CPC(HEP & NP), 2010, **34**(7): 929–936 Chinese Physics C Vol. 34, No. 7, Jul., 2010

J/ψ inclusive production in $\chi_{bJ}(nP)$ decays

LI Rong(李荣)[1] HE Zhi-Guo(何志国)[2,3;1)]

[1] Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

[2] Institute of High Energy Physics, Chinese Academy of Science, P.O. Box 918(4), Beijing 100049, China

[3] Theoretical Physics Center for Science Facilities, (CAS), Beijing 100049, China

Abstract We study J/ψ inclusive production in the decays of P-wave bottomonium $\chi_{bJ}(1P,2P)$ for J=0,1,2. Within the framework of the non-relativistic QCD (NRQCD) factorization method we calculate the contributions coming from four relevant processes including one color-singlet process and three color-octet ones, which are $b\bar{b}(^3P_J,\underline{1}) \rightarrow c\bar{c}(^3S_1,\underline{8})+c+\bar{c}$, $b\bar{b}(^3S_1,\underline{8}) \rightarrow c\bar{c}(^3S_1,\underline{1})+g+g$, $b\bar{b}(^3S_1,\underline{8}) \rightarrow c\bar{c}(^3S_1,\underline{1})+c+\bar{c}$ and $b\bar{b}(^3S_1,\underline{8}) \rightarrow c\bar{c}(^3S_1,\underline{8})+g$. Our calculation shows that the color-octet processes, especially the gluon fragmentation one, contribute the most in the decays of χ_{b0} and χ_{b1}. However, the J/ψ production in the χ_{b2} decay is dominated by the color-singlet process. Furthermore, the gluon fragmentation process gives a δ function in the energy fraction distribution of J/ψ, which can be considered to be another characteristic for its identification. From our estimation the branching ratio for these processes is about $10^{-4}-10^{-5}$, which indicates that J/ψ inclusive production is detectable at B-factories. Studying these processes would help us to gain a deeper understanding of the color-octet mechanism.

Key words NRQCD, heavy quarkonium, inclusive decay

PACS 12.38.Bx, 12.39.Jh, 13.20.Gd

1 Introduction

Heavy quarkonium, constituted by heavy quark and anti-quark pair, plays an important role in the study of high energy physics. On the one hand, a large amount of such $Q\bar{Q}$ states could be produced at various colliders, such as e^+e^-, ep, $p\bar{p}$ or the forthcoming large hadron collider (LHC). This large number of heavy quarkonium states make the accurate study on this system possible. On the other hand, the large scale of the heavy quark mass allows people to investigate some of its properties within perturbative QCD. The early work on the production and decay of heavy quarkonium has been done within the color-singlet model (CSM) [1]. In the 1980s, Barbieri, et al. calculated the QCD corrections to P-wave heavy quarkonia decays [2] and found infrared singularities, which were expressed in terms of a logarithmic dependence on the binding energy. In the calculation of the relativistic corrections to S-wave heavy quarkonium decay, the singularity also appears [3]. To cure these infrared divergence problems, Bodwin, Braaten and Lepage [4] proposed that these infrared divergences could be absorbed into the color-octet matrix elements, which describe the transition of a color-octet heavy quark pair into the physical state. They then put forward the non-relativistic QCD (NRQCD) effective theory [5], which has been verified to be equal to the full QCD theory at one-loop level, to describe the heavy quarkonium system. By introducing the color-octet mechanism (COM), the NRQCD has made much impressive progress, such as solving the infrared divergence problems in P-wave quarkonia decay calculations [4], reconciling the discrepancies between the experimental data and theoretical predictions on the ψ′ surplus problem at Tevatron [6, 7], J/ψ photoproduction at LEP [8], and so on. More references and detailed descriptions of NRQCD progress can be found in Ref. [9].

Recently many works on the next-to-leading-order (NLO) QCD correction in the production of heavy quarkonium have been done. The NLO results on the inclusive J/ψ production at B-factories in e^+e^- annihilation indicate that the cross sections can be described within the CSM [10]. This may give us a hint that some of the long-distance matrix elements

Received 15 June 2009, Revised 28 August 2009

1) E-mail: hzgzlh@gmail.com

(LDMEs) in NRQCD are smaller than expected. The NLO calculations of J/ψ production at Tevatron show that the NLO results in the CSM are not large enough to explain the experimental data [11]. Moreover, neither the NLO results in the CSM nor those including the contributions from NLO results in the COM can properly describe the J/ψ transverse momentum (p_t) distribution and polarization at Tevatron [12, 13] simultaneously. The NLO result on the production of J/ψ at HERA in the CSM cannot give a good description not only on the p_t distribution but also on the polarization [14].

