm MeV\cdot fm^3}\,. \tag{8}$$

The M3Y force range was chosen to assure a long-range tail of the one-pion exchange potential as well as a short-range repulsive part. In order to consider the higher-order exchange effects and Pauli blocking effects, a density-dependent factor F was inserted into the density-independent NN interaction. So the general expression of the density-dependent M3Y effective NN interaction is given by

$$v(s,\rho,E)=F(\rho,E)v^{\rm (M3Y)}(s,E)\,, \tag{9}$$

$$F(\rho,E)=C[1-\beta(E)\rho_1^{2/3}][1-\beta(E)\rho_2^{2/3}], \tag{10}$$

where ρ_1 and ρ_2 are the matter density distributions of an α particle and the daughter nucleus. The parameter $\beta(E)$ is related to the mean free path in the nuclear medium, and is considered to remain constant and independent of energy; here its value remains the same $\sim$1.6 fm^2[10] obtained from the optimum fit to data, while the parameter C is basically the overall normalization constant; its value is kept fixed and equal to unity. According to Eq. (1), the folding nuclear potential is obtained by folding the matter densities of the α particle and the daughter nucleus with the general density-dependent effective NN interaction. The Coulomb potential is calculated between the α particle and the residual daughter nucleus by the pointlike plus uniform model, and it is given by

$$V_{\rm C}(R)=Z_1Z_2e^2\begin{cases}\dfrac{1}{R} & (R>R_{\rm C})\\[2ex] \dfrac{1}{2R_{\rm C}}\left[3-\left(\dfrac{R}{R_{\rm C}}\right)^2\right] & (R<R_{\rm C})\end{cases}, \tag{11}$$

where assuming spherical charge distribution for the residual daughter nucleus and the point particle for the emitted α particle and $e^2=1.44$ MeV·fm, $R_{\rm C}=c_\alpha+c_{\rm d}$ where c_α and $c_{\rm d}$ are calculated by Eq. (4), Z_1 and Z_2 represent the atomic numbers of the emitted the α particle and the residual daughter nucleus, respectively. The total interaction potential $V(R)$ between α particle and the daughter nucleus is equal to the sum of the nuclear potential $V_{\rm N}(R)$, the Coulomb potential $V_{\rm C}(R)$ and the centrifugal barrier. So

$$V(R)=V_{\rm N}(R)+V_{\rm C}(R)+\frac{l(l+1)\hbar^2}{2\mu R^2}\,, \tag{12}$$

where $\mu=\dfrac{m_\alpha m_{\rm d}}{m_\alpha+m_{\rm d}}$ is the reduced mass of the α-daughter nucleus system, m_α and $m_{\rm d}$ denote the

masses of the α particle and the daughter nucleus, respectively, and all are in the unit of MeV/c^2.

The half-life of a parent nucleus decaying by the mode of emitting an α particle is calculated with WKB barrier penetration probability. The decay half-life $T_{1/2}$ is given by[11]

$$T_{1/2} = [(\pi\hbar \ln 2)/E_{\rm v}][1+\exp(K)]. \quad (13)$$

The wave number K is given by

$$K = \frac{2}{\hbar}\int_{R_1}^{R_2}\sqrt{2\mu(V(R)-E_{\rm v}-Q)}{\rm d}R\,, \quad (14)$$

where μ is the reduced mass of the α particle and the daughter nucleus. Q is the release energy of the emitted α particle. $E_{\rm v}$ is the zero-point vibration energy, which implicitly considers the shell effect because it is proportional to the Q value which is at its maximum when the daughter nucleus has a magic number of neutrons and protons. R_1 and R_2 are the two turning points of the WKB approximation integral obtained from the equation

$$V(R_1) = Q + E_{\rm v} = V(R_2). \quad (15)$$

3 Numerical calculations and results

According to the above method, we can calculate the α decay half-life of a parent nucleus by Eq. (13) using the microscopic double folding model with Eq. (1) and acquiring the Coulomb potential from Eq. (11) along with the total energy $V(R)$ from Eq. (12). In the present calculation the zero-point vibration energy used is the same as in Ref. [12] determined by

$$\text{even}(Z)\text{-even}(N)\ \text{nuclei}: E_{\rm v} = 0.1045Q; \quad (16)$$

$$\text{odd}(Z)\text{-even}(N)\ \text{nuclei}: E_{\rm v} = 0.0962Q; \quad (17)$$

$$\text{even}(Z)\text{-odd}(N)\ \text{nuclei}: E_{\rm v} = 0.0907Q; \quad (18)$$

$$\text{odd}(Z)\text{-odd}(N)\ \text{nuclei}: E_{\rm v} = 0.0767Q, \quad (19)$$

where Q is the release energy of the emitted α particle. Z and N represent the proton and neutron numbers of the parent nuclei, respectively. We can see that the proportional value between the zero-point vibration energy $E_{\rm v}$ and the release energy Q is the largest for even-even parent nuclei and is the smallest for odd-odd parent nuclei.

