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Can a non-unitary effect be prominent in neutrino oscillation measurements?*

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Abstract Subject to neutrino experiments, the mixing matrix of ordinary neutrinos can still have small violation from unitarity. We introduce a quasi-unitary matrix to interpret this violation and propose a natural scheme to parameterize it. A quasi-unitary factor Δ_{QF} is defined to be measured in neutrino oscillation experiments and the numerical results show that the improvement in experimental precision may help us figure out the secret of neutrino mixing.

Key words neutrino, mixing angle, unitarity, oscillation

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1 Introduction

The definite evidence of neutrino oscillation has confirmed that neutrinos are massive and they exhibit non-trivial mixing, which has strengthened our belief in new physics beyond the Standard Model (SM). The compelling experiments verifying neutrino masses and mixing angles are from the neutrino oscillation measurements that include solar (SNO, KamLAND) [1, 2], atmosphere (SK) [3] and reactor (CHOOZ) [4] neutrino experiments. These can give the mass-squared splitting and trigonometric function of mixing angles. Up to now, all other experiments to measure the absolute values of neutrino masses have only been able to give the upper limits and relatively rough mixing angles.

To explain neutrino masses, adding the neutrino Yukawa coupling (which gives the Dirac mass of neutrino) to the SM Lagrangian is straightforward and causes no anomaly, but the real problem is the huge hierarchy between the up component (ν_e, ν_μ, ν_τ) and the down component (e, μ, τ) of the lepton isospin doublet, which is very different from the hierarchy between the up-type and down-type quarks. Among the mechanisms proposed to generate very light neutrino masses, the most popular one is the seesaw mechanism [5, 6]. In seesaw models, heavy right-handed Majorana neutrino mass terms are added to the SM Lagrangian as they are a complete singlet of gauge transformation in the SM. The Dirac masses of neutrinos, which generally at the electro-weak scale are suppressed by Majorana mass terms to be ultra-light Majorana neutrinos, are (primarily) left-handed. This mechanism embodies not only the mixing of ordinary light neutrino flavors, which is similar to quark mixing, but also the mixing between ordinary and additional heavy neutrinos, which is different from charged fermions. On the other side, current experimenters and data analyzers still use the three-flavor neutrino model in which the neutrino eigenstates are transformed by a 3×3 unitary matrix, namely the PMNS matrix [7], just the same as the CKM matrix.

If there do exist more than three neutrino species, the matrix that transforms the ordinary neutrino mass eigenstate to the flavor eigenstate should not be inherently unitary [8, 9], as the additional heavy neutrinos can mix with the three ordinary ones and make the unitarity of the PMNS matrix deviated. The non-unitary effects of ordinary neutrino mixing have been extensively studied in the literature, giving possible correction to unitarity and further with new parameterizations for the neutrino mixing matrix [10–14]. While models like these emphasize obtaining a more physical interpretation of neutrino mixing, there is still one pressing question that needs to be answered: can the non-unitary effects stand out within the cur-

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rent or forthcoming measurement precision? That is to say whether the violation from unitarity tests will be more of experimental error or theoretical non-unitary effects itself. The answer to the question is more fundamental for the parameterization of non-unitary neutrino mixing.

In this work, we start from a relatively simpler yet reasonable parameterization and try to answer the question. A quasi-unitary matrix is proposed to be theoretically deviated from unitarity, thus this theoretical effect may account for the violation of unitarity. Then, to determine whether the neutrino mixing is unitary or quasi-unitary is important for the parameterization of neutrino mixing matrix. Furthermore, such tests will give us more hints about the lepton sector of the SM because a unitary neutrino mixing implies that neutrinos are Dirac fermions or are Majorana fermions with no mixing with the heavy degrees of freedom in some seesaw models [14], but a non-unitary mixing matrix will tell us other stories about this.

2 Quasi-unitary mixing and parameterization

To test the unitarity of neutrino mixing is important because unitary mixing implies only three Dirac neutrinos or only three special Majorana neutrinos, while quasi-unitary mixing will imply the existence of more than three neutrino species and a new mechanism for the generation of neutrino mass. For a unitary mixing matrix $\boldsymbol{U}$,

$$\boldsymbol{U}\boldsymbol{U}^{\dagger} = \boldsymbol{I}. \tag{1}$$

Subject to constraints from weak decays, the mixing matrix of ordinary neutrinos can be violated from unitarity at the order of 1% level [9]. If this mixing matrix is inherently quasi-unitary with the definition

$$\boldsymbol{Q} = (\boldsymbol{I}+\boldsymbol{X})\boldsymbol{U}, \tag{2}$$

where $\boldsymbol{X}$ is a small matrix at subleading order (1st order), strictly speaking, matrix $\boldsymbol{X}$ should be hermitian. Many parameters need to be introduced for its parameterization, as done by many previous works. Considering that only a few neutrino experimental measurements are available, it is sufficient to give a relatively simpler, yet reasonable parameterization for this matrix. An ideal parameterization will be $\boldsymbol{X}+\boldsymbol{X}^{\dagger}=0$,

$$\boldsymbol{Q}\boldsymbol{Q}^{\dagger} = \boldsymbol{I}+\boldsymbol{X}\boldsymbol{X}^{\dagger}, \tag{3}$$

thus $\boldsymbol{Q}\boldsymbol{Q}^{\dagger}$ equates to $\boldsymbol{I}$ at the first order and violates the unitarity slightly at the second order. However, this scheme seems to be ideal since on the theoretical side, the unitarity is violated at the first order in many neutrino mixing models, such as the seesaw models; and on the experimental side, we cannot determine whether the error is from the experimental measurements or theoretically from the second order violation by the test of $\boldsymbol{N}\boldsymbol{N}^{\dagger}$ ($\boldsymbol{N}$ is the neutrino mixing matrix measured from experiments). The second order violation implies that the neutrino mixing matrix can violate the unitarity greatly. So to determine the order of the violation of unitarity is important for the unitarity test. In our paper, we change the ideal scheme in Eq. (3) slightly and propose a simple parameterization for neutrino mixing to try to make the problem clear.

If neutrinos are Majorana fermions, as predicted by the seesaw mechanism, the mixing matrix for the diagonalization of the left- and right-handed neutrinos should be an overall unitary matrix $\boldsymbol{O}$ instead of the 3×3 PMNS matrix for the left-handed neutrinos, which is only part of $\boldsymbol{O}$ and no longer unitary. To be more clear, we denote the mixing matrix $\boldsymbol{O}$ as

$$\boldsymbol{O}_{2n\times2n} = \begin{pmatrix} \boldsymbol{A}_{n\times n} & \boldsymbol{C}_{n\times n} \\ \boldsymbol{D}_{n\times n} & \boldsymbol{B}_{n\times n} \end{pmatrix}, \tag{4}$$

where $\boldsymbol{A}$ is the matrix that transforms the mass eigenstates of three left-handed neutrinos to the flavor eigenstates with n as the flavor number. The unitarity of $\boldsymbol{O}$ requires that

$$\boldsymbol{A}\boldsymbol{A}^{\dagger}+\boldsymbol{C}\boldsymbol{C}^{\dagger}=\boldsymbol{I}. \tag{5}$$

We also denote $\boldsymbol{A}=(\boldsymbol{I}+\boldsymbol{X})\boldsymbol{U}$ as the quasi-unitary definition in Eq. (2), where $\boldsymbol{X}$ is a small arbitrary matrix at the subleading order. Thus we have

$$\boldsymbol{A}\boldsymbol{A}^{\dagger}=\boldsymbol{I}+\boldsymbol{X}+\boldsymbol{X}^{\dagger}+\boldsymbol{X}\boldsymbol{X}^{\dagger}=\boldsymbol{I}-\boldsymbol{C}\boldsymbol{C}^{\dagger}. \tag{6}$$