The inclusive production of charmed hadrons and charmonium in the decays of bottomonium states is also an interesting topic and has been widely studied. In Ref. [15] the authors considered the production of charm quark and J/ψ in the decay of Υ within the color-evaporation model. The charm hadron production in the decays of S and P-wave bottomonium is systemically investigated in 1979 [16] and the η_c production in the radiative decay of Υ has been studied by Guberina [17]. Since the NRQCD factorization approach came out, the production of charmonium, especially the J/ψ state, in bottomonium decay has been studied by many authors using this method [18]. The inclusive and exclusive production of charmonium and charmed hadrons in the decay of η_b was proposed to hunt and study the ground bottomonium state in Refs. [19–21]. The exclusive double charmonia production in the decays of Υ, η_b and P-wave bottomonium χ_{bJ} was studied in Refs. [22, 23]. Recently calculations of $c\bar{c}$ production in bottomonium decays have been performed at leading order [24, 25] and next-to-leading order [26].

At B-factories more than 10^8 data of $\Upsilon(2S)$ and $\Upsilon(3S)$ have been accumulated [27]. Because of the large branch ratio of $\Upsilon(2S,3S)\to\chi_{bJ}(1P,2P)+\gamma$, the B-factories can provide many χ_{bJ} decay data. Moreover, the dilepton signal makes J/ψ a good candidate to be studied. So in this work, we will investigate the J/ψ inclusive production in χ_{bJ} decays within the NRQCD framework. The rest of this paper is organized as follows. In Section 2, we briefly describe the factorization formula for $\chi_{bJ}\to J/\psi+X$ within the NRQCD formalism and covariant projection method used to calculated the short distance part of the processes under consideration. The analytical and numerical results of the color-singlet and the color-octet processes will be given in Section 3. Finally, we draw our conclusion and give some discussions about our results.

2 Basic formulae

According to NRQCD the physical heavy quarkonium states χ_{bJ} and J/ψ have the following expansions into Fock states [5]:

$$|\chi_{bJ}\rangle = O(1)|b\bar{b}[^3P_J,\underline{1}]\rangle+O(v_b)|b\bar{b}[^3S_1,\underline{8}]g\rangle+\dots, \tag{1a}$$

$$|J/\psi\rangle = O(1)|c\bar{c}[^3S_1,\underline{1}]\rangle+O(v_c)|c\bar{c}[^3P_J,\underline{8}],g\rangle+ O(v_c^2)|c\bar{c}[^3S_1,\underline{8}],gg\rangle+\dots, \tag{1b}$$

where v_b and v_c are the average relative velocities of the $b\bar{b}$ and $c\bar{c}$ in χ_{cJ} and J/ψ respectively. So the inclusive decay rate of $\chi_{bJ}\to J/\psi+X$ can be factorized into the product of short-distance-coefficients (SDCs) and the LDMEs given by

$$d\Gamma[\chi_{bJ}\to J/\psi+X]=\sum_{m,n}d\Gamma_{mn}(b\bar{b}[m]\to c\bar{c}[n]+ X)\langle\chi_{bJ}|\mathcal{O}(m)|\chi_{bJ}\rangle\langle\mathcal{O}^{J/\psi}(n)\rangle, \tag{2}$$

where $d\Gamma_{mn}(b\bar{b}[m]\ \to\ c\bar{c}[n]+X)$, the short distance part of the corresponding subprocess, could be calculated perturbatively, and $\langle\chi_{bJ}|\mathcal{O}(m)|\chi_{bJ}\rangle$ and $\langle\mathcal{O}^{J/\psi}(n)\rangle$ the LDMEs could be determined phenomenologically or by lattice simulation. m and n in the above expression indicate the quantum states. In the practical calculation the summation is truncated according to the required theoretical accuracy. The velocity scaling rule of the corresponding LDMEs tells us that the color-octet contributions are of the same order in the v_b expansion as the color-singlet ones in the initial state χ_{bJ} case. That is to say we should consider both the $b\bar{b}[^3P_J,\underline{1}]$ and $b\bar{b}[^3S_1,\underline{8}]$ in leading order of v_b. As for the J/ψ, there is only the color-singlet process at leading order in v_c. However, χ_{bJ} can annihilate into two gluons, one of which can fragment into a color-octet $c\bar{c}$ pair which will then evolve into a J/ψ by emitting soft gluons. Although the color-octet LDME has a v_c^4 suppression compared with the color-singlet one, this color-octet process can happen at the order of α_s^3 and has only two final states, both of which may provide enhancement effects. So in our calