CPC(HEP & NP), 2009, **33**(9): 759—763 Chinese Physics C Vol. 33, No. 9, Sep., 2009

Influence of angular momentum in axially symmetric potentials with octupole deformation*

JIN Hua(金华)[1;1)] SUN Zhen-Wu(孙振武)[1] ZHENG Ren-Rong(郑仁蓉)[2;2)]

1 (Institute of Applied Sciences, Shanghai Dianji University, Shanghai 200240, China)
2 (Department of Physics, Shanghai Normal University, Shanghai 200234, China)

Abstract The chaotic classical single-particle motion in an oblate octupole deformed potential with a non-zero z-component of angular momentum L_z is investigated. The stability analysis of the trajectories shows that with increasing rotation of the system, the unstable negative curvature regions of the effective potential surface decrease, which converts the chaotic motion of the system into a regular one.

Key words octupole deformation, angular momentum, chaotic motion

PACS 21.60.Ev, 24.60.Lz, 05.45.Pq

1 Introduction

The analysis of classical dynamics is an effective way to explain the origin of chaos in the corresponding quantum systems. Single-particle motion under different axially symmetric potentials has been discussed from the classical point of view demonstrating that the deformation of the potentials could be a possible source of irregularities in atomic nuclei[1—4]. W. D. Heiss et al.[2] found that in oblate axially symmetric potentials with octupole deformation the classical single-particle motions have a higher tendency to become chaotic than in prolate potentials. Gu Jian-Zhong et al. studied the classical dynamic behaviour of a nucleon in heavy nuclei within the framework of the two-center shell model. He found that classical chaotic motion occurs even for prolate nuclei with shapes having a pronounced octupole-like deformation and a considerable neck[4]. The stability of such axially symmetric systems can be determined by the Gaussian curvature of the potential surfaces[1, 5—7]. The appearance of a negative curvature is treated as a criterion for the chaotic motion of these systems.

Recently, P. Cejnar et al.[8, 9] investigated classical chaos from the collective vibrations and rotations of atomic nuclei within the geometric collective model. They found that rotations could influence the classical motion of the system. Depending on the values of the control parameters in the model, they demonstrated a tendency for overall suppression or enhancement of chaos with angular momentum[9]. Their research displayed a rich picture of chaos caused by nuclear collective motions. As the z-component of angular momentum L_z is constant for systems with axially symmetric deformed potentials, which is similar to the geometric collective model, we are able to study the single-particle motion in multipole deformed potentials with non-zero L_z.

In Ref. [6] the single-particle's chaotic motions were systematically investigated in an axially symmetric octupole deformed potential with angular momentum $L_z = 0$. In this paper non-zero angular momentum is considered. The text is organized as follows: In Sect. 2 we investigate the characteristics of the effective potential for the octupole deformation with non-zero L_z. In Sect. 3 the stability analysis of trajectories is discussed by the criterion of the Gaussian curvature of the effective potential surfaces. The numerical results of the Poincaré sections and the maximum Lyapunov exponent for the trajectories are also given in this section. Sect. 4 gives our conclusion.

Received 8 December 2008

* Supported by Shanghai Education Committee (SDJ05012), and Shanghai Dianji University Reserch Grant (06C312)

1) E-mail: jinhua@sdju.edu.cn

2) E-mail: rrzheng@online.sh.cn

2 Effective potential for the octupole deformation with non-zero L_z

Using cylindrical coordinates,(r, z, φ), the single-particle Hamiltonian in the axially symmetric octupole deformed potential can be written as:

$$H = \frac{p_r^2}{2}+\frac{p_z^2}{2}+\frac{L_z^2}{2r^2}+V(r,z)= \frac{p_r^2}{2}+\frac{p_z^2}{2}+\frac{L_z^2}{2r^2}+\frac{1}{2}\left(r^2+\frac{z^2}{b^2}+\lambda\frac{2z^3-3zr^2}{\sqrt{r^2+z^2}}\right), \tag{1}$$

where we put the mass equal to unity and $r=\sqrt{x^2+y^2}$ in the Cartesian coordinates. The momenta are given by $p_r=\dot{r}$ and $p_z=\dot{z}$. $L_z=r^2\dot{\varphi}$ is the z-component of angular momentum which is conserved due to the axial symmetry. In the potential $V(r,z)$[2, 6], λ measures the strength of the octupole deformation. If $\lambda=0$, V is the harmonic oscillator potential, where the parameter b describes the quadrupole deformation which is prolate for $b>1$ and oblate for $b<1$. λ has the critical value λ_c for a given quadrupole deformation. If $\lambda>\lambda_c$, the particle's motion is not restricted by the potential.

When dealing with $L_z\neq 0$ in axially symmetric systems one introduces the effective potential[10] $U=V+L_z^2/2r^2$. Then the energy $E=p_r^2/2+p_z^2/2+U$ and the particle's motion takes place in a four-dimensional phase space (r,p_r,z,p_z) which can be reduced to a three-dimensional one due to energy conservation. Considering that the sign of L_z does not change the value of the effective potential, we only discuss the case $L_z>0$ in the text.

In Eq. (1), the effective potential U takes the form

$$U=\frac{1}{2}\left(r^2+\frac{z^2}{b^2}+\lambda\frac{2z^3-3zr^2}{\sqrt{r^2+z^2}}\right)+L_z^2/2r^2\,. \tag{2}$$

Let $r=R\sin\theta$, $z=R\cos\theta$, where R, θ are spherical coordinates (R,θ,φ) and $\theta\in(0,\pi)$ as $r>0$. Then Eq. (2) transforms into

$$U = \frac{R^2}{2}\left[\sin^2\theta+\frac{\cos^2\theta}{b^2}+\lambda(2\cos^3\theta- 3\cos\theta\sin^2\theta)\right]+\frac{L_z^2}{2R^2\sin^2\theta}\,. \tag{3}$$

Denoting $A=[\sin^2\theta+\cos^2\theta/b^2+\lambda(2\cos^3\theta-3\cos\theta\cdot\sin^2\theta)]/2$ (here A is everywhere $A>0$), $B=1/(2\sin^2\theta)$ and $f=A\cdot B$, we get Eq. (4)

$$AR^4-UR^2+B\cdot L_z^2=0\,. \tag{4}$$

If θ and U are given, the condition for a real, positive solution of R is

$$U^2-4f\cdot L_z^2\geqslant 0\,. \tag{5}$$

So the values of U and L_z are confined by Eq. (5). For a given L_z, U has a minimal value $U_{\min}$ at

$$U_{\min}=2L_z\sqrt{f_{\min}}\,, \tag{6}$$

where $f_{\min}$ is the minimal value of the function f, which is easily computed numerically. For a given energy E, if $U=E$, L_z will have the maximum value $L_{z\max}$,

$$L_{z\max}=\frac{E}{2}\sqrt{\frac{1}{f_{\min}}}\,. \tag{7}$$

Then with a certain energy, the degree of the particle's rotation can be characterized by the ratio of the actual angular momentum J_z and the maximal angular momentum $J_{z\max}$[9],

$$j=\frac{L_z}{L_{z\max}}\,. \tag{8}$$

Consider the scale transformation $r\to\gamma r$, $z\to\gamma z$, $L_z\to\gamma^2L_z$, where γ is the scaling constant. One obtains the scaling behaviour $U\to\gamma^2U$, $j\to j$. Therefore equipotential lines of U with the same j will have similar surfaces.

In the present work we investigate the single-particle motion in an oblate octupole deformed potential with parameters $b=0.5$, $\lambda_c\approx 1.64$ and $\lambda=0.5\lambda_c$[2, 6]. Such a small deformation strength can cause a strong chaos if $L_z=0$[6]. Figs. 1—2 show the equipotential lines of the effective potential U for different energy E and angular momenta L_z. As we set the value of U equal to the particle's energy E, the region inside the equipotential lines corresponds to a space of physically allowed motion, where $U\in[U_{\min},E]$. We can also clearly see the potential scales from these two figures by comparing the different equipotential lines with the same j. In the following we will study the influence of the non-zero angular momentum L_z on the single-particle motion.

