$B\to 0^+(1^+)+$ missing energy in unparticle physics*

M. Jamil Aslam[1,2] LÜ Cai-Dian(吕才典)[1,3;1)]

1 (Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, China)
2 (Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan)
3 (School of Physics and Technology, Nanjing Normal University, Nanjing 210097, China)

Abstract We examine the effects of an unparticle $\mathcal{U}$ as a possible source of missing energy in the p-wave decays of a B meson. The dependence of the differential branching ratio on the $K_0^*(K_1)$ – meson's energy is discussed in the presence of scalar and vector unparticle operators and significant deviation from the standard model value is found after addition of these operators. Finally, we have shown the dependence of the branching ratio for the above-mentioned decays on the parameters of unparticle stuff like effective couplings, cutoff scale $\Lambda_{\mathcal{U}}$ and the scale dimensions $d_{\mathcal{U}}$.

Key words unparticle, missing energy, B physics

PACS 14.80.-j, 13.20.He

1 Introduction

Flavor changing neutral current (FCNC) processes induced by $b\to s$ transitions are not allowed at tree level in the Standard Model (SM), but are generated at loop level and are further suppressed by the CKM factors. Therefore, these decays are very sensitive to the physics beyond the SM via the influence of new particles in the loop. Though the branching ratios of FCNC decays are small in the SM, quite interesting results are obtained from the experiments both for the inclusive $B\to X_s l^+l^-$ [1] and exclusive decay modes $B\to K l^+l^-$ [2—4] and $B\to K^* l^+l^-$ [5]. These results are in good agreement with theoretical estimates[6—8].

Among different semileptonic decays induced by $b\to s$ transitions, $b\to s\nu\bar{\nu}$ decays are of particular interest, because of absence of a photonic penguin contribution and hadronic long distance effects gives much smaller theoretical uncertainties. But experimentally, it is too difficult to measure the inclusive decay modes $B\to X_s\nu\bar{\nu}$ as one has to sum over all the X_s's. Therefore, exclusive $B\to K(K^*)\nu\bar{\nu}$ decays play a peculiar role both from the experimental and theoretical points of view. The theoretical estimates of the branching ratio of these decays are $Br(B\to K\nu\bar{\nu}\sim 10^{-5})$ and $Br(B\to K^*\nu\bar{\nu}\sim 10^{-6})$[9] whereas the experimental bounds given by the B-factories, BELLE and BaBar, on these decays are[10, 11]:

$$\begin{aligned} &Br(B\to K\nu\bar{\nu})<1.4\times 10^{-5},\\ &Br(B\to K^*\nu\bar{\nu})<1.4\times 10^{-4}. \end{aligned} \tag{1}$$

These processes, based on $b\to s\nu\bar{\nu}$, are very sensitive to new physics and have been studied extensively in the literature in the context of large extra dimension model and Z' models[12, 13]. Any new physics model which can provide a relatively light new source of missing energy (which is attributed to neutrinos in the SM) can potentially enhance the observed rates of $B\to K(K^*)+$ missing energy. Recently, H. Georgi proposed one such model of unparticles, which is one of the tantalizing issues these days[14]. The main idea of Georgi's model is that at a very high energy our theory contains the fields of the standard model and the fields of a theory with a nontrivial infrared fixed point, which he called BZ (Banks-Zaks) fields[15]. The interaction among the two sets is through the exchange of particles with a large mass scale $M_{\mathcal{U}}$. The coupling between the SM fields and BZ fields are nonrenormalizable below this

Received 6 August 2008, Revised 31 October 2008
* Supported by National Science Foundation of China (10735080 and 10625525), and China Postdoctoral Science Foundation
1) E-mail: lucd@ihep.ac.cn

scale and are suppressed by the powers of $M_{\mathcal{U}}$. The renormalizable couplings of the BZ fields then produce dimensional transmutation and the scale invariant unparticle emerges below an energy scale $\Lambda_{\mathcal{U}}$. In the effective theory below the scale $\Lambda_{\mathcal{U}}$ the BZ operators match unparticle operators, and the renormalizable interaction matched a new set of interactions between the standard model and the unparticle fields. The outcome of this model is the collection of unparticle stuff with scale dimension $d_{\mathcal{U}}$, which is just like a non-integral number of invisible massless particles, whose production might be detectable in missing energy and momentum distributions[16].

This idea has promoted a lot of interest in unparticle physics and its signatures have been discussed at colliders[16—20], in low energy physics[21], Lepton Flavor Violation[22], unparticle physics effects in B_s mixing[23], and also in cosmology and astrophysics[24]. Aliev et al. have studied $B\rightarrow K(K^*)+$ missing energy in unparticle physics[25]. They studied the effects of an unparticle $\mathcal{U}$ as a possible source of missing energy in these decays. They found the dependence of the differential branching ratio on the $K(K^*)$ -meson's energy in the presence of scalar and vector unparticle operators and then, using the upper bounds on these decays, they put stringent constraints on the parameters of the unparticle stuff.

The studies are even more complete if similar studies for the p-wave decays of a B meson such as $B\rightarrow K_0^*(1430)+\not{E}$ ($\not{E}$ is missing energy) and $B\rightarrow K_1(1270)+\not{E}$, where $K_0^*(1430)$ and $K_1(1270)$ are the pseudoscalar and axial vector mesons respectively, are carried out. In this paper, we have studied these p-wave decays of B mesons in unparticle physics using the framework of Aliev et al.[25] We have considered the decay $B\rightarrow K_0^*(K_1)\nu\bar{\nu}$ in the SM although for these modes no signals have been observed so far, but in future B-factories where enough data are expected, these decays will be observed. These Super B-factories will measure these processes by analyzing the spectra of the final state hadrons. In doing this measurement a cut at high momentum on the hadron is imposed, in order to suppress the background. Therefore, the unparticle would give us a unique distribution of the high energy hadrons in the final state, such that in future B-factories one will be able to distinguish the presence of an unparticle by observing the spectrum of the final state hadrons in $B\rightarrow(K,\ K^*,\ K_0^*,\ K_1)+\not{E}$[25].

This work is organized as follows. In section 2, after giving the expression for the effective Hamiltonian for the decay $b\rightarrow s\nu\bar{\nu}$, we define the scalar and vector unparticle physics operators for $b\rightarrow s\mathcal{U}$. Then using these expressions we calculate the various contributions to the decay rates of $B\rightarrow K_0^*(K_1)+\not{E}$ both from the SM and unparticle theory in Section 3. Recently, Grinstein et al. made comments on the unparticle[26], mentioning that Mack's unitarity constraint lowers the bounds on CFT operator dimensions, e.g $d_{\mathcal{U}}\geqslant 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. We will also give the expressions for the decay rate using these modified vector operators in the same section. Finally, section 4 contains our numerical results and conclusions.

2 Effective Hamiltonian in the SM and unparticle operators

The flavor changing neutral current $b\rightarrow s\nu\bar{\nu}$ is of particular interest both from the theoretical and experimental point of view. One of the main reasons of interest is the absence of long distance contributions related to four-quark operators in the effective Hamiltonian. In this respect, the transition to the neutrino represents a clean process even in comparison with $b\rightarrow s\gamma$ decay, where long-distance contributions, though small, are expected to be present [27]. In the Standard Model these processes are governed by the effective Hamiltonian

$$\mathcal{H}_{\rm eff}=\frac{G_{\rm F}}{\sqrt{2}}\frac{\alpha}{2\pi}V_{\rm tb}V_{\rm ts}^*C_{10}\bar{s}\gamma^{\mu}(1-\gamma_5)b\bar{\nu}\gamma_{\mu}(1-\gamma_5)\nu, \tag{2}$$

where $V_{\rm tb}V_{\rm ts}^*$ are the elements of the Cabbibo-Kobayashi Maskawa Matrix and C_{10} is obtained from the Z^0 penguin and box diagrams where the dominant contribution corresponds to a top quark intermediate state and it is

$$C_{10}=\frac{D(x_{\rm t})}{\sin^2\theta_{\rm w}}. \tag{3}$$

$\theta_{\rm w}$ is the Weinberg angle and $D(x_{\rm t})$ is the usual Inami-Lim function, given by

$$D(x_{\rm t})=\frac{x_{\rm t}}{8}\left\{\frac{x_{\rm t}+2}{x_{\rm t}-1}+\frac{3x_{\rm t}-6}{(x_{\rm t}-1)^2}\ln(x_{\rm t})\right\}, \tag{4}$$

with $x_{\rm t}=m_{\rm t}^2/m_{\rm W}^2$.

The unparticle transition at the quark level can be described by $b\rightarrow s\mathcal{U}$, where one can consider the following operators.

1) Scalar unparticle operator

$$C_{\rm s}\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{s}\gamma_{\mu}b\partial^{\mu}O_{\mathcal{U}}+C_{\rm P}\frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{s}\gamma_{\mu}\gamma_5 b\partial^{\mu}O_{\mathcal{U}}\,. \tag{5}$$

2) Vector unparticle operator

$$C_V \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s}\gamma_\mu b O_{\mathcal{U}}^\mu + C_A \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s}\gamma_\mu\gamma_5 b O_{\mathcal{U}}^\mu . \qquad (6)$$

The propagator for the scalar unparticle field can be written as[14, 16, 17]

$$\int d^4x e^{iP\cdot x} \langle 0|T O_{\mathcal{U}}(x) O_{\mathcal{U}}(0)|0\rangle = i\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}(-P^2)^{d_{\mathcal{U}}-2} \qquad (7)$$

with

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \qquad (8)$$

3 Differential decay widths

In the Standard Model the decay $B\to K_0^*(K_1)+\not{E}$ is described by the decay $B\to K_0^*(K_1)\nu\bar{\nu}$. At quark level this process is governed by the effective Hamiltonian defined in Eq. (2) which when sandwiched between B and $K_0^*(K_1)$ involves the hadronic matrix elements for the exclusive decay $B\to K_0^*(K_1)\nu\bar{\nu}$. They can be parameterized by the form factors and the non-vanishing matrix elements for $B\to K_0^*$[27]:

$$\langle K_0^*(p')|\bar{s}\gamma_\mu\gamma_5 b|B(p)\rangle = -i\left[f_+(q^2)(p+p')_\mu + f_-(q^2)q_\mu\right], \qquad (9)$$

where $q_\mu=(p+p')_\mu$. Using the above definition and taking into account the three species of neutrinos in the Standard Model, the differential decay width as a function of K_0^* energy $(E_{K_0^*})$ can be written as[27]:

$$\frac{d\Gamma^{SM}}{dE_{K_0^*}} = \frac{G_F^2\alpha^2}{2^7\pi^5 M_B^2}|V_{tb}V_{ts}^*|^2|C_{10}|^2 f_+^2(q^2)\times \sqrt{\lambda^3\left(M_B^2, M_{K_0^*}^2, q^2\right)} \qquad (10)$$

with $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2bc-2ca$ and $q^2 = M_B^2+M_{K_0^*}^2-2M_B E_{K_0^*}$. Here $f_+(q^2)$ and $f_-(q^2)$ are the form factors which are non-perturbative quantities and can be calculated using some models. The model used here was calculated by using the Light Front Quark Model (LFQR) by Cheng et al.[27] and can be parameterized as:

$$F(q^2) = \frac{F(0)}{1-aq^2/M_B^2+b(q^2/M_B^2)^2} .$$

The fitted parameters are given in Table 1.

Table 1. Parameters for the $B\to K_0^*$ form factors.