We can resort to the details of a seesaw mechanism to decide which situation is appropriate for neutrino mixing. For example, in the Type-I seesaw model, the ultralight values of $\boldsymbol{m}_{\nu}$ can be obtained by

$$\boldsymbol{m}_{\nu}=-\boldsymbol{m}_{\mathrm{D}}\left(\boldsymbol{m}_{\mathrm{M}}^{\mathrm{R}}\right)^{-1}\boldsymbol{m}_{\mathrm{D}}^{\mathrm{T}}, \tag{7}$$

where $\boldsymbol{m}_{\mathrm{D}}$ is the Dirac mass matrix arising from Yukawa coupling and $\boldsymbol{m}_{\mathrm{M}}^{\mathrm{R}}$ is the Majorana mass matrix, which is unforbidden by gauge transformation of the SM. For the transformation matrix $\boldsymbol{O}$, it diagonalizes the neutrino mass matrix in two steps. The first step is a block diagonalization, which reduces the problem to a two by two problem and the rotation is essentially a generalized 2×2 Euler rotation. The second step is the diagonalization of a light neutrino mass matrix by a unitary matrix $\boldsymbol{U}_1$ and that of the heavy one by $\boldsymbol{U}_2$. Then the overall matrix is given by

$$\boldsymbol{O}_{2n\times 2n}=\begin{pmatrix}\cos\boldsymbol{\Theta}\ \boldsymbol{U}_1 & \sin\boldsymbol{\Theta}\ \boldsymbol{U}_1\\ -\sin\boldsymbol{\Theta}\ \boldsymbol{U}_2 & \cos\boldsymbol{\Theta}\ \boldsymbol{U}_2\end{pmatrix}, \qquad (8)$$

where the $\sin\boldsymbol{\Theta}$ and $\cos\boldsymbol{\Theta}$ are to be interpreted as series expansions of a small matrix $\boldsymbol{\Theta}=\boldsymbol{m}_{\rm D}/\boldsymbol{m}_{\rm M}^{\rm R}$. From Eq. (4),

$$\boldsymbol{A}=\cos\boldsymbol{\Theta}\boldsymbol{U}_1=\left(\boldsymbol{I}-\frac{\boldsymbol{\Theta}^2}{2!}+\frac{\boldsymbol{\Theta}^4}{4!}-\cdots\right)\boldsymbol{U}_1, \qquad (9)$$

$$\boldsymbol{C}=\sin\boldsymbol{\Theta}\boldsymbol{U}_1=\left(\boldsymbol{\Theta}-\frac{\boldsymbol{\Theta}^3}{3!}+\frac{\boldsymbol{\Theta}^5}{5!}-\cdots\right)\boldsymbol{U}_1, \qquad (10)$$

we can see that $\boldsymbol{X}+\boldsymbol{X}^\dagger$ is at the same order of $\boldsymbol{C}\boldsymbol{C}^\dagger$, thus $\boldsymbol{A}$ violates unitarity at the first order. Because $\sin\boldsymbol{\Theta}$ is a small matrix at $\mathcal{O}(\boldsymbol{m}_{\rm D}/\boldsymbol{m}_{\rm M}^{\rm R})$ and we know nothing about $\boldsymbol{m}_{\rm D}$ and $\boldsymbol{m}_{\rm M}^{\rm R}$, in order to simplify the problem, we consider that

$$\boldsymbol{C}\sim\boldsymbol{U}/\eta, \qquad (11)$$

where $1/\eta$ is a small number. Here we just use an approximate matrix to substitute the transformation matrix $\boldsymbol{O}$, and the diagonalization is not consistent with the seesaw mechanism, but it can simplify the parameterization for the quasi-unitary matrix and give us a way to test the order of unitarity violation. Now we have

$$\boldsymbol{A}\boldsymbol{A}^\dagger\approx\boldsymbol{I}+\boldsymbol{X}+\boldsymbol{X}^\dagger\approx\boldsymbol{I}-\frac{\boldsymbol{I}}{\eta^2}. \qquad (12)$$