3 Analysis of the classical dynamics of the trajectories

As mentioned in the introduction, the stability of the single-particle motion in an axially symmetric potential is connected with the Gaussian curvature[1, 5—7]. For a two-dimensional potential surface the Gaussian curvature G is defined as,

$$G(q_1,q_2)=\frac{\partial^2V}{\partial q_1^2}\frac{\partial^2V}{\partial q_2^2}-\left(\frac{\partial^2V}{\partial q_1\partial q_2}\right)^2\,. \tag{9}$$

The regions with negative Gaussian curvature $G<0$ are unstable. If the single-particle motions through such regions are continuous, neighbouring trajectories in the phase space will separate exponentially and the system tends to be chaotic. On the other hand, the system will be regular if $G > 0$ everywhere in the potential surface.

We investigate the Gaussian curvature of the effective potential U for a given oblate octupole deformation by increasing the degree of the system's rotation j. In Fig. 1, we let the single-particle energy E be a constant and increase the angular momentum L_z in order to enhance j. The shaded area represents the regions of negative Gaussian curvature ($G < 0$) for a given L_z. We find that there are considerable regions of negative Gaussian curvature in the potential surface for small j. With increasing j, the regions of negative Gaussian curvature decrease. For $j = 0.4$

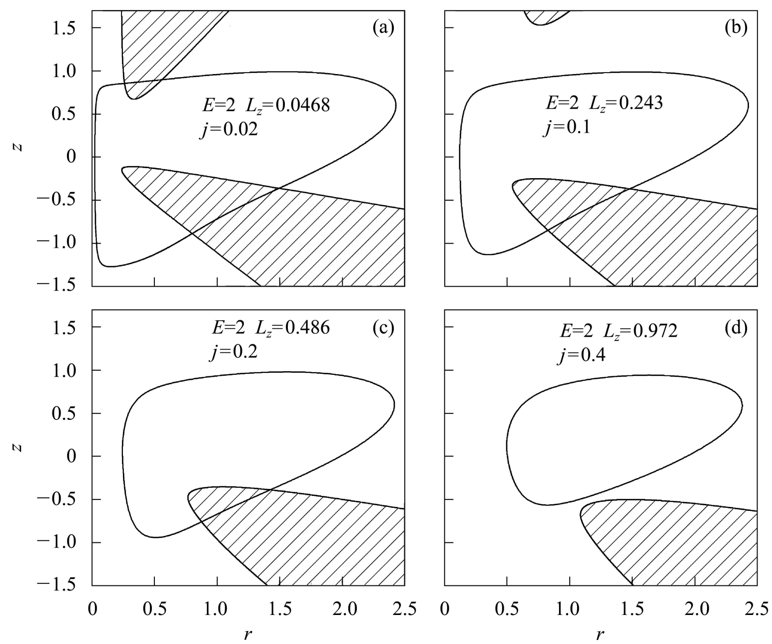

Fig. 1. Equipotential lines of the effective potential U for the oblate octupole deformation ($b = 0.5$, $\lambda = 0.5\lambda_c$) with the same energy $E = 2$ and different angular momenta L_z. The shaded areas correspond to the regions with negative Gaussian curvature ($G < 0$) of the effective potential surface for a given L_z.

