Radiative leptonic decay $B_c^- \to \tau^- \bar{\nu}_\tau \gamma$ in the standard model and the two-Higgs-doublet model*

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Abstract We analyze the radiative leptonic B_c decay $B_c^- \to \tau^- \bar{\nu}_\tau \gamma$ in the Standard Model and the two-Higgs-doublet model using the non-relativistic constituent quark model. The results confirm that this channel is experimentally promising in view of the large number of B_c mesons which are expected to be produced at future hadron facilities. We also find that this decay is sensitive to the parameters of the two-Higgs-doublet model, and it can be tested in future experiments.

Key words radiative decay, standard model, two-Higgs-doublet model, branching ratio

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1 Introduction

Consisting of two different heavy flavors, the B_c meson is believed to offer a unique probe for both strong and weak interactions. The physics of the B_c meson has stimulated much work on its production [1], spectroscopy [2, 3] and different decay modes [4, 5]. B_c was observed in the CDF detector through the semileptonic decay mode $B_c^\pm \to J/\Psi l^\pm X$ at the 1.8 TeV $p\bar{p}$ Fermilab's Tevatron collider [6]. The measured mass and lifetime of the meson are $M_{B_c} = 6.40 \pm 0.39(\text{stat}) \pm 0.13(\text{syst})$ GeV and $\tau_{B_c} = 0.46^{+0.18}_{-0.16}(\text{stat}) \pm 0.03(\text{syst})$ ps, respectively. Further detailed experimental studies can be performed at B factories such as Fermilab's Tevatron and CERN's Large Hadron Collider (LHC). This is the main motivation for extensive theoretical studies of this meson. In particular, with luminosity $L = 10^{-34}$ cm^{-2}·s^{-1} and $\sqrt{s} = 14$ TeV at LHC, the number of $B_c^\pm$ events is expected to be about 2×10^8 per year. Therefore, some interesting rare decays can be studied experimentally in the foreseeable future. We can classify B_c decays into three types at the quark level: b quark decays through $b \to \bar{c}W$ with c quark as a spectator, c quark decays through $\bar{c} \to \bar{s}W$ with b as a spectator, and the annihilation mode $\bar{c}b \to W$.

In view of the annihilation process, leptonic decay is the simplest and the most straightforward way to determine the decay constant f_{B_c}. The leptonic decay width of B_c in the Standard Model (SM) is given by

$$\Gamma(B_c^- \to l^- \bar{\nu}_l) = \frac{G_F^2}{8\pi}|V_{cb}|^2 f_{B_c}^2 M_{B_c}^3 \frac{m_l^2}{M_{B_c}^2}\left(1-\frac{m_l^2}{M_{B_c}^2}\right)^2, \quad (1)$$

where G_F is the Fermi constant and $M_{B_c}(m_l)$ is the mass of the B_c meson (charged lepton), and V_{cb} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Since the leptonic decays are helicity suppressed by a factor of $\frac{m_l^2}{M_{B_c}^2}$ (l represents an electron or muon), the determination of f_{B_c} through $B_c^- \to l^- \bar{\nu}_l$ is very difficult. Although $B_c^- \to \tau^- \bar{\nu}_\tau$ does not suffer so much from helicity suppression, the produced τ will decay promptly, generating at least one more neutrino. This makes such decay more difficult to observe.

The leptonic decay of B_c has been studied in Ref. [7] in the two-Higgs-doublet model (2HDM) [8] and its decay width has the following expression,

$$\Gamma(B_c^- \to l^- \bar{\nu}_l) = \frac{G_F^2}{8\pi}|V_{cb}|^2 M_{B_c} f_{B_c}^2 m_l^2 \left(1-\frac{m_l^2}{M_{B_c}^2}\right)^2 \times\left(1-\tan^2\beta\frac{m_{B_c}^2}{M_{H^\pm}^2}\right)^2, \quad (2)$$

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where $\tan\beta$ is the ratio of the vacuum expectation values of the two Higgs doublets and $M_{H^\pm}$ denotes the mass of the charged Higgs boson in 2HDM. Their results show that the branching ratio changes significantly with the parameters $\tan\beta$ and $M_{H^\pm}$ in 2HDM.