For simplicity, we treat $\boldsymbol{X}$ as a real matrix, thus $\boldsymbol{X}$ can be parameterized as

$$\boldsymbol{X}=\begin{pmatrix}-\epsilon & x & y & \vdots\\ -x & -\epsilon & z & \vdots\\ -y & -z & -\epsilon & \vdots\\ \cdots & \cdots & \cdots & \ddots\end{pmatrix}, \qquad (13)$$

where $\epsilon=1/2\eta^2$. We should note that here we just treat ϵ as a parameter that can be set as any real number, even zero, to make $\boldsymbol{A}$ a quasi-unitary matrix, thus our parameterization is not totally based on a seesaw mechanism but on a much simpler and more generalized way to parameterize the quasi-unitary matrix. In such parameterization, $\boldsymbol{A}\boldsymbol{A}^\dagger$ deviates from $\boldsymbol{I}$ only in the diagonal elements at the first order when $\epsilon\neq 0$. This is consistent with the result of Ref. [9], in which $\boldsymbol{N}\boldsymbol{N}^\dagger$ deviates from $\boldsymbol{I}$ apparently in the diagonal elements. $\boldsymbol{A}$ is thus given by

$$\boldsymbol{A}_{2\times 2}=\begin{pmatrix}1-\epsilon & x\\ -x & 1-\epsilon\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix} \qquad (14)$$

for the two-flaver case and

$$\boldsymbol{A}_{3\times 3}=\begin{pmatrix}1-\epsilon & x & y\\ -x & 1-\epsilon & z\\ -y & -z & 1-\epsilon\end{pmatrix}\times$$
$$\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12} & s_{13}\\ -c_{23}s_{12}-s_{23}s_{13}c_{12} & c_{23}c_{12}-s_{23}s_{13}s_{12} & s_{23}c_{13}\\ s_{23}s_{12}-c_{23}s_{13}c_{12} & -s_{23}c_{12}-c_{23}s_{13}s_{12} & c_{23}c_{13}\end{pmatrix} \qquad (15)$$

for the three-flavor case. Here, θ (θ_{ij}) is the rotation angle used to parameterize the unitary matrix $\boldsymbol{U}_{2\times 2}$ ($\boldsymbol{U}_{3\times 3}$) and s_{ij} (c_{ij}) means $\sin\theta_{ij}(\cos\theta_{ij})$. Note that we do not consider the CP phase δ in the three-flavor case. In addition to the unitary parameterization, there are two more parameters for the 2×2 quasi-unitary matrix and four more for the 3×3 one.

In the seesaw interpretation, parameters ϵ, x, y, z are too small (typically around $\mathcal{O}(10^{-26})$) to have any implication for the neutrino measurements, thus the neutrino mixing matrix $\boldsymbol{A}$ can be safely parameterized as a unitary matrix. But if ϵ, x, y, z are not so small, they can make unitary parameterization deviated with observable errors, which should be theoretical errors. In the next section, we will show the implications for such parameterization in neutrino oscillation experiments.

3 Implications for neutrino oscillation

Ever since the issue of neutrino mixing was on the table, it has been assumed that the neutrino flavor eigenstates $|\nu_\alpha\rangle$ (α = e, μ, τ) are a linear superposition of the neutrino mass eigenstates $|\nu_i\rangle$ ($i=1,\ 2,\ 3$) through a unitary leptonic mixing matrix: $U^*_{\alpha i}$ and $|\nu_\alpha\rangle=\sum_i U^*_{\alpha i}|\nu_i\rangle$. By oscillating when propagating in vacuum, the oscillation probabilities can be expressed as (two flavors case $n=2$) [15]

$$P(\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta)\approx S_{\alpha\beta}\sin^2[1.27\Delta M^2(L/E)] \qquad (16)$$