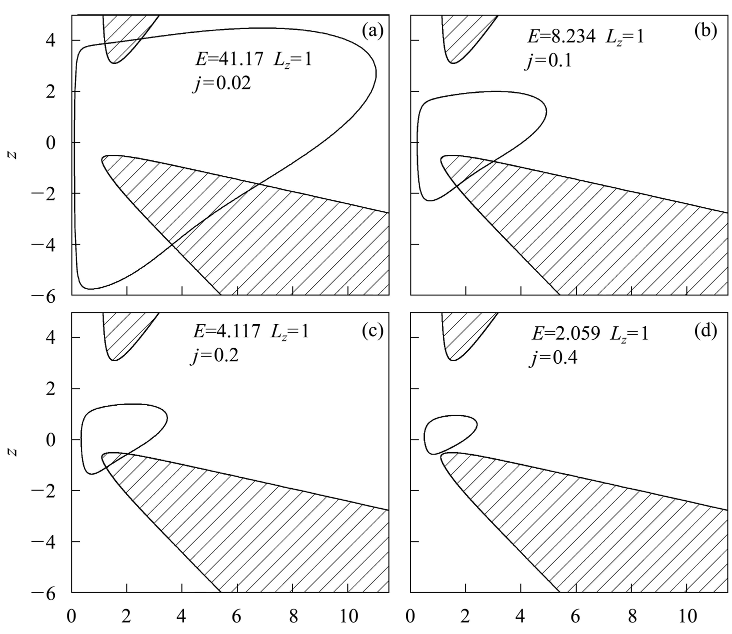

Fig. 2. Equipotential lines of the effective potential U for the oblate octupole deformation ($b = 0.5$, $\lambda = 0.5\lambda_c$) with the same angular momenta $L_z = 1$ and different energy E. The shaded areas correspond to the regions with negative Gaussian curvature ($G < 0$) of the effective potential surface for $L_z = 1$.

there is no region of negative Gaussian curvature. Another way of enhancing j is to set L_z invariable and decrease the single-particle's energy E. From Fig. 2 we can draw the same conclusion that the regions of negative Gaussian curvature become less if j increases. The discussion above implies that the system can undergo a transition from chaos to regularity by increasing the degree of the system's rotation j. By numerical analysis we find that the critical value of j for this transition in the investigated system is about 0.37.

The Poincaré section is also a reliable criterion to judge the stability of trajectories when the single-particle motions occur due to energy conservation in a reduced three-dimensional phase space. The section can be generated by solving the Hamilton-Jacobi equations of the system numerically. Fig. 3 shows the sections of the r-p_r plane at $z=0.4$ for different angular momenta L_z and constant energy of the system. In Fig. 3(a), there are large chaos areas in the surface of the section under the condition of small j. With increasing the degree of the single-particle's rotation, the chaos areas decrease gradually (Fig. 3(b) and Fig. 3(c)). In Fig. 3(d), $j=0.4$, chaos areas disappear and the system becomes regular. In Fig. 4 we plot the sections of the z-p_z plane at $r=1$ for different energies E and a given $L_z=1$. Similarly, the chaos areas in the surfaces of the sections get smaller if j increases. Apparently, the dynamic characteristics of the system showed by the Poincaré section are in agreement with the analysis of the Gaussian curvature of the effective potential U.

We also calculate the maximum Lyapunov exponent of the system. As is well-known, a positive maximum Lyapunov exponent means that the system is chaotic[11]. For the regular system the maximum Lyapunov exponent is zero. In Fig. 5, the maximum Lyapunov exponent corresponding to Figs. 3—4 is plotted. We can see that the two curves almost overlap, which indicates that the system has the same dynamics with the same j due to the scaling property. Fig. 5 shows that the positive maximum Lyapunov exponent decreases in general if j increases. Between $j=0.36$ and 0.38, the maximum Lyapunov exponent tends to be zero, which is also consistent with the critical value of the system for the transition from chaos to regularity obtained by the Gaussian curvature analysis.