In the zero-range pseudo term $\hat{J}_{00}(E)$, E/A is the laboratory energy per nucleon of projectile with the unit of MeV/nucleon. For the α decay process, the experimental α decay energy is almost equal to zero, so the energy dependence E/A of the exchange term is very weak. The zero-range pseudo term $\hat{J}_{00}(E)$ is practically independent of energy for the α decay process and is taken to be -276 MeV·fm^3. We perform the zero angular-momentum transfer for the α decay process in all calculations. In order to easily compare between the experimental half-life and the theoretically calculated one a hindrance factor (HF) is defined as the ratio of the experimental $T_{1/2}$ to the theoretical $T_{1/2}$[13], $HF = T_{1/2}(\text{exp.})/T_{1/2}(\text{cal})$. The results are shown for the variation of HF with mass number for Z = 103—118 isotopes which are in the range of superheavy elements in Fig. 1, where the experimental α decay half-lives of the different isotopes

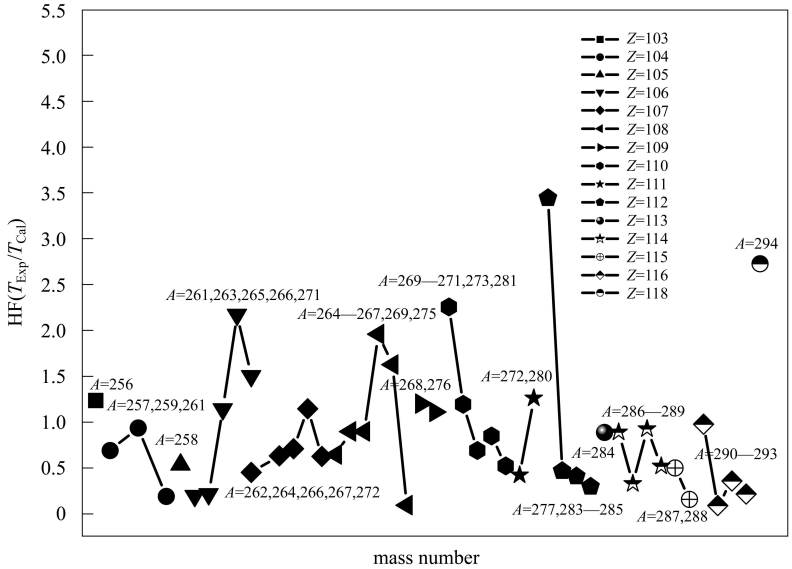

Fig. 1. The HF for Z = 103—118 isotopes.

are from Ref. [14] for $Z = 103$—105, Refs. [14—16] for $Z = 106$, Refs. [14, 17—19] for $Z = 107$, Refs. [14, 20—23] for $Z = 108$, Refs. [17, 18] for $Z = 109$, Refs. [14, 22, 24] for $Z = 110$, Refs. [14, 17] for $Z = 111$, Refs. [16, 21, 25] for $Z = 112$, Ref. [17] for $Z = 113$, 115 and Ref. [16] for $Z = 114$, 116, 118. As one can see in Fig. 1, the quantitative description is reasonable between the calculated results and the experimental data. The HF is within 2.5 for many nuclei except for seven cases which are slightly small and two cases which are slightly large. It indicates that the half-lives of α decay for ^{261}104, ^{261,263}106, ^{275}108, ^{285}112, ^{288}115 and ^{291}116 elements are largely overestimated. However, the theoretical Viola-Seaborg systematics (VSS)[26] estimates the half-lives as largely than the present calculation for the above seven elements and then the hindrance factors of VSS are very small: 0.048, 0.058, 0.02, 0.06, 0.09 and 0.02 for ^{261}104, ^{261,263}106, ^{275}108, ^{285}112, ^{288}115 and ^{291}116, respectively, showing inconsistencies with the experimental data. So the present calculation can still estimate much better the α decay half-life of superheavy elements than that estimated by VSS. As one can see in Fig. 1, the hindrance factors of the two cases are

Table 1. Comparison between the experimental and calculated half-lives of α decay for superheavy elements.