for $\alpha\neq\beta$ and

$$P(\overset{(-)}{\nu}_\alpha\to\overset{(-)}{\nu}_\beta)\approx 1-4T_\alpha(1-T_\alpha)\sin^2[1.27\Delta M^2(L/E)] \qquad (17)$$

for $\alpha=\beta$, where E is the energy of neutrinos, L is the distance they travel, ΔM^2 is the neutrino mass squared splitting and

$$S_{\alpha\beta}\equiv 4\left|\sum_{i\ \rm Up}U^*_{\alpha i}U_{\beta i}\right|^2,\quad T_\alpha\equiv\sum_{i\ \rm Up}|U_{\alpha i}|^2. \qquad (18)$$

Here, 'i up' denotes a sum over only those neutrino mass eigenstates that lie above ΔM^2 or, alternatively, only those that lie below it. The unitarity of U guarantees that $S_{\alpha\beta}$ and $4T_\alpha(1-T_\alpha)$ get exactly the same results in Eq. (16) and Eq. (17), which yield

$$4T_\alpha(1-T_\alpha)=S_{\alpha\beta}=\sin^2 2\theta. \tag{19}$$

This is the two-flavor case and the same result occurs for the three-flavor case.

Inspired by Eq. (19), we define a 'Quasi-unitary factor' to test the unitarity of neutrino mixing,

$$\varDelta_{\rm QF}=4T_\alpha(1-T_\alpha)-S_{\alpha\beta}. \tag{20}$$

For the unitary mixing, $\varDelta_{\rm QF}=0$ theoretically, so if we calculate this factor from the experimental data, a non-zero result must come from (and only from) the experimental errors. But in the quasi-unitary case, a non-zero result should be partially from the quasi-unitarity. If $\epsilon=0$, $\varDelta_{\rm QF}$ still equates to zero at the leading order since

$$4T_\alpha(1-T_\alpha)\approx S_{\alpha\beta}\approx\sin^2 2\theta+2x\sin 4\theta, \tag{21}$$

which is similar to Eq. (19), but the situation changes when $\epsilon\neq 0$, where we have

$$\begin{aligned}4T_\alpha(1-T_\alpha)\ \approx\ &\sin^2 2\theta+2x\sin 4\theta\\ &-4\epsilon\sin^2 2\theta+8\epsilon\sin^2\theta,\end{aligned} \tag{22}$$

$$S_{\alpha\beta}\ \approx\ \sin^2 2\theta+2x\sin 4\theta-4\epsilon\sin^2 2\theta, \tag{23}$$

therefore $\varDelta_{\rm QF}$ no longer equates to zero,

$$\varDelta_{\rm QF}\approx 8\epsilon\sin^2\theta+\mathcal{O}(x^2,\ \epsilon^2,\cdots). \tag{24}$$

Thus, by comparing the neutrino oscillation data between $P(\nu_\alpha\to\nu_\alpha)$ and $P(\nu_\alpha\to\nu_\beta)$, we can test the unitarity of neutrino mixing. From Eq. (22) and Eq. (23), one should realize that the mixing angle θ here has a different meaning from the ordinary one measured under the unitary parameterization, which is essentially an effective mixing angle. To discriminate the quasi-unitary mixing angles and those in unitary scheme, we denote the quasi-unitary ones as $\theta^{\rm Q}$ in subsequent discussions. Meanwhile, as indicated by Eq. (24), it is the diagonal parameter ϵ in $\boldsymbol{X}$ that dominates $\varDelta_{\rm QF}$ while the off-diagonal effect by $x,\ y,\ z$ appears at subleading order, thus it can be absorbed into the effective unitary parameterization of the mixing matrix. But from Eqs. (21, 22, 23), we can see that $x,\ y,\ z$ together with ϵ can make $\theta^{\rm Q}$ (maybe greatly) deviated from θ and make $\varDelta_{\rm QF}$ big enough to be measured in the neutrino oscillation experiments.