If there is an additional particle, such as one photon in the final state, the leptonic decays will become radiative ones, and hence the helicity suppression moves away. Enlightened by the leptonic decay of B_c in SM and 2HDM, we will study the radiative leptonic decay $B_c^- \to \tau^-\nu_\tau\gamma$ in SM and analyze how it is influenced by the parameters in 2HDM.

The paper is organized as follows. After the introduction, we give the details of formalism for $B_c^- \to \tau^-\nu_\tau\gamma$ decay in the SM and in the 2HDM in Sec. 2. Numerical results and discussions are presented in Sec. 3.

2 Formalism for $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ decay

2.1 In the standard model

As we have noted in the introduction, the leptonic decays $B_c^- \to l^-\bar{\nu}_l$ ($l = e, \mu$) are helicity suppressed and $B_c^- \to \tau^-\bar{\nu}_\tau$ is not helicity suppressed in principle. The relevant Feynman diagrams for the decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ are shown in Fig. 1.

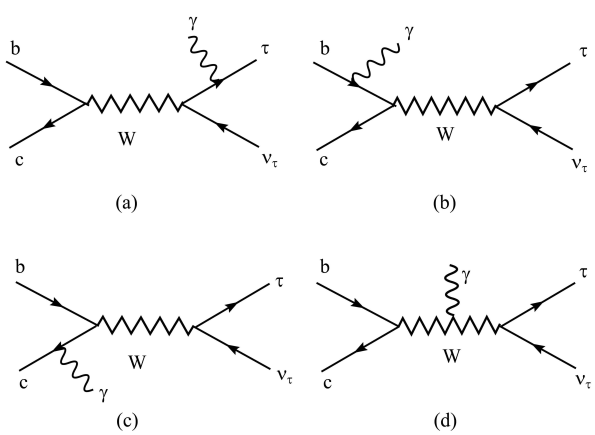

Fig. 1. Feynman diagrams for $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ decay in the SM.

The contribution from Fig. 1(d) can be neglected due to a suppression factor $\frac{m_{B_c}^2}{M_W^2}$ (M_W is the mass of boson W) when the photon is radiated from the mediated $W^\pm$ boson. The main contributions to decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ in the SM come from the diagrams when the photon is radiated from initial quarks or the final lepton. We will use the non-relativistic quark model which was previously used in Ref. [9] to write down the amplitudes. Since the bottom and charm quarks inside the B_c meson are heavy, it is reasonable to neglect the relative momentum of the quark constituents and their binding energy relative to their masses. In this non-relativistic limit, the constituents are on the mass shell and move together with the same velocity. This means that the following equations are valid to quite good accuracy,

$$M_{B_c} = m_b + m_c, \quad p_b = \frac{m_b}{M_{B_c}} p, \quad p_c = \frac{m_c}{M_{B_c}} p, \qquad (3)$$

where p, p_b and p_c represent the momentum of the B_c meson, b quark and c quark, respectively, and $m_b(m_c)$ denotes the mass ofthe b(c) quark.

With these approximations, we obtain the amplitudes for the decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ in SM corresponding to Fig. 1(a), 1(b) and 1(c) as follows,

$$\begin{aligned}\mathcal{M}_a = & \frac{-iG_F e V_{cb}}{2\sqrt{2} p_1 \cdot p_3} \langle 0|\bar{c}\gamma_\mu(1-\gamma_5)b|B_c(p)\rangle \bar{u}(p_1) \\ & \times \not{\epsilon}(\not{p}_1 + \not{p}_3 + m_\tau)\gamma^\mu(1-\gamma_5)v(p_2), \end{aligned} \qquad (4)$$

$$\begin{aligned}\mathcal{M}_b = & \frac{-iG_F e Q_b V_{cb}}{2\sqrt{2} p_b \cdot p_3} \langle 0|\bar{c}\gamma_\mu(1-\gamma_5)(\not{p}_b - \not{p}_3 + m_b) \\ & \times \not{\epsilon} b|B_c(p)\rangle \bar{u}(p_1)\gamma^\mu(1-\gamma_5)v(p_2), \end{aligned} \qquad (5)$$

$$\begin{aligned}\mathcal{M}_c = & \frac{-iG_F e Q_c V_{cb}}{2\sqrt{2} p_c \cdot p_3} \langle 0|\bar{c}\,\not{\epsilon}(\not{p}_3 - \not{p}_c + m_c)\gamma_\mu(1-\gamma_5) \\ & \times b|B_c(p)\rangle \bar{u}(p_1)\gamma^\mu(1-\gamma_5)v(p_2), \end{aligned} \qquad (6)$$

where $Q_b(Q_c)$ is the charge of the constituent b(c) quark, m_τ and p_1 are the mass and the momentum of tau, respectively, p_2 is the momentum of the anti-tau neutrino, and ϵ and p_3 denote the polarization and momentum of the photon, respectively.

Using the identity

$$\gamma^\mu\gamma^\alpha\gamma^\beta = \gamma^\mu g^{\alpha\beta} + \gamma^\beta g^{\mu\alpha} - \gamma^\alpha g^{\mu\beta} - i\epsilon^{\mu\alpha\beta\delta}\gamma_\delta\gamma_5, \quad (7)$$

the Dirac equality $(\not{p} - m)u(p) = 0$, the condition $p_3 \cdot \epsilon = 0$, and the definition of the meson decay constant

$$\langle 0|\bar{c}\gamma^\mu\gamma_5 b|B_c(p)\rangle = i f_{B_c} p^\mu, \qquad (8)$$

with Eqs. (4)−(6), we can obtain the gauge invariant amplitudes,

$$\begin{aligned}\mathcal{M}_1 = & \frac{G_F f_{B_c} V_{cb} e m_\tau}{\sqrt{2}} \bar{u}(p_1)\Big[-\frac{\epsilon \cdot p}{p \cdot p_3} \\ & + \frac{(\not{\epsilon}\not{p}_3 + 2\epsilon \cdot p_1)}{2p_1 \cdot p_3}\Big](1-\gamma_5)v(p_2), \end{aligned} \qquad (9)$$

$$\begin{aligned}\mathcal{M}_2 = & -\frac{G_F f_{B_c} V_{cb} e}{6\sqrt{2} p \cdot p_3}\big[(6 - s_1)(\epsilon \cdot p\ p_{3\mu} - p \cdot p_3 \epsilon_\mu) \\ & + i s_2 \varepsilon_{\mu\nu\alpha\beta} p^\nu \epsilon^\alpha p_3^\beta\big]\bar{u}(p_1)\gamma^\mu(1-\gamma_5)v(p_2), \end{aligned} \qquad (10)$$

where

$$s_1=\frac{M_{\mathrm{B_c}}}{m_{\mathrm{b}}}+2\frac{M_{\mathrm{B_c}}}{m_{\mathrm{c}}},\quad s_2=\frac{M_{\mathrm{B_c}}}{m_{\mathrm{b}}}-2\frac{M_{\mathrm{B_c}}}{m_{\mathrm{c}}}.$$

So the total amplitude for the decay $\mathrm{B_c^-}\rightarrow\tau^-\bar{\nu}_\tau\gamma$ in SM is

$$\mathcal{M}_{\mathrm{SM}}=\mathcal{M}_1+\mathcal{M}_2, \tag{11}$$

where $\mathcal{M}_1$ is called the internal bremsstrahlung (IB) part and $\mathcal{M}_2$ is called the structure dependent (SD) part.

2.2 In the two-Higgs-doublet model

We proceed to consider the radiative leptonic decay $\mathrm{B_c^-}\rightarrow\tau^-\bar{\nu}_\tau\gamma$ in the so-called model II of the two-Higgs-doublet model [8] in which one Higgs doublet couples to down-type quarks and charged leptons and the other to up-type quarks.