4 Conclusion

In this paper we have studied the influence of the non-zero z-component of the angular momentum L_z on the classical single-particle motion in an oblate octupole deformed potential. We find that the unstable Gaussian negative curvature regions of the effective

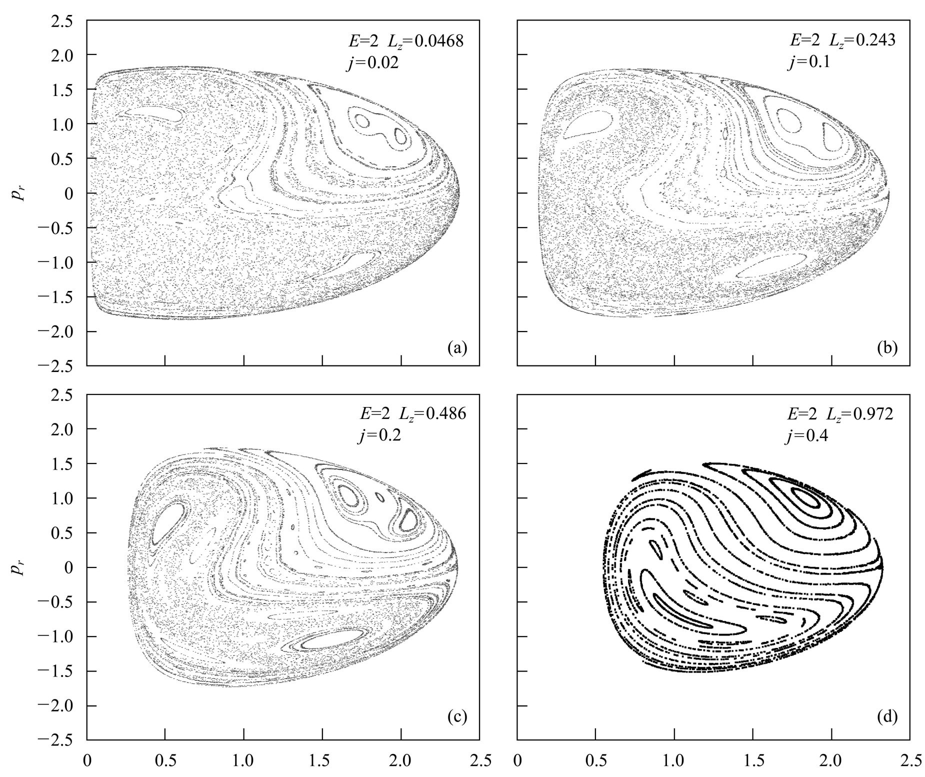

Fig. 3. Poincaré sections of the r-p_r plane at $z=0.4$ for different angular momenta L_z and energy $E=2$.