parent nuclei		this work	MM model	Möller	Exp.	
Z	A	$\log_{10}T$/ms	$\log_{10}T$/ms	$\log_{10}T$/ ms	Q_α	$\log_{10}T$/ms
108	274	2.637	2.796	2.941	9.50	2.825
108	273	1.912	3.433	3.951	9.90	2.602
108	272	0.990	2.258	3.710	10.10	1.175
108	271	1.972	2.307	5.789	9.90	2.602
108	270	3.345	2.370	5.318	9.30	3.410
109	275	0.531	2.202	2.176	10.48	1.089
109	274	1.082	1.835	2.541	10.50	1.404
109	273	−0.279	1.316	2.828	10.82	0.256
109	272	0.871	0.911	4.306	10.60	1.156
109	271	1.540	1.189	4.326	10.14	1.963
110	280	3.907	3.152	4.871	9.30	4.098
110	279	3.436	3.395	3.874	9.60	4.103
110	278	1.884	1.766	1.007	10.0	2.089
110	277	1.493	2.055	1.199	10.30	2.185
110	276	0.336	0.553	0.207	10.60	0.528
110	275	−0.496	1.150	1.10	11.10	0.218
110	274	−1.554	−0.132	0.753	11.40	−1.359
111	279	1.081	1.894	0.622	10.52	1.612
111	278	1.169	1.675	−0.107	10.72	1.476
111	277	−0.531	1.036	−0.719	11.18	0.008
111	276	−0.256	1.037	−0.398	11.32	0.052
111	275	−1.341	0.195	−0.314	11.55	−0.830
113	283	2.452	2.948	5.546	10.26	2.921
113	282	3.364	3.558	4.626	10.16*	
113	281	−0.574	−0.049	1.799	11.46*	
113	280	−0.246	0.045	0.488	11.58*	
113	279	−3.110	−2.567	−2.508	12.65*	
117	293	0.537	1.088	0.599	11.47*	
117	292	0.849	1.454	0.919	11.60	1.150
117	291	−0.427	0.622	0.508	11.90	0.101
117	290	0.115	0.40	2.128	11.93*	
117	289	−1.292	−0.792	−0.077	12.31*	
117	288	−1.384	−1.092	0.201	12.63*	
117	287	−2.201	−1.722	−0.318	12.76*	

∗ denotes Q_α is from the macroscopic-microscopic (MM) model[27].

3.4 and 2.7 for ^{277}112 and ^{294}118, respectively. From the definition of hindrance factor, one can see that the half-lives of the two cases are underestimated, and may not consider the spin-parity conservation of the decay nuclei. In the calculation the angular momentum is equal to zero. The term $\hbar^2 l(l+1)/(2\mu R^2)$ in Eq. (12) does not contribute to the barrier. This results in the tunnelling probability veiny increased and the half-life veiny underestimated. For the two cases ^{277}112 and ^{294}118 it is shown that the angular momentum carried by the α particle is non-zero.

According to the above process, we calculate the α decay half-lives of some superheavy elements, which will be possibly synthesized in future experiments. Since the release energy Q is entered into the integral Eq. (14) and the zero-point vibration energy $E_{\rm v}$ is proportional to Q, the half-life is sensitive to the release energy Q. In Ref. [27] one can see that the calculated α decay energies agree well with the experimental data. We can use the calculated α decay energies to obtain the α decay half-lives for some possibly synthesized superheavy elements. In Table 1 the calculated results are shown in comparison with the macroscopic-microscopic (MM) model[27], Möller's results and the experimental data. It is shown that our results are close to the experimental data and better than Möller's results, and the results of some cases are also better than those of the MM model. The results of ^{270}108, ^{279,280}110 in comparison with the experimental data is in the order of seconds, indicating that these nuclei live long enough to be easily detected after synthesis in the current setup. The lifetimes of the rest isotopes of $Z = 108$, 109 and 110 are basically in the order of milliseconds. For the predicted lifetimes of the $Z = 111$, 113, 117 isotopes, one can see that the half-life of the ^{282}113 element is in the order of seconds. The element also lives long to be easily detected after enough synthesis in the current setup. The half-lives of the $Z = 111$ and 117 isotopes stay in the order of milliseconds, indicating that these nuclei are hard to detect in the current setup.

4 Summary and concluding remarks

In conclusion, the nuclear potential is obtained by using the density-dependent nucleon-nucleon interaction with the zero-range pseudo exchange term in the frame of the double folding model. In the frame of the WKB approximation, α decay half-lives of superheavy elements are calculated in comparison with the experimental data through the defined hindrance factor ($\mathrm{HF} = T_{\rm exp}/T_{\rm cal}$). The results of the present calculation can reproduce well α decay half-lives in the superheavy region. The procedure successfully provides theoretical predictions of α decay half-lives for possibly synthesized superheavy elements in future experiments.

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