	$F(0)$	a	b
f_+	−0.26	1.36	0.86
f_-	0.21	1.26	0.93

Similarly, for the $B\to K_1$ transition the matrix elements can be parameterized as[28]

$$\langle K_1(k,\varepsilon)|V_\mu|B(p)\rangle = i\varepsilon_\mu^*(M_B+M_{K_1})V_1(q^2) - (p+k)_\mu(\varepsilon^*\cdot q)\frac{V_2(q^2)}{M_B+M_{K_1}} - q_\mu(\varepsilon\cdot q)\frac{2M_{K_1}}{s}[V_3(q^2)-V_0(q^2)], \qquad (11)$$

$$\langle K_1(k,\varepsilon)|A_\mu|B(p)\rangle = \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_B+M_{K_1}}\varepsilon^{*\nu}p^\alpha k^\beta A(q^2), \qquad (12)$$

where $V_\mu=\bar{s}\gamma_\mu b$ and $A_\mu=\bar{s}\gamma_\mu\gamma_5 b$ are the vector and axial vector currents respectively and ε_μ^* is the polarization vector for the final state axial vector meson. In this case we have used the form factors that were calculated by Paracha et al.[28] and the corresponding expressions are:

$$A(s) = \frac{A(0)}{(1-s/M_B^2)(1-s/M_B'^2)},$$

$$V_1(s) = \frac{V_1(0)}{\left(1-s/M_{B_A^*}^2\right)\left(1-s/M_{B_A^*}'^2\right)}\times\left(1-\frac{s}{M_B^2-M_{K_1}^2}\right),$$

$$V_2(s) = \frac{\tilde{V}_2(0)}{\left(1-s/M_{B_A^*}^2\right)\left(1-s/M_{B_A^*}'^2\right)} - \frac{2M_{K_1}}{M_B-M_{K_1}}\frac{V_0(0)}{(1-s/M_B^2)(1-s/M_B'^2)} \qquad (13)$$

with

$$A(0) = -(0.52\pm0.05),$$
$$V_1(0) = -(0.24\pm0.02), \qquad (14)$$
$$\tilde{V}_2(0) = -(0.39\pm0.03).$$

The differential decay rate can be calculated as[25]:

$$\frac{d\Gamma^{SM}}{dE_{K_1}} = \frac{G_F^2\alpha^2}{2^9\pi^5 M_B^2}|V_{tb}V_{ts}^*|^2|C_{10}|^2\lambda^{1/2}|M_{SM}|^2 \qquad (15)$$

where

$$|M_{SM}|^2 = \frac{8q^2\lambda|A(q^2)|^2}{(M_B+M_{K_1})^2} + \frac{1}{M_{K_1}^2}\left[\lambda^2\frac{|V_2(q^2)|^2}{(M_B+M_{K_1})^2} + (M_B+M_{K_1})^2\left(\lambda+12M_{K_1}^2q^2\right)|V_1(q^2)|^2 - \lambda\left(M_B^2-M_{K_1}^2-q^2\right)\mathrm{Re}(V_1^*(q^2)V_2(q^2) + V_2^*(q^2)V_1(q^2))\right] \qquad (16)$$

and $\lambda=\lambda\left(M_B^2, M_{K_1}^2, q^2\right)$ with $q^2 = M_B^2+M_{K_1}^2-2M_B E_{K_1}$.

Now in the decay mode $B\to K_0^*(K_1)+\not{E}$, the missing energy $\not{E}$ can also be attributed to the unparticle and hence the unparticle can also contribute to these decay modes. Therefore, the signature of the two decay modes $B\to K_0^*(K_1)\nu\bar{\nu}$ and $B\to K_0^*(K_1)\mathcal{U}$ should be similar to that for $B\to K(K^*)\nu\bar{\nu}$ and $B\to K(K^*)\mathcal{U}$ given in Ref. [25].

3.1 The scalar unparticle operator

Using the scalar unparticle operator defined in Eq. (5) the matrix element for $B\to K_0^*\mathcal{U}$ can be written as

$$\mathcal{M}^{S\mathcal{U}}_{K_0^*} = \frac{1}{\Lambda^{d_\mathcal{U}}}\langle K_0^*(p')|\bar{s}\gamma_\mu(\mathcal{C}_S+\mathcal{C}_P\gamma_5)b|B(p)\rangle\partial^\mu O_\mathcal{U} = \frac{1}{\Lambda^{d_\mathcal{U}}}\mathcal{C}_P[f_+(q^2)\left(M_B^2-M_{K_0^*}^2\right)+f_-(q^2)q^2]O_\mathcal{U}\ . \tag{17}$$

Now the decay rate for $B\to K_0^*\mathcal{U}$ can be evaluated to be:

$$\frac{d\Gamma^{S\mathcal{U}}}{dE_{K_0^*}} = \frac{1}{8\pi^2 m_B}\sqrt{E_{K_0^*}^2-M_{K_0^*}^2}\left|\mathcal{M}^{S\mathcal{U}}\right|^2, \tag{18}$$

where

$$\left|\mathcal{M}^{S\mathcal{U}}\right|^2 = |\mathcal{C}_P|^2\frac{A_{d_\mathcal{U}}}{\Lambda^{2d_\mathcal{U}}}\left(M_B^2+M_{K_0^*}^2-2M_BE_{K_0^*}\right)^{d_\mathcal{U}-2}\times \left[f_+(q^2)\left(M_B^2-M_{K_0^*}^2\right)+ f_-(q^2)\left(M_B^2+M_{K_0^*}^2-2M_BE_{K_0^*}\right)\right]^2. \tag{19}$$

