# Triaxial deformation in ${ }^{104} \mathrm{Rh}$ of the chiral doublet bands＊ 

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#### Abstract

Triaxial relativistic mean field（RMF）approaches are used to study the properties of ${ }^{104} \mathrm{Rh}$ ．The existence of multiple chiral doublets is suggested for ${ }^{104} \mathrm{Rh}$ based on the triaxial deformations and their corre－ sponding proton and neutron configurations．


Key words chirality，triaxial deformation，relativistic mean field
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## 1 Introduction

Handedness or chirality is common and has im－ portant consequence in molecular physics，elementary particles，and optical physics．The concept of chiral－ ity was first given by Lord Kelvin in $1904^{[1]}$ ，its occur－ rence in nuclear physics was suggested by Frauendorf and Meng Jie in $199{ }^{[2]}$ ，and the predicted patterns of spectra exhibiting chirality were experimently ob－ served in $2001^{[3]}$ ．

In nuclear physics，the chirality exists in many particles as having a parallel or antiparallel orienta－ tion of spin and momentum．The chirality may exist in nuclei when a particular angular momentum cou－ pling scheme appears，where three angular momenta are mutually perpendicular so that a left－and a right－ handed system can be formed．Hence the chirality of the nuclear rotation results from a combination of dy－ namics（the angular momentum）and geometry（the triaxial shape）．It was suggested to occur in odd－odd triaxial nuclei ${ }^{[4]}$ when the Fermi level is located in the lower part of the valence neutron high－j subshell re－ sulting in its angular momentum oriented along the short axis of the triaxial core，and in the upper part of the valence proton subshell leading to its angular mo－ mentum aligned with the long axis，while the angular momentum of the core is along the intermediate axis due to its largest moment of inertia for irrotational－
like flow ${ }^{[5]}$ ．
In this paper，we study the chiral doublet bands in ${ }^{104} \mathrm{Rh}$ by triaxial relativistic mean－field（RMF）and $\beta^{2}$ constrained calculations to search for the basic fea－ tures of its ground states．

## 2 Triaxial relativistic mean－field theory

The interaction between nucleons is described as the exchanging of mesons $\sigma$（provides an attractive interaction），$\omega$（provides a repulsive interaction）， $\rho$（provides the isospin－dependent effects）and pho－ ton（provides electromagnetic interaction）．The star－ ing point of the RMF theory is the following La－ grangian density ${ }^{[6-8]}$ ：

$$
\begin{align*}
\mathcal{L}= & \bar{\psi}\left[\mathrm{i} \gamma^{\mu} \partial_{\mu}-M-g_{\sigma} \sigma-g_{\omega} \gamma^{\mu} \omega_{\mu}-g_{\rho} \gamma^{\mu} \boldsymbol{\tau} \cdot \boldsymbol{\rho}_{\mu}-\right. \\
& \left.e \gamma^{\mu} A_{\mu} \frac{1-\tau_{3}}{2}\right] \psi+\frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma-\frac{1}{2} m_{\sigma}^{2} \sigma^{2}- \\
& \frac{1}{3} g_{2} \sigma^{3}-\frac{1}{3} g_{3} \sigma^{4}-\frac{1}{4} \omega^{\mu \nu} \omega_{\mu \nu}+\frac{1}{2} m_{\omega}^{2} \omega^{\mu} \omega_{\mu}+ \\
& \frac{1}{4} c_{3}\left(\omega^{\mu} \omega_{\mu}\right)^{2}-\frac{1}{4} \rho^{\mu \nu} \cdot \rho_{\mu \nu}+ \\
& \frac{1}{2} m_{\rho}^{2} \rho^{\mu} \cdot \rho_{\mu}-\frac{1}{4} A^{\mu \nu} A_{\mu \nu}, \tag{1}
\end{align*}
$$

[^0]where $\psi$ is the dirac spinor, $\bar{\psi}=\psi^{+} \gamma^{0}$. The nucleon mass $M$ and $m_{\sigma}\left(g_{\sigma}\right), m_{\omega}\left(g_{\omega}\right)$ and $m_{\rho}\left(g_{\rho}\right)$ are the masses(coupling constants) of the respective mesons. The field tensors for vector mesons $\omega, \rho$ and photon fields are defined as
\[

$$
\begin{align*}
\omega^{\mu \nu} & =\partial^{\mu} \omega^{\nu}-\partial^{\nu} \omega^{\mu} \\
\rho^{\mu \nu} & =\partial^{\mu} \rho^{\nu}-\partial^{\nu} \rho^{\mu}-2 g_{\rho}\left(\rho^{\mu} \times \rho^{\nu}\right)  \tag{2}\\
A^{\mu \nu} & =\partial^{\mu} A_{\nu}-\partial^{\nu} A_{\mu}
\end{align*}
$$
\]