Now we inspect the actual three-neutrino mixing situation and try to give the 'quasi-unitary correction' to certain explicit measurements. As mentioned above, in order to test the unitarity, we need to measure $\varDelta_{\rm QF}$ dependent on the same $\theta^{\rm Q}_{ij}$, which needs data from different oscillation patterns. In the case of $n=3$, however, such checks are very difficult, as all the current accelerator, reactor, solar and atmospheric neutrino data are described within the framework of the 3×3 PMNS matrix. And different experiments measure different θ_{ij} and Δm^2_{ij} without considering the non-unitarity of neutrino mixing. We find that short distance $\nu_{\rm e}$ oscillation measurements may give us some hints. For a short distance ($L<5$ km) it is a good approximation to express the $\nu_{\rm e}$ oscillation probabilities in the unitary scheme as [15]

$$\begin{aligned}P(\bar\nu_{\rm e}\to\bar\nu_{\rm e})\ &\approx\ 1-\sin^2(2\theta_{13})\sin^2(\Delta m^2_{32}L/4E),\\ P(\nu_{\rm e}\to\nu_\mu)\ &\approx\ \sin^2(2\theta_{13})\sin^2(\theta_{23})\sin^2(\Delta m^2_{32}L/4E),\\ P(\nu_{\rm e}\to\nu_\tau)\ &\approx\ \sin^2(2\theta_{13})\cos^2(\theta_{23})\sin^2(\Delta m^2_{32}L/4E).\end{aligned} \tag{25}$$

This takes the similar two-neutrino form with θ_{13} and Δm^2_{32}. If the neutrino mixing is quasi-unitary, $\varDelta_{\rm QF}$ can be defined as

$$\begin{aligned}\varDelta_{\rm QF}\ &=\ 4T_{\rm e}(1-T_{\rm e})-S_{\rm e\mu}-S_{\rm e\tau}\\ &\approx\ 8\epsilon\sin^2\theta^{\rm Q}_{13}+\mathcal{O}(x^2,\ xy,\ x\epsilon,\cdots),\end{aligned} \tag{26}$$

which has a similar form to Eq. (24). The measurements of $P(\bar\nu_{\rm e}\to\bar\nu_{\rm e})$ have already been taken in the reactor neutrino experiments. However, $P(\nu_{\rm e}\to\nu_\mu)$ and $P(\nu_{\rm e}\to\nu_\tau)$ are difficult to measure in accelerator neutrino experiments. Thus we need more precise experiments to measure $\varDelta_{\rm QF}$ for the test of unitarity. Though there are no data for such a test, the current experiment data can give us some information about the quasi-unitary mixing. In the next section we will show some numerical results from current experimental constraints.

4 Testing unitarity by experiment

Solar, atmospheric and reactor neutrino experiments are sensitive to different neutrino oscillations, which give us the best fit mixing angles and the errors in the unitary scheme. In the numerical calculation, our general method was to require the new matrix elements $A_{\alpha\beta}$ in Eq. (15) to satisfy the corresponding constraints derived from the latest data [16].

In the last section, we know that it is ϵ that dominates the quasi-unitary factor while $x,\ y,\ z$ and ϵ can make the mixing angles deviate. Therefore, our first step is to see how these parameters could influence the mixing angles. In order to compare the unitary scheme and the quasi-unitary scheme, we scatter θ_{ij}^{Q} from 0° to 90°. For parameters x, y, z and ϵ, we scanned them randomly between −0.1 and 0.1 and kept all of the experimentally allowed points to see the range of θ_{ij}^{Q}. The results are listed in Table 1 in comparison with the unitary scheme data derived from Ref. [16].