The Yukawa interaction between fermions and the charged Higgs field is determined by $\tan\beta$, fermion masses and CKM matrix elements. The relevant Lagrangian has the following form,

$$\begin{aligned}L\ =\ &\frac{g_{\mathrm{W}}}{2\sqrt{2}M_{\mathrm{W}}}\{V_{ij}m_{u_i}X\bar{u}_i(1-\gamma_5)d_j+V_{ij}m_{d_j}Y\bar{u}_i\\&\times(1+\gamma_5)d_j+m_lZ\bar{\nu}(1+\gamma_5)l\}H^{\pm}+\mathrm{h.c.},\end{aligned} \tag{12}$$

where g_{W} represents the weak coupling constant, $H^{\pm}$ is the charged physical field, $u_i(d_j)(i,j{=}1,2,3)$ represents the field operators of up(down)-type quarks and V_{ij} is the relevant CKM matrix element. In model II of 2HDM, $X=\cot\beta$ and $Y=Z=\tan\beta$.

The Feynman diagrams for the decay $\mathrm{B_c^-}\rightarrow\tau^-\bar{\nu}_\tau\gamma$ in 2HDM are depicted in Fig. 2.

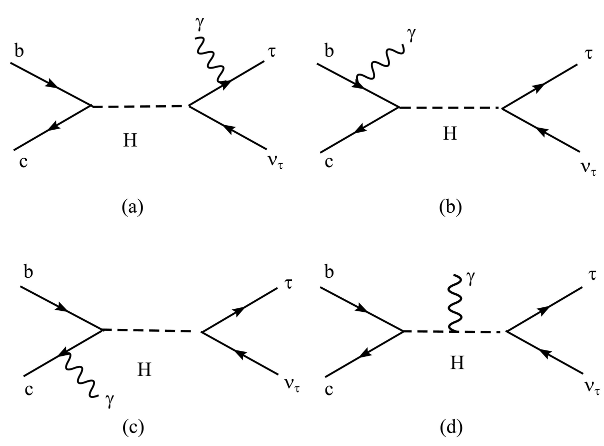

Fig. 2. Feynman diagrams for $\mathrm{B_c^-}\rightarrow\tau^-\nu_\tau\gamma$ decay in 2HDM.

From the virtual Higgs Boson contribution to $\mathrm{B}\rightarrow\mathrm{X_s}\gamma$, one finds $M_{\mathrm{H}}>300$ MeV [10]. As we have analyzed in the SM, the photon emitted from $H^{\pm}$ is similarly suppressed by a factor $\frac{m^2_{\mathrm{B_c}}}{M^2_{\mathrm{H^\pm}}}$. So we can safely neglect the contribution from Fig. 2(d). The amplitudes corresponding to Fig. 2(a), 2(b), 2(c) can be written as

$$\begin{aligned}\tilde{\mathcal{M}}_{\mathrm{a}}\ =\ &\frac{\mathrm{i}G_{\mathrm{F}}V_{\mathrm{cb}}eR}{\sqrt{2}}\langle0|\bar{c}(1+\gamma_5)b|B_{\mathrm{c}}(p)\rangle\ \bar{u}(p_1)\\&\times\not{\epsilon}\frac{(\not{p}_1+\not{p}_3+m)}{2p_1\cdot p_3}(1-\gamma_5)v(p_2),\end{aligned} \tag{13}$$

$$\begin{aligned}\tilde{\mathcal{M}}_{\mathrm{b}}\ =\ &\frac{\mathrm{i}G_{\mathrm{F}}V_{\mathrm{cb}}eQ_{\mathrm{b}}R}{\sqrt{2}}\langle0|\bar{c}(1+\gamma_5)\frac{(\not{p}_{\mathrm{b}}-\not{p}_3+m_{\mathrm{b}})}{2p_{\mathrm{b}}\cdot p_3}\\&\times\not{\epsilon}b|B_{\mathrm{c}}(p)\rangle\bar{u}(p_1)(1-\gamma_5)v(p_2),\end{aligned} \tag{14}$$

$$\begin{aligned}\tilde{\mathcal{M}}_{\mathrm{c}}\ =\ &\frac{\mathrm{i}G_{\mathrm{F}}V_{\mathrm{cb}}eQ_{\mathrm{c}}R}{\sqrt{2}}\langle0|\bar{c}\not{\epsilon}\frac{(\not{p}_3-\not{p}_{\mathrm{c}}+m_{\mathrm{c}})}{2p_{\mathrm{c}}\cdot p_3}\\&\times(1+\gamma_5)b|B_{\mathrm{c}}(p)\rangle\bar{u}(p_1)(1-\gamma_5)v(p_2).\end{aligned} \tag{15}$$

Using the relation

$$\langle0|\bar{c}\gamma_5b|B_{\mathrm{c}}(p)\rangle=-\mathrm{i}f_{\mathrm{B_c}}\frac{M^2_{\mathrm{B_c}}}{m_{\mathrm{b}}+m_{\mathrm{c}}}\approx-\mathrm{i}f_{\mathrm{B_c}}M_{\mathrm{B_c}}, \tag{16}$$

and Eqs. (13)–(15), we can get the gauge invariant amplitude for the radiative leptonic decay $\mathrm{B_c^-}\rightarrow\tau^-\bar{\nu}_\tau\gamma$ in 2HDM,

$$\begin{aligned}\mathcal{M}_{\mathrm{2HDM}}\ =\ &\frac{G_{\mathrm{F}}f_{\mathrm{B_c}}V_{\mathrm{cb}}eM_{\mathrm{B_c}}R}{\sqrt{2}}\bar{u}(p_1)\left\{-\frac{\epsilon\cdot p}{p\cdot p_3}\right.\\&\left.+\frac{\not{\epsilon}\not{p}_3+2\epsilon\cdot p_1}{2p_1\cdot p_3}\right\}(1-\gamma_5)\ v(p_2),\end{aligned} \tag{17}$$

where

$$R=r^2m_\tau m_{\mathrm{b}},\quad r=\frac{\tan\beta}{M_{\mathrm{H^\pm}}}.$$

While deriving Eq. (17), we have neglected a term proportional to m_{c} as in Ref. [11] because it is suppressed by $\frac{m_{\mathrm{c}}}{m_{\mathrm{b}}}$. From Eq. (17), we find that only the internal bremsstrahlung part contributes in 2HDM.

Then, the total amplitude for the radiative leptonic decay including the contributions from both the SM and 2HDM is

$$\mathcal{M}_{\mathrm{total}}=\mathcal{M}_{\mathrm{SM}}+\mathcal{M}_{\mathrm{2HDM}}. \tag{18}$$

We define two variables,

$$x_2=\frac{2E_\nu}{M_{\mathrm{B_c}}},\quad x_3=\frac{2E_\gamma}{M_{\mathrm{B_c}}}, \tag{19}$$

where E_ν and E_γ are the energies of neutrino and photon, respectively. The physical regions of x_2 and x_3 are given as

$$\begin{aligned}1-x_3-\frac{m^2_\tau}{M^2_{\mathrm{B_c}}}\ \leqslant\ x_2\leqslant1-\frac{m^2_\tau}{M^2_{\mathrm{B_c}}(1-x_3)},\\0\ \leqslant\ x_3\leqslant1-\frac{m^2_\tau}{M^2_{\mathrm{B_c}}}.\end{aligned} \tag{20}$$

The expression for the differential decay width can be written as

$$d\Gamma = \frac{M_{B_c}}{256\pi^3}\sum_{\text{spins}}|\mathcal{M}_{\text{total}}|^2 dx_2 dx_3. \quad (21)$$

3 Numerical results and discussions

The mass of B_c was measured as $M_{B_c} = 6.276 \pm 0.004$ GeV in 1.96 TeV $p\bar{p}$ collisions using the CDF detector at Fermilab's Tevatron Collidor [12]. For numerical results, we use the following set of parameters $M_{B_c} = 6.276$ GeV, $m_b = 4.2$ GeV, $m_c = 1.27$ GeV, $m_\tau = 1.777$ GeV, $|V_{cb}| = 0.0412$, $\tau_{B_c} = 0.46$ ps [12], $f_{B_c} = 0.36$ GeV [13] and $\alpha_{em} = 1/132$.