From Eq. (1), we can obtain the Dirac equations of motion for nucleon

$$
\begin{equation*}
[-\mathrm{i} \boldsymbol{\alpha} \cdot \boldsymbol{\nabla}+V(\boldsymbol{r})+\beta(M+S(\boldsymbol{r}))] \psi_{\mathrm{i}}(\boldsymbol{r})=\epsilon_{\mathrm{i}} \psi_{\mathrm{i}}(\boldsymbol{r}) \tag{3}
\end{equation*}
$$

and equations of motion for mesons and the photon are

$$
\left\{\begin{align*}
\left(-\Delta+\partial_{\sigma} U(\sigma)\right) \sigma(\boldsymbol{r}) & =-g_{\sigma} \rho_{s}(\boldsymbol{r})  \tag{4}\\
\left(-\Delta+m_{\omega}^{2}\right) \omega^{0}(\boldsymbol{r}) & =g_{\omega} \rho_{v}(\boldsymbol{r}) \\
\left(-\Delta+m_{\rho}^{2}\right) \rho^{0}(\boldsymbol{r}) & =g_{\rho} \rho_{3}(\boldsymbol{r}) \\
-\Delta A^{0}(\boldsymbol{r}) & =e \rho_{p}(\boldsymbol{r})
\end{align*}\right.
$$

where the vector and scale potentials are $V(\boldsymbol{r})=$ $g_{\omega} \omega^{0}(\boldsymbol{r})+g_{\rho} \tau_{3} \rho^{0}(\boldsymbol{r})+e \frac{1-\tau_{3}}{2} A^{0}(\boldsymbol{r})$ and $S(\boldsymbol{r})=g_{\sigma} \sigma(\boldsymbol{r})$ respectively.

In order to get the potential energy surface, we use constrained mass quadruple moment $\left\langle\hat{Q}_{2}\right\rangle$ to a given value $\lambda$,

$$
\begin{equation*}
\left\langle H^{\prime}\right\rangle=\langle H\rangle+\frac{1}{2} C\left(\left\langle\hat{Q}_{2}\right\rangle-\lambda\right)^{2} \tag{5}
\end{equation*}
$$

with

$$
\begin{equation*}
\left\langle\hat{Q}_{2}\right\rangle=\left\langle\hat{Q}_{2}\right\rangle_{\mathrm{n}}+\left\langle\hat{Q}_{2}\right\rangle_{\mathrm{p}} \tag{6}
\end{equation*}
$$

where $\left\langle\hat{Q}_{2}\right\rangle_{\mathrm{n}, \mathrm{p}}=\left\langle 2 r^{2} P_{2}(\cos \theta)\right\rangle_{\mathrm{n}, \mathrm{p}}=\left\langle 2 z^{2}-x^{2}-y^{2}\right\rangle_{\mathrm{n}, \mathrm{p}}$ and $C$ is the curvature constant number. By varying
$\lambda$, the binding energy at different deformations can be obtained.

The total energy of the system can be described as

$$
\begin{equation*}
E_{\mathrm{total}}=E_{\mathrm{part}}+E_{\sigma}+E_{\omega}+E_{\rho}+E_{\mathrm{c}}+E_{\mathrm{pair}}+E_{\mathrm{cm}} \tag{7}
\end{equation*}
$$

where $E_{\text {part }}$ is the total single-particle energy, $E_{\mathrm{i}}(\mathrm{i}=$ $\sigma, \omega, \rho, c)$ stand for the energies of respective mesons and coulomb energy, $E_{\text {pair }}$ is the pairing energy and $E_{\mathrm{cm}}$ center of mass correction. The contribution of paring relation effects is so small in odd-odd nuclei, so we neglect $E_{\text {pair }}$ in this paper. The center of mass correction to the binding energy from projection-after-variation in first-order approximation is given by $E_{\mathrm{cm}}=-\frac{1}{2 m A}\left\langle\hat{P}_{\mathrm{cm}}^{2}\right\rangle$.

## 3 Numerical details and discussions

In this work we use PK1 ${ }^{[9]}$ effective interaction. It provides a better description of the properties of both stable and exotic nuclei. The oscillator frequency is $\hbar \omega=41 A^{-1 / 3}$. In order to get the proper number of expanded oscillator shell for fermions $n_{0 f}$, the binding energy and the deformations ( $\beta$ and $\gamma$ ) are checked. It is found that as long as $n_{0 f} \geqslant 10$, the binding energies as well as the deformations $\beta$ and $\gamma$ obtained are almost same ${ }^{[10]}$. Meanwhile, results are not sensitive with the number of expanded oscillator shell for bosons. Furthermore, they are independent on the deformation $\beta_{0}$ of the basis. In the following, a spherical basis with 12 major oscillator shells for fermions and 10 shells for bosons will be used, which gives an error less than $0.1 \%$ for the binding energy ${ }^{[11]}$.