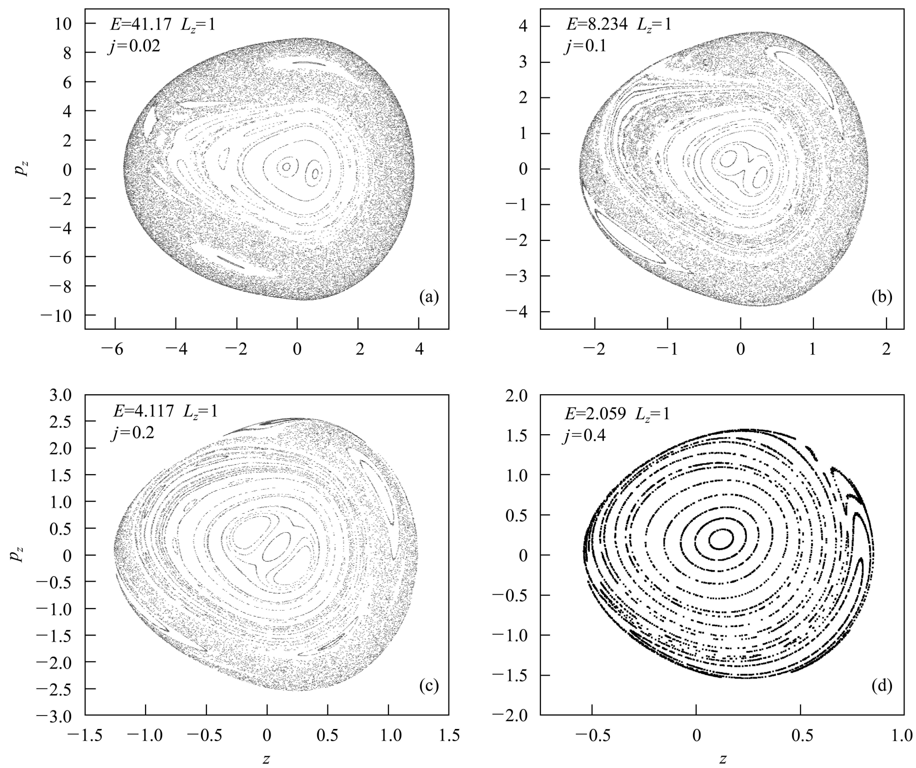

Fig. 4. Poincaré sections of the z-p_z plane at $r=1$ for different energy E and angular momentum $L_z=1$.

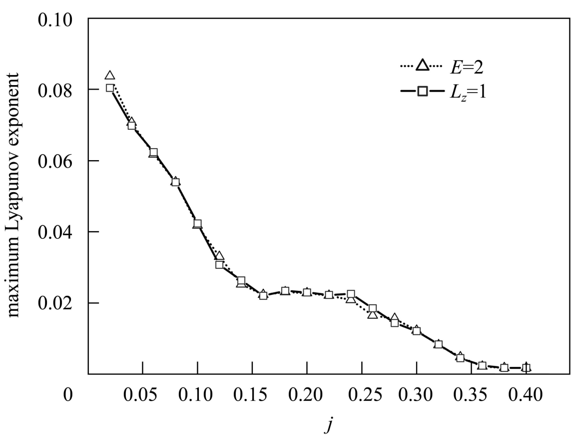

Fig. 5. The maximum Lyapunov exponent for $E=2$ and $L_z=1$.

potential surface decrease by increasing the degree of the system's rotation and the chaos of the system can be suppressed. This conclusion is also true for prolate octupole deformations, which we have not discussed here. The significance of the present work is to show that the oblate octupole deformed system can also be regular within a certain region of angular momenta. Since octupole deformations occur frequently in atomic nuclei, we believe that the transition from prolate deformations to oblate ones[12, 13] is connected with the relatively high angular momentum of the single-particle orbits in the corresponding transition region.

References

1 LI Jun-Qing, HUANG Wei-Qi . Phys. Rev. C, 1994, **50**: 1632

2 Heiss W D, Nazmitdinov R G, Radu S. Phys. Rev. Lett., 1994, **72**: 2351

3 Heiss W D, Nazmitdinov R G, Radu S. Phys. Rev. C, 1995, **52**: 3032

4 GU Jian-Zhong, ZHAO En-Guang, ZHUO Yi-Zhong et al. Eur. Phys. J. A, 1998, **2**: 115; WU Xi-Zhen, GU Jian-Zhong, ZHUO Yi-Zhong et al. Phys. Rev. Lett., 1997, **79**: 4542

5 LI Jun-Qing. Phys. Rev. Lett., 1997, **79**: 2387

6 LI Jun-Qing. J. Phys. G, 1998, **24**: 1021; LI Jun-Qing. HEP & NP, 1997, **21**(9): 818 (in Chinese)

7 GU Jian-Zhong, ZHUO Yi-Zhong, ZHAO En-Guang et al. Eur. Phys. J. A, 1998, **3**: 217

8 Cejnar P, Stránský P. Phys. Rev. Lett., 2004, **93**: 102502

9 Stránský P, Kurian M, Cejnar P. Phys. Rev. C, 2006, **74**: 014306

10 Guéron E, Letelier P S. Phys. Rev. E, 2001, **63**: 035201

11 Wolf A, Swift J B, Swinney H L et al. Physica D, 1985, **16**: 285

12 Stolk A, Narakeh M N, Hesselink W H A et al. Phys. Rev. C, 1989, **40**: R2454

13 SUN Yang, Walker P M, XU Fu-Rong et al. Phys. Lett. B, 2008, **659**: 165