We firstly estimate the branching ratio of the radiative leptonic decay using Eqs. (18)−(21). Since the total decay width is singular when the photon energy approaches zero, we impose a cut for the photon energy, as in Ref. [14]. The photon energy $E_\gamma \geqslant 50$ MeV corresponds to $x_3|_{\min} = 1.593\times10^{-2}$ and $E_\gamma \geqslant 100$ MeV corresponds to $x_3|_{\min} = 3.187\times10^{-2}$. We let x_3 be 1.593×10^{-2} for $B_c^- \to l^-\bar{\nu}_l\gamma$ (l=e, μ) and 1.593×10^{-2} or 3.187×10^{-2} for the decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$. The results for branching ratios for radiative leptonic decays in the SM are collected in Table 1, where "IN" represents the interference between the IB part and the SD part, and "SUM" denotes the sum of IB, SD and IN contributions.

Table 1. Branching ratios for the B_c radiative leptonic decays in the SM.

decay Br	IB	SD	IN	SUM
$B_c^- \to e^-\bar{\nu}_e\gamma$	1.72×10^{-10}	4.59×10^{-5}	3.04×10^{-11}	4.59×10^{-5}
$B_c^- \to \mu^-\bar{\nu}_\mu\gamma$	2.72×10^{-6}	4.58×10^{-5}	5.13×10^{-7}	4.91×10^{-5}
$B_c^- \to \tau^-\bar{\nu}_\tau\gamma$				
$x_3 = 1.593\times10^{-2}$	9.06×10^{-5}	3.05×10^{-5}	2.11×10^{-5}	1.42×10^{-4}
$x_3 = 3.187\times10^{-2}$	6.90×10^{-5}	3.03×10^{-5}	2.08×10^{-4}	1.20×10^{-4}

Table 2. Comparison of the branching ratios for the B_c radiative leptonic decays.

decay Br	Ref.	IB	SD	IN	SUM
$B_c^- \to e^-\bar{\nu}_e\gamma$	[15]				4.9×10^{-5}
	[16]				4.4×10^{-5}
	[18]				1.0×10^{-5}
	[19]$x_3 = 0.016$	1.7×10^{-10}	2.0×10^{-5}	1.5×10^{-11}	2.0×10^{-5}
$B_c^- \to \mu^-\bar{\nu}_\mu\gamma$	[15]				4.9×10^{-5}
	[16]				4.4×10^{-5}
	[18]				1.0×10^{-5}
	[19]$x_3 = 0.016$	2.8×10^{-10}	2.0×10^{-5}	2.5×10^{-11}	2.3×10^{-5}
$B_c^- \to \tau^-\bar{\nu}_\tau\gamma$	[19]$x_3 = 0.016$	8.7×10^{-5}	1.2×10^{-5}	0.8×10^{-5}	1.07×10^{-4}
	[14]$x_3 = 0.016$	8.6×10^{-5}	7.24×10^{-6}	2.17×10^{-6}	9.54×10^{-5}
	[20]$x_3 = 0.03182$				3.44×10^{-4}
	[19]$x_3 = 0.032$	6.8×10^{-5}			8.8×10^{-5}
	[14]$x_3 = 0.032$	6.53×10^{-5}	7.24×10^{-6}	2.18×10^{-6}	7.47×10^{-5}

In the following, we also present the predictions of the branching ratios for the leptonic decays with the same set of parameters,

$$Br(B_c^- \to e^-\bar{\nu}_e) \approx 1.36\times10^{-9},$$
$$Br(B_c^- \to \mu^-\bar{\nu}_\mu) \approx 5.83\times10^{-5},$$
$$Br(B_c^- \to \tau^-\bar{\nu}_\tau) \approx 1.40\times10^{-2}. \quad (22)$$

From a comparison between the numbers in Table 1 and those in Eq. (22), we find that the branching ratio of the radiative leptonic decay is of the same order as the leptonic decay for a muon, while the branching ratio is enhanced by about 4 orders for an electron. Due to the fine-structure constant α_{em} suppression, the branching ratio for $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ is 2 orders smaller than the leptonic decay.