Following the same lines, the corresponding matrix element for $B\to K_1\mathcal{U}$ is

$$\mathcal{M}^{S\mathcal{U}}_{K_1} = \frac{1}{\Lambda^{d_\mathcal{U}}}\langle K_1(p')|\bar{s}\gamma_\mu(\mathcal{C}_S+\mathcal{C}_P\gamma_5)b|B(p)\rangle\partial^\mu O_\mathcal{U} = \frac{i}{\Lambda^{d_\mathcal{U}}}\mathcal{C}_S(\varepsilon^*\cdot q)\Big[(M_B+M_{K_1})V_1(q^2)- (M_B-M_{K_1})V_2(q^2)- 2M_{K_1}(V_3(q^2)-V_0(q^2))\Big]O_\mathcal{U}, \tag{20}$$

and the differential decay rate is

$$\frac{d\Gamma^{S\mathcal{U}}}{dE_{K_1}} = \frac{M_B}{2\pi^2}\frac{A_{d_\mathcal{U}}}{\Lambda^{2d_\mathcal{U}}}|\mathcal{C}_S|^2|V_0(q^2)|^2\left(E_{K_1}^2-M_{K_1}^2\right)^{3/2}\times \left(M_B^2+M_{K_1}^2-2M_BE_{K_1}\right)^{d_\mathcal{U}-2}. \tag{21}$$

One can see from Eq. (18) and Eq. (21) that the scalar unparticle contribution to the decay rate depends on $\mathcal{C}_P$, $\mathcal{C}_S$, $d_\mathcal{U}$ and $\Lambda_\mathcal{U}$. Therefore one can see the behavior of the decay rates for the said decays on these parameters, for which we hope to get constraints once experimental data for these decays become available. This we will do in a separate section.

3.2 The vector unparticle operator

The matrix element for $B\to K_0^*\mathcal{U}$ using the vector unparticle operator defined in Eq. (6) and the definition of the form factors given in Eq. (9) can be calculated as:

$$\mathcal{M}^{V\mathcal{U}}_{K_0^*} = \frac{1}{\Lambda^{d_\mathcal{U}-1}}\langle K_0^*(p')|\bar{s}\gamma_\mu(\mathcal{C}_V+\mathcal{C}_A\gamma_5)b|B(p)\rangle O^\mu_\mathcal{U} = \frac{1}{\Lambda^{d_\mathcal{U}-1}}\mathcal{C}_A[f_+(q^2)(p+p')_\mu+f_-(q^2)q_\mu]O^\mu_\mathcal{U}\ . \tag{22}$$

The differential decay rate is then

$$\frac{d\Gamma^{V\mathcal{U}}}{dE_{K_0^*}} = \frac{1}{8\pi^2 m_B}\frac{A_{d_\mathcal{U}}}{\Lambda^{2d_\mathcal{U}-2}}|\mathcal{C}_A|^2|f_+(q^2)|^2\times \left(M_B^2+M_{K_0^*}^2-2M_BE_{K_0^*}\right)^{d_\mathcal{U}-2}\sqrt{E_{K_0^*}^2-M_{K_0^*}^2}\times \left\{-\left(M_B^2+M_{K_0^*}^2+2M_BE_{K_0^*}\right)+ \frac{\left(M_B^2-M_{K_0^*}^2\right)^2}{\left(M_B^2+M_{K_0^*}^2-2M_BE_{K_0^*}\right)}\right\}. \tag{23}$$

For $B\to K_1$ case the matrix element for $B\to K_1\mathcal{U}$ is

$$\mathcal{M}^{V\mathcal{U}}_{K_1} = \frac{1}{\Lambda^{d_\mathcal{U}-1}}\langle K_1(p')|\bar{s}\gamma_\mu(\mathcal{C}_V+\mathcal{C}_A\gamma_5)b|B(p)\rangle O^\mu_\mathcal{U} = \left[\frac{\mathcal{C}_V}{\Lambda^{d_\mathcal{U}-1}}(i\varepsilon^*_\mu(M_B+M_{K_1})V_1(q^2)- i(p+p')_\mu(\varepsilon^*\cdot q)\frac{V_2(q^2)}{M_B+M_{K_1}}- iq_\mu(\varepsilon^*\cdot q)\frac{2M_{K_1}}{q^2}(V_3(q^2)-V_0(q^2)))+ \frac{\mathcal{C}_A}{\Lambda^{d_\mathcal{U}-1}}\left(\frac{2A(q^2)}{M_B+M_{K_1}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{\nu*}p^\alpha p'^\beta\right)\right]O^\mu_\mathcal{U} \tag{24}$$

and the differential decay rate will be:

$$\frac{d\Gamma^{V\mathcal{U}}}{dE_{K_1}} = \frac{1}{8\pi^2 m_B}\frac{A_{d_\mathcal{U}}}{\Lambda^{2d_\mathcal{U}-2}}\sqrt{E_{K_1}^2-M_{K_1}^2}(q^2)^{d_\mathcal{U}-2}\times \Bigg[8|\mathcal{C}_A|^2M_B^2(E_{K_1}^2-M_{K_1}^2)\frac{A(q^2)}{(M_B+M_{K_1})^2}+ |\mathcal{C}_V|^2\frac{1}{M_{K_1}^2(M_B+M_{K_1})^2q^2}\times \Big[(M_B+M_{K_1})^4(3M_{K_1}^4+2M_B^2M_{K_1}^2- 6M_BM_{K_1}^2E_{K_1}+M_B^2E_{K_1}^2)|V_1(q^2)|^2+ 2M_B^4\left(E_{K_1}^2-M_{K_1}^2\right)|V_2(q^2)|^2+4(M_B+M_{K_1})^2\times \left(M_BE_{K_1}-M_{K_1}^2\right)\left(M_{K_1}^2-E_{K_1}^2\right)\times M_B^2(V_1V_2^*+V_2V_1^*)\Big]\Bigg]. \tag{25}$$

The total decay width can be obtained if we integrate over the energy of the final state meson in the range $M_{\mathrm{K(K_1)}} < E_{\mathrm{K(K_1)}} < \left(M_{\mathrm{B}}^2+M_{\mathrm{K(K_1)}}^2\right)/2M_{\mathrm{B}}$ for $\mathrm{B}\to\mathrm{K}(\mathrm{K}_1)+\not{E}$.