Table 1. Comparison of the mixing angle ranges under different schemes.

	range	
	unitary scheme	quasi-unitary scheme
θ_{12}^{Q}	$31.8° - 36.4°$	$24.8° - 44.1°$
θ_{23}^{Q}	$37.2° - 50.9°$	$31.8° - 56.3°$
θ_{13}^{Q}	$< 10.9°$	$< 18.4°$

As seen in Table 1, with $x,\ y,\ z, \epsilon \neq 0$ and the same required constraints for neutrino mixing, the allowed ranges for neutrino mixing angles are expanded. If the parameters $x,\ y,\ z, \epsilon$ are comparable to 10^{-1}, there will be highly deviated mixing angles. Though such mixing angles have no direct physical meaning (the physical value is $A_{\alpha\beta}$ in Eq. (15)), we can see from Eq. (21) that the measurement of trigonometric function has been changed, which means that the angles we measured under the unitary scheme are just the effective values.

Since the quasi-unitary effect in Eq. (26) can be reflected in the discrepancies of different measurements of θ_{13}^{Q}, we can also calculate the quasi-unitary factor from the simulation metioned above. The numerical result is shown as the green points (gray ×) in Fig. 1. As the figure indicates that under the quasi-unitary parameterization required by the seesaw mechanism, nowadays experimental data cannot allow ϵ bigger than 0.05 and Δ_{QF} can reach up to 2.6×10^{-2} at most, corresponding to the deviated $\theta_{13}^{\mathrm{Q}} \lesssim 18.35°$.

If ϵ and x, y, z in $\boldsymbol{A}$ are big enough, we can measure a sizable Δ_{QF} in the quasi-unitary scheme. However, the error ranges in Ref. [16] cannot be totally from the non-unitary effect because the experimental errors also take a share. In fact, the seesaw-predicted scales of $x,\ y,\ z,\ \epsilon$ are typically too small to reach the order of 10^{-2} because the non-unitary effect is very likely to be so trivial in comparison with the experimental error, which makes this theoretical effect very hard to stand out within the precision capacity of today. However, as long as the neutrino mixing matrix is inherently non-unitary as predicted in the seesaw framework, Δ_{QF} will remain non-zero in the measurements because it contains the theoretical deviation from unitarity. We expect that the precision improvement of experiments will eliminate the experimental background as much as possible, leaving a relatively pure constitution of Δ_{QF}.

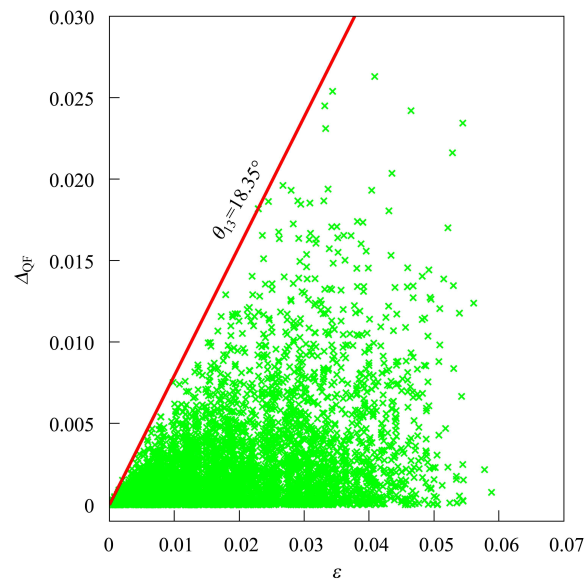

Fig. 1. The magnitude of quasi-unitary factor versus ϵ. The green points (gray ×) are obtained by scattering ϵ, x, y, z randomly between −0.1 and 0.1, corresponding to the deviation (Δ_{QF}) from unitarity theoretically.

5 Conclusion

As the mixing matrix of ordinary neutrinos may be unitary or quasi-unitary, we parameterize the mixing as a quasi-unitary matrix with four additional parameters, ϵ, x, y, z, and define a quasi-unitary factor Δ_{QF} for the test of unitarity. Our numerical analysis gives the possible deviations in mixing angles without violating any current experimental constraints, as well as the magnitude of Δ_{QF}, which separately describes the theoretical sensibility of discrepancies in future θ_{13} measurements. The improvement in experimental precision may help us to figure out the secret of unitarity of neutrino mixing and tell us whether the seesaw framework is a rational model for neutrino masses.

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