The radiative leptonic decay width for $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ has been calculated by several authors. For comparison, these results are collected together in Table 2. The authors of Ref. [15] and Ref. [16] use a non-relativistic constituent model and non-relativistic QCD, respectively. It can be seen from Table 1 and Table 2 that the results for the branching ratio for $B_c^- \to \mu^-\bar{\nu}_\mu\gamma$ are all of the same order, 10^{-5}. From Table 1, when the final lepton is an electron or a muon, one can also find that the branching ratio for the IB part is smaller than that for the SD part because the IB is suppressed by helicity and $\alpha_{\rm em}$ [17], while the SD part is only reduced by $\alpha_{\rm em}$, so the SD part provides the main contribution, as our results predict. As far as τ is concerned, the IB part is not suppressed by helicity any more while the SD part is still reduced by $\alpha_{\rm em}$, so the main contribution comes from the IB part. It can also be seen from Table 2 that the branching ratios for $\rm B_c^- \to l^-\bar{\nu}_l\gamma$ ($\rm l$=e, μ) in our calculation are 4–5 times larger than those predicted by the QCD sum rule in Ref. [18] and about 2 times larger than those given in the light front framework in Ref. [19].

As we noted earlier, approximately 2×10^8 $\rm B_c$ mesons will be produced at LHC per year. With this, we can obtain the numbers of expected events for the decay $\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ at LHC as $N \approx 2.7\times10^4$ (for $x_3 = 1.593\times10^{-2}$) and $N \approx 2.3\times10^4$ (for $x_3 = 3.187\times10^{-2}$) using the branching ratio in Table 1. We also depict the differential decay width $\mathrm{d}\Gamma(\rm B_c^- \to l^-\bar{\nu}_l\gamma)/\mathrm{d}x_3$ in Figs. 3, 4 and 5 corresponding to e, μ and τ, respectively. It is easy to find that $\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ has a large contribution from the soft photon emission, while the $\rm B_c^- \to e^-\bar{\nu}_e\gamma$ and $\rm B_c^- \to \mu^-\bar{\nu}_\mu\gamma$ are dominated by the hard photon emission.

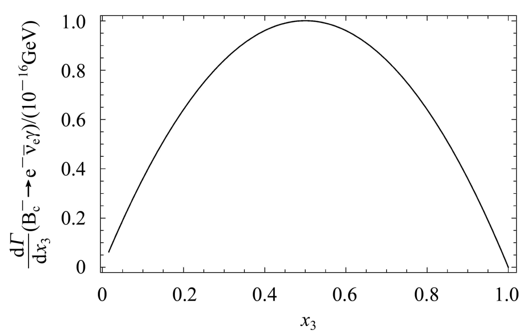

Fig. 3. The differential decay width $\mathrm{d}\Gamma(\rm B_c^- \to e^-\bar{\nu}_e\gamma)/\mathrm{d}x_3$ as a function of $x_3(=2E_\gamma/M_{\rm B_C})$.

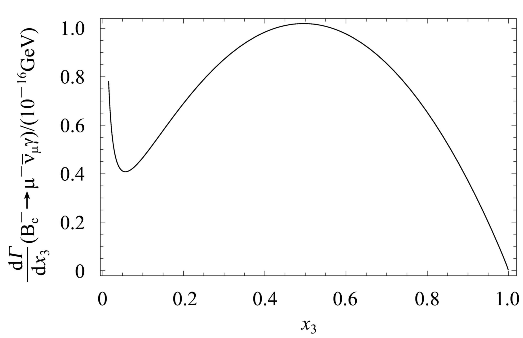

Fig. 4. The differential decay width $\mathrm{d}\Gamma(\rm B_c^- \to \mu^-\bar{\nu}_\mu\gamma)/\mathrm{d}x_3$ as a function of $x_3(=2E_\gamma/M_{\rm B_C})$.

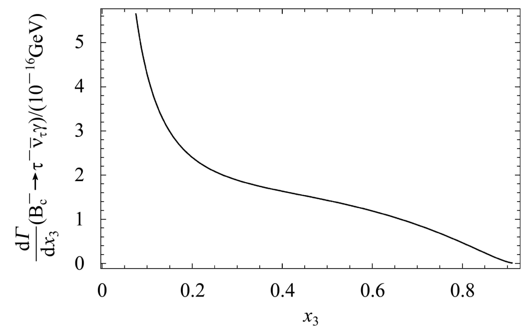

Fig. 5. The differential decay width $\mathrm{d}\Gamma(\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma)/\mathrm{d}x_3$ as a function of $x_3(=2E_\gamma/M_{\rm B_C})$.