Recently, Grinstein et al. have made a comment on the unparticle[26] in which they mentions that Mack's unitarity constraint lower the bounds on the CFT operator dimensions, e.g. $d_{\mathcal{U}}\geqslant 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. The modified vector propagator is

$$\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}Px}\langle 0|T(O_{\mathcal{U}}^{\mu}(x)O_{\mathcal{U}}^{\nu}(x))|0\rangle = A_{d_{\mathcal{U}}}(-g^{\mu\nu}+aP^{\mu}P^{\nu}/P^2)(P^2)^{d_{\mathcal{U}}-2}. \tag{26}$$

Here P is the momentum of the unparticle, $A_{d_{\mathcal{U}}}$ is defined in Eq. (8) and $a\neq 1$ (in contrast to the value $a=1$ which was considered by Georgi[14]) but is defined as:

$$a=\frac{2(d_{\mathcal{U}}-2)}{(d_{\mathcal{U}}-1)}. \tag{27}$$

By incorporating this factor a in the vector unparticle operator Eqs. (23) and (25) are modified and the modified result of the decay rate for $\mathrm{B}\to\mathrm{K}_0^*\mathcal{U}$ is

$$\begin{aligned}\frac{\mathrm{d}\Gamma^{\mathrm{V}\mathcal{U}}}{\mathrm{d}E_{\mathrm{K}_0^*}} = {} & \frac{1}{8\pi^2 m_{\mathrm{B}}}\frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}}|\mathcal{C}_{\mathrm{A}}|^2|f_+(q^2)|^2\times \\ & \left(M_{\mathrm{B}}^2+M_{\mathrm{K}_0^*}^2-2M_{\mathrm{B}}E_{\mathrm{K}_0^*}\right)^{d_{\mathcal{U}}-2}\sqrt{E_{\mathrm{K}_0^*}^2-M_{\mathrm{K}_0^*}^2}\times \\ & \left[|f_+(q^2)|^2\left(-\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_0^*}^2+2M_{\mathrm{B}}E_{\mathrm{K}_0^*}\right)+\right.\right. \\ & \left.\frac{a\left(M_{\mathrm{B}}^2-M_{\mathrm{K}_0^*}^2\right)^2}{\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_0^*}^2-2M_{\mathrm{B}}E_{\mathrm{K}_0^*}\right)}\right)+ \\ & |f_-(q^2)|^2(a-1)\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_0^*}^2-2M_{\mathrm{B}}E_{\mathrm{K}_0^*}\right)+ \\ & \left.2(a-1)(f_+(q^2)f_-(q^2))\left(M_{\mathrm{B}}^2-M_{\mathrm{K}_0^*}^2\right)\right].\end{aligned} \tag{28}$$

Similarly, for $\mathrm{B}\to\mathrm{K}_1\mathcal{U}$ the result becomes

$$\begin{aligned}\frac{\mathrm{d}\Gamma^{\mathrm{V}\mathcal{U}}}{\mathrm{d}E_{\mathrm{K}_1}} = {} & \frac{1}{8\pi^2 m_{\mathrm{B}}}\frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}}\sqrt{E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2}(q^2)^{d_{\mathcal{U}}-2}\times \\ & \left[|\mathcal{M}_{11}|^2+|\mathcal{M}_{22}|^2+|\mathcal{M}_{33}|^2+|\mathcal{M}_{44}|^2+\right. \\ & \left.|\mathcal{M}_{23}|^2+|\mathcal{M}_{24}|^2+|\mathcal{M}_{34}|^2\right]\end{aligned} \tag{29}$$

with

$$\begin{aligned}
|\mathcal{M}_{11}|^2 = {} & 8|\mathcal{C}_{\mathrm{A}}|^2M_{\mathrm{B}}^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)\frac{A(q^2)}{(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2}, \\
|\mathcal{M}_{22}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[(M_{\mathrm{B}}+M_{\mathrm{K}_1})^4\left(3M_{\mathrm{K}_1}^2\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_1}^2-2M_{\mathrm{B}}E_{\mathrm{K}_1}\right)-\right.\right. \\
& \left.\left.a\left(M_{\mathrm{B}}^2M_{\mathrm{K}_1}^2-M_{\mathrm{B}}^2E_{\mathrm{K}_1}^2\right)\right)|V_1(q^2)|^2\right], \\
|\mathcal{M}_{33}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[M_{\mathrm{B}}^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)\left(a\left(M_{\mathrm{B}}^2-M_{\mathrm{K}_1}^2\right)^2+(2M_{\mathrm{B}}E_{\mathrm{K}_1})^2-\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_1}^2\right)^2\right)|V_2(q^2)|^2\right], \\
|\mathcal{M}_{44}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[4M_{\mathrm{B}}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)(a-1)M_{\mathrm{K}_1}^2|V_3(q^2)-V_0(q^2)|^2\right], \\
|\mathcal{M}_{23}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[M_{\mathrm{B}}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)\left(M_{\mathrm{B}}^2+M_{\mathrm{K}_1}^2-2M_{\mathrm{B}}E_{\mathrm{K}_1}-\right.\right. \\
& \left.\left.a\left(M_{\mathrm{B}}^2-M_{\mathrm{K}_1}^2\right)\right)(V_1(q^2)V_2^*(q^2)+V_2(q^2)V_1^*(q^2))\right], \\
|\mathcal{M}_{24}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[2M_{\mathrm{K}_1}(M_{\mathrm{B}}+M_{\mathrm{K}_1})^3\left((1-a)M_{\mathrm{B}}^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)\right)\times\right. \\
& \left.(V_1(V_3-V_0)^*+(V_3-V_0)V_1^*)\right], \\
|\mathcal{M}_{34}|^2 = {} & |\mathcal{C}_{\mathrm{V}}|^2\frac{1}{M_{\mathrm{K}_1}^2(M_{\mathrm{B}}+M_{\mathrm{K}_1})^2q^2}\left[2M_{\mathrm{K}_1}(M_{\mathrm{B}}+M_{\mathrm{K}_1})\times\right. \\
& \left.\left(M_{\mathrm{B}}^2-M_{\mathrm{K}_1}^2\right)M_{\mathrm{B}}^2\left(E_{\mathrm{K}_1}^2-M_{\mathrm{K}_1}^2\right)(a-1)\left(V_2(V_3-V_0)^*+(V_3-V_0)V_2^*\right)\right].
\end{aligned} \tag{30}$$

One can easily see that Eqs. (28) and (29) reduce to Eqs. (23) and (25) respectively, if one sets $a=1$.