Fig. 1. Energy surface(a) and $\gamma$ deformation as functions of deformation $\beta$ in constrained triaxial calculation with PK1 for ${ }^{104} \mathrm{Rh}$. The minima in the energy surface are presented and labeled by A-F with their corresponding deformation $\beta$ and $\gamma$ and energies.

The unrestricted RMF calculation can give a local minimum on the energy surface. To obtain the energy surface and deformation $\gamma$ as a function of $\beta$, a $\beta^{2}$ constrained triaxial RMF calculation with PK1 should be performed. The energy surface and deformation $\gamma$ as functions of $\beta$ in triaxial RMF calculations with PK1 for ${ }^{104} \mathrm{Rh}$ are presented in Figs. 1(a) and 1(b), respectively. In comparison, the minima in the energy surfaces of the configuration-fixed constrained calculations become obvious, which are represented by labeled A-F. Their corresponding deformations $\beta$ and $\gamma$, and their binding energies are given in Figs. 1(a) and $1(\mathrm{~b})$, respectively. The total energy minima as a function of the parameters are very shallow and their differences for these minima are within 3.6 MeV to each other but correspond to different deformations $\beta$ and $\gamma$.

The calculations indicate a mechanism of generating chirality that is quite different from the previous examples for chirality. It is the triaxial shape coexistence for the ground states A , the excited $\mathrm{B}, \mathrm{C}$ and D. The states $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D have deformation $\beta$ and $\gamma$ suitable for chirality. As these states are all in one single nucleus, the building of chiral bands on them may lead to a new phenomenon, the existence of $M_{\chi} D$ in one single nucleus. And in Fig. 1 (b), it is interesting to note that for each fixed configuration, the deformation is approximately a constant. It means that the deformation is mainly determined by it corresponding configuration.

Here we will investigate the photon and neutron configurations in detail to see whether the particle
and hole configurations required by chirality are available. Fig. 2 shows us the neutron and proton singleparticle levels obtained in triaxial RMF calculations. The positive (negative) parity states are marked by solid (dashed) lines, and the occupations which corresponding to the minima in Fig. 2 are marked by filled circles and stars. The corresponding quantum numbers for the spherical case are labeled at the left side of the levels.

Except the state D, in which there is no high-j neutron valence particle, we found the high-j proton and neutron configuration for the ground states for A, B, C: state A as $\pi\left(1 g_{9 / 2}\right)^{-1} \otimes v\left(1 d_{5 / 2}\right)^{1}$, state B as $\pi\left(1 g_{9 / 2}\right)^{-3} \otimes v\left(1 h_{11 / 2}\right)^{1}$, state C as $\pi\left(1 g_{9 / 2}\right)^{-1} \otimes$ $v\left(1 d_{5 / 2}\right)^{1}$. All of them have high-j proton holes and high-j neutron particle configurations, which together with triaxial deformations favor the construction of chiral doublets bands. We can see states A and C compete strongly with each other in energy, but they do not mix up because of different parities, so they can produce the chiral doublets $M_{\chi} D$. Here we can get the conclusion that configuration $\pi g_{9 / 2}^{-1} \otimes \mu h_{11 / 2}$ exists in the ${ }^{104} \mathrm{Rh}$, where the valence proton has the hole character with the angular momentum aligning along the long axis and the valence neutron has a character with the angular momentum along the short axis. They are perpendicular to each other and perpendicular to the related collective angular momentum. The chiral doublet bands have identical or very similar energies, spin alignments, shapes and electromagnetic transition probabilities.


Fig. 2. Neutron and proton single-particle levels obtained in constrained triaxial RMF calculation with PK1 for ${ }^{104} \mathrm{Rh}$ as a function of deformation $\beta$.

## 4 Summary

In this paper, we have performed triaxial relativistic mean field(RMF) calculations with PK1 to
search for chiral doublet bands in ${ }^{104} \mathrm{Rh}$. The existence of multiple chiral doublets $\left(M_{\chi} D\right)$ is suggested in ${ }^{104} \mathrm{Rh}$ from the results of our work of the deformation and the corresponding configurations. Apart
from ${ }^{104} \mathrm{Rh}$, the favorable deformation has been found in ${ }^{102,106,108,110} \mathrm{Rh},{ }^{108-112} \mathrm{Ag}$, and ${ }^{112} \mathrm{In}$, which implies that more chiral doublet bands can be expected in the $A \sim 100$ mass region ${ }^{[11]}$. Besides these nuclei, other

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nuclei in the $A \sim 100$ region may also exhibit chiral doublets characters as well, which needs further experimental and theoretical studies.

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