We proceed to analyze radiative leptonic decays including the contribution from 2HDM which has a parameter r to be fixed. The BABAR collaboration [21] measured the branching ratio for $\rm B \to \tau\nu_\tau$,

$$\mathcal{B}(\rm B \to \tau\nu_\tau) = (1.8^{+0.9}_{-0.8} \pm 0.4 \pm 0.2)\times10^{-4}.$$

It also measured the 90% CL upper limit using the $\rm CL_s$ method [22] to be

$$\mathcal{B}(\rm B \to \tau\bar{\nu}_\tau) < 3.4\times10^{-4}. \tag{23}$$

Assuming $f_{\rm B} = 0.216 \pm 0.022$ GeV [23], using $V_{\rm ub} = (3.93\pm0.36)\times10^{-3}$ [12] and Eq. (23), we get the constraint for r,

$$r = \frac{\tan\beta}{M_{\rm H^\pm}} < 0.307\ \mathrm{GeV}^{-1}, \tag{24}$$

. The large uncertainties lie in $f_{\rm B}$ and $V_{\rm ub}$, which are difficult to extract.

With the above constraint for r, we show the dependence of decay width for the decay $\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma$, including the contributions from both SM and 2HDM on the parameter r in the range $0 \leqslant r \leqslant 0.4$ in Fig. 6.

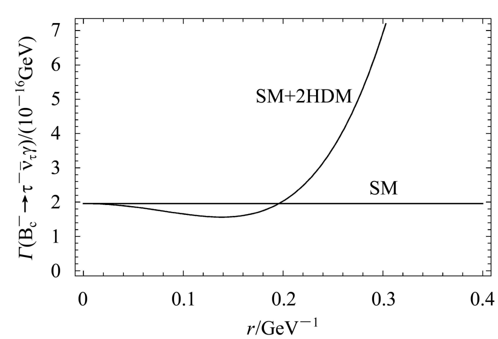

Fig. 6. The decay width $\Gamma(\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma)$ in the SM and in the SM+2HDM. The curved line represents the change in $\Gamma(\rm B_c \to \tau\bar{\nu}_\tau\gamma)$ with the parameter r in the SM+2HDM. The horizontal line represents the decay width in the SM.

It can be seen from Fig. 6 that $\Gamma(\rm B_c^- \to \tau^-\bar{\nu}_\tau\gamma)$ is sensitive to the values of parameter r. The decay width increases when r is larger than 0.20. This behavior is not consistent with the prediction in Ref. [24] where the authors considered different values of $\tan\beta$ and $M_{H^\pm}$. However, this tendency is very similar to that for the leptonic decay in Ref. [7]. We obtain the following branching ratio upper bound when r reaches the upper limit 0.307,

$$Br(B_c^- \to \tau^-\bar{\nu}_\tau\gamma) \leqslant 4.52\times10^{-4}. \quad (25)$$

We note that the authors in Ref. [20] gave a broader range with the upper bound 9.05×10^{-3} in the multi Higgs doublet model than ours.

In conclusion, we analyze the radiative leptonic decay modes $B_c^- \to e^-\bar{\nu}_e\gamma$, $B_c^- \to \mu^-\bar{\nu}_\mu\gamma$ and $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ in the Standard Model using the non-relativistic constituent quark model as well as the decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ in the two-Higgs-doublet model. We find that the branching ratios for $B_c^- \to e^-\bar{\nu}_e\gamma$ and $B_c^- \to \mu^-\bar{\nu}_\mu\gamma$ in the SM are at the level 10^{-5}. The branching ratio for the decay $B_c^- \to \tau^-\bar{\nu}_\tau\gamma$ in SM is at the level 10^{-4}, which may be detectable at the LHC and this decay has a strong dependence on the parameter of 2HDM. When enough B_c samples are collected, the radiative leptonic decays will be alternate channels for measuring the decay constant f_{B_c}. To enhance the accuracy of the theoretical predictions, much more careful calculations of the B_c radiative leptonic decays with different models are needed. All the calculations will be tested in the forthcoming experiment.

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