4 Results and discussion

In this section we present our numerical study for $B \to K_0^*(K_1)+\not{E}$ where we try to distinguish unparticle physics effects from those of the SM. In the Standard Model $\not{E}$, which is the missing energy, is attributed to the neutrinos whereas in the case under consideration, this is attributed to the unparticle. Therefore the total decay rate can be written as

$$\Gamma = \Gamma^{\rm SM} + \Gamma^{\mathcal{U}} . \tag{31}$$

Here $\Gamma^{\rm SM}$ is the Standard Model contribution ($B \to K_0^*(K_1)\nu\bar{\nu}$) whereas $\Gamma^{\mathcal{U}}$ comes from the unparticle ($B \to K_0^*(K_1)\mathcal{U}$) according $B \to K_0^*(K_1)+\not{E}$. In Ref. [25] it is pointed out that the SM process $B \to K(K^*)\nu\bar{\nu}$ provides a unique energy distribution spectrum of final state hadrons and gives experimental limits for the branching ratio of these processes that are about an order of magnitude below the respective SM expectation values. The authors of Ref. [25] have used an experimental upper limit on the branching ratio of the $B \to K(K^*)\nu\bar{\nu}$ decay to estimate the constraints on the unparticle properties.

In the case of $B \to K_0^*(K_1)\nu\bar{\nu}$ there is no experimental limit on the branching ratio of these decays, but these will be expected to be measured at future Super B-factories where they will analyze the spectra of the final state hadron by imposing a cutoff on the high momentum of the hadron to reduce the background. To calculate the numerical value of the branching ratio for $B \to K_0^*(K_1)\nu\bar{\nu}$ in the SM we have to integrate Eqs. (10) and (15) over the energy of the final state hadron. Thus, after the integration, the values of the branching ratios in the SM are:

$$\begin{aligned} \mathcal{B}r(B \to K_0^*\nu\bar{\nu}) &= 1.12\times 10^{-6}, \\ \mathcal{B}r(B \to K_1\nu\bar{\nu}) &= 1.77\times 10^{-6}. \end{aligned} \tag{32}$$

With these values at hand, we have plotted the differential decay width for $B \to K_0^*(K_1)+\not{E}$ as a function of the energy of the final state hadron $E_{K_0^*}(E_{K_1})$ and by fixing the parameters of the unparticle from Ref. [25] in Fig. 1. One can easily see from the figure that the signatures of the unparticle operators are very distinctive from the SM ones when plotted as a function of the final state hadron's energy. Just as in the case of $B \to K(K^*)+\not{E}$ the distribution of the unparticle contribution is quite different if a vector operator ($a=1$) for the high energetic final state hadron is included. The issue of using other values of a will be discussed separately. Thus the Super B-factories will be able to clearly distinguish the presence of an unparticle by observing the spectrum of the final state hadrons in $B \to K_0^*(K_1)+\not{E}$ in complement to $B \to K(K^*)+\not{E}$.

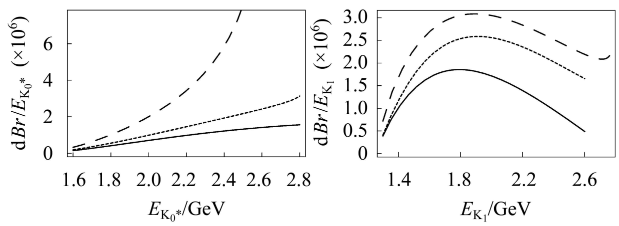

Fig. 1. The differential branching ratio for $B \to K_0^*(K_1)+\not{E}$ as a function of hadronic energy $E_{K_0^*}(E_{K_1})$ is plotted. The left panel is for $B \to K_0^*+\not{E}$ and the right one is for $B \to K_1+\not{E}$. The other parameters are $d_{\mathcal{U}}=1.9$, $\Lambda_{\mathcal{U}}=1000$ GeV, $C_P=C_S=2\times 10^{-3}$ and $C_V=C_A=10^{-5}$. Solid lines are for the SM, dashed lines for the scalar operator and long-dashed lines are for the vector operator.

In Fig. 2 and Fig. 3 we have shown the sensitivity of the branching ratio on the scaling dimension $d_{\mathcal{U}}$ for different values of the cutoff scale $\Lambda_{\mathcal{U}}$ by using the same values of C_S, C_P, C_V and C_A as in Fig. 1. We can see from these figures that the branching ratio is very sensitive to the variable $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$. The constraints on the vector operator are stronger than those on the scalar operators and the constraints for $B \to K_0^*+\not{E}$ are more suitable than those for the $B \to K_1+\not{E}$ decays.

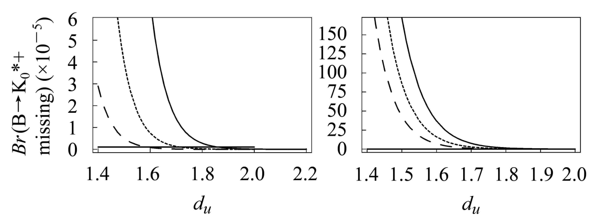

Fig. 2. The branching ratio for $B \to K_0^*+\not{E}$ as a function of $d_{\mathcal{U}}$ for various values of $\Lambda_{\mathcal{U}}$. The left panel is for the scalar operator and the right one is for the vector operator. The values of the coupling constants are the same as in Fig. 1. Solid lines correspond to $\Lambda_{\mathcal{U}}=1000$ GeV, dashed lines to $\Lambda_{\mathcal{U}}=2000$ GeV and the long-dashed lines to $\Lambda_{\mathcal{U}}=5000$ GeV. The horizontal solid line represents the SM result.

After showing the dependence of the branching ratio on $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$ we show in Fig. 4 the sensitivity of the branching ratio of $B \to K_0^*+\not{E}$ on the effective coupling constants of the scalar and vector unparticle operators. One can see that "$B \to K_0^*+$ scalar unparticle operator" constrains the parameter C_P and

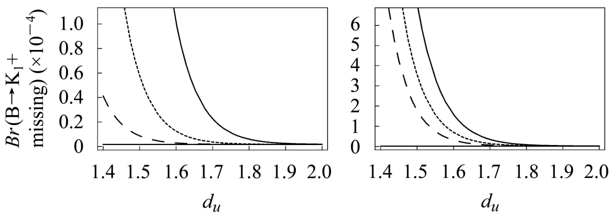

Fig. 3. The branching ratio for $B \to K_1 + \not{E}$ as a function of $d_{\mathcal{U}}$ for various values of $\Lambda_{\mathcal{U}}$. Left panel: scalar operator, right panel: vector operator. The values for the coupling constants are the same as in Fig. 1. Solid line: $\Lambda_{\mathcal{U}} = 1000$ GeV, dashed line: $\Lambda_{\mathcal{U}} = 2000$ GeV, long-dashed line: $\Lambda_{\mathcal{U}} = 5000$ GeV. The horizontal solid line is the SM result.

"$B \to K_0^* +$ vector unparticle operator" constrains C_A. Thus observing this decay we can get some useful constraints on C_P and C_A which provide us with a signature of the unparticle physics. Similarly, we have shown the dependence of the branching ratio of $B \to K_1 + \not{E}$ on the effective coupling constants in Fig. 5. It is seen that if we consider the scalar operator then the only dependence is on C_S, whereas if the vector operators are considered then the decay rate depends on both C_V and C_A.

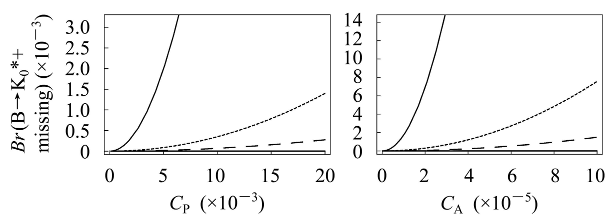

Fig. 4. The branching ratio for $B \to K_0^* + \not{E}$ as a function of C_P (left panel) and C_A (right panel). The cutoff scale has been taken to be $\Lambda_{\mathcal{U}} = 1000$ GeV. Solid lines are for $d_{\mathcal{U}} = 1.5$, dashed lines are for $d_{\mathcal{U}} = 1.7$ and long-dashed lines are for $d_{\mathcal{U}} = 1.9$. The horizontal solid line is the SM result.

We have already mentioned that in a recent publication on the unparticle, Grinstein et al.[26] reported that Mack's unitarity constraint lowers the bounds on the CFT operator dimensions, e.g., $d_{\mathcal{U}} \geqslant 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. The modified expressions of the decay rate for the processes under consideration are given in Eq. (28) and Eq. (29). The results incorporating the modification in the vector unparticle operator are shown in Fig. 6. There the fractional error

$$\Delta \equiv \frac{\left(\dfrac{1}{\Gamma}\dfrac{\mathrm{d}\Gamma}{\mathrm{d}E_{K_0^*(K_1)}}\right)_{a=1} - \left(\dfrac{1}{\Gamma}\dfrac{\mathrm{d}\Gamma}{\mathrm{d}E_{K_0^*(K_1)}}\right)_{a}}{\left(\dfrac{1}{\Gamma}\dfrac{\mathrm{d}\Gamma}{\mathrm{d}E_{K_0^*(K_1)}}\right)_{a=1}} \qquad (33)$$

is depicted, defined as the difference between the spectrum of $B \to K_0^*(K_1)\mathcal{U}$ using the vector unparticle operator with $a = 1$ and that with $a = 2(d_{\mathcal{U}}-2)/(d_{\mathcal{U}}-1)$ with $3 < d_{\mathcal{U}} < 3.9$. It is clear from the graph that with increasing unparticle scaling dimensions $d_{\mathcal{U}}$ the contribution of the vector unparticle operator to the decay rate decreases significantly because the increase is proportional to the inverse power of the cutoff scale $\Lambda_{\mathcal{U}}$ (see Eqs. (28) and (29)).

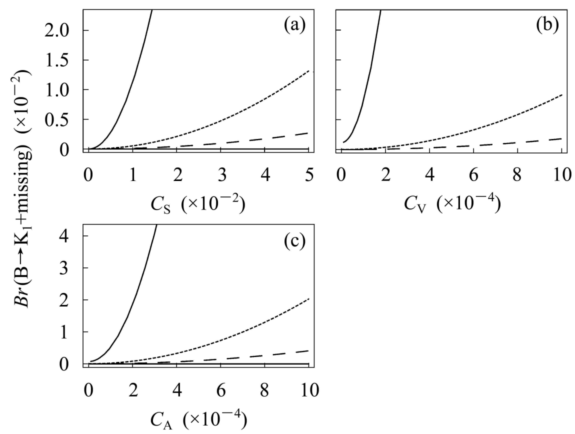

Fig. 5. The branching ratio for $B \to K_1 + \not{E}$ as a function of C_S (a), C_A (b) and C_V (c). The cutoff scale has been taken to be $\Lambda_{\mathcal{U}} = 1000$ GeV. Solid lines are for $d_{\mathcal{U}} = 1.5$, dashed lines are for $d_{\mathcal{U}} = 1.7$ and long-dashed lines are for $d_{\mathcal{U}} = 1.9$. The horizontal solid line is the SM result.

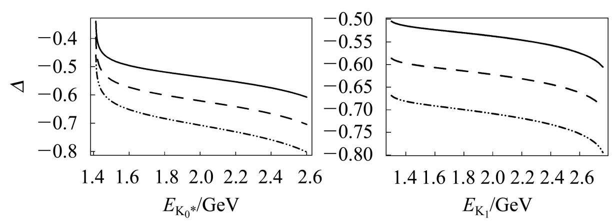

Fig. 6. Fractional error Δ in the spectrum for the decay $B \to K_0^*(K_1)+$ vector unparticle operator as a function of energy of the final state hadron. The left panel shows the values for $B \to K_0^*$ and the right panel those for $B \to K_1$. The values for the coupling constants and cutoff scale are the same as in Fig. 1. Solid lines are for $d_{\mathcal{U}} = 3.2$, dashed lines are for $d_{\mathcal{U}} = 3.4$ and dashed-double dotted lines are for $d_{\mathcal{U}} = 3.6$.

In conclusion, the study of the considered p-wave decays of B mesons will not only provide us with information on the SM but it may also indicate possible physics beyond it. In future, when enough data have been accumulated from the Super B-factories, we believe that these decays will take us a step forward to the study of the unparticle as a source of missing energy in flavor physics.

The authors would like to thank W. Wang and Yu-Ming Wang for useful discussions.

References

1 Aubert B et al. (BaBar Collaboration). Phys. Rev. Lett., 2004, **93**: 081802

2 Iwasaki M I et al. (BELLE Collaboration). Phys. Rev. D, 2005, **72**: 092005

3 Abe K et al. (BELLE Collaboration). Phys. Rev. Lett., 2002, **88**: 021801

4 Aubert B et al. (BaBar Collaboration). Phys. Rev. Lett., 2003, **91**: 221802

5 Ishikawa A et al. (BELLE Collaboration). Phys. Rev. Lett., 2003, **91**: 261601

6 Colangelo P, De Fazio F, Santorelli P, Scrimieri E. Phys. Rev. D, 1996, **53**: 3672; 1998, **57**: 3186(E); Ali A, Ball P, Handoko L T, Hiller G. Phys. Rev. D, 2000, **61**: 074024; Ali A, Lunghi E, Greub C, Hiller G. Phys. Rev. D, 2002, **66**: 034002

7 Aliev T M, Koru H, Zpineci A Ö, Savc M. Phys. Lett. B, 1997, **400**: 194; Aliev T M, Zpineci A Ö, Savc M. Phys. Rev. D, 1997, **56**: 4260; Melikhov D, Nikitin N, Simula S. Phys. Rev. D, 1998, **57**: 6814

8 Burdman G. Phys. Rev. D, 1995, **52**: 6400; Hewett J L, Walls J D. Phys. Rev. D, 1997, **55**: 5549; CHEN C H, GENG C Q. Phys. Rev. D, 2001, **63**: 114025

9 Buchalla G, Hiller G, Isidori G. Phys. Rev. D, 2001, **63**: 014015

10 CHEN Kai-Feng. (BELLE Collaboration). Talk given at FPCP07, Slovenia, May 12—16, 2007

11 Aubert B et al. (BaBar Collaboration). Phys. Rev. Lett., 2005, **94**: 101801

12 Kim C S, Kim Y G, Morozumi T. Phys. Rev. D, 1999, **60**: 094007 [arXiv: hep-ph/9905528]

13 Mahajan N. Phys. Rev. D, 2003, **68**: 034012

14 Georgi H. Phys. Rev. Lett., 2007, **98**: 221601 [arXiv: hep-ph/0703260]

15 Banks T, Zaks A. Nucl. Phys. B, 1982, **196**: 189

16 Georgi H. Phys. Lett. B, 2007, **650**: 275—278 [arXiv: hep-ph/07042457]

17 CHEUNG K, KEUNG W Y, YUAN T C. Phys. Rev. Lett., 2007, **99**: 051803 [arXiv: hep-ph/0704.2588]

18 CHEUNG K, KEUNG W Y, YUAN T C. Phys. Rev. D, 2007, **76**: 055003 [arXiv: hep-ph/0706.3155]

19 Fox P J, Rajaraman A, Shirman Y. arXiv:0705.3092; Greiner N. arXiv:0705.3518; CHEN S L, HE X G. arXiv:0705.3946; DING G J, YAN M L. arXiv:0705.0794

20 Kikuchi T, Okada N. arXiv: hep-ph/0707.0893; arXiv: hep-ph/0801.0018

21 LUO M, ZHU G. arXiv:0704.3532; CHEN C H, GENG C Q. arXiv:0705.0689; LIAO Y. arXiv: hep-ph/0705.0837; LI X Q, WEI Z T. arXiv:hep-ph/0705.1821

22 Choudhury D, Ghosh D K, Mamta. arXiv: hep-ph/0705.3637; Aliev T M, Cornell A S, Gaur N. Phys. Lett. B, 2007, **657**: 77: arXiv: hep-ph/0705.1326; LÜ C D, WANG W, WANG Y M. Phys. Rev. D, 2007, **76**: 077701: arXiv: hep-ph/0705.2909

23 Lenz A. Phys. Rev. D, 2007, **76**: 065006: arXiv: hep-ph/0707.1535

24 Davoudiasl H. arXiv: hep-ph/0705.3636; Kikuchi T, Okada N. arXiv: hep-ph/0711.1506

25 Aliev T M, Cornell A S, Gaur N. JHEP, 2007, **07**: 072 [arXiv: hep-ph/07054542]

26 Grinstein B, Intrilligator K, Rothstein I Z. arXiv: hep-ph/08011140

27 CHEN Chuan-Hung, GENG Chao-Qiang, LIH Chong-Chung, LIU Chun-Chu. arXiv: hep-ph/0703106

28 Paracha M A, Ahmed I, Aslam M J. Eur. Phys. J. C, 2007, **52**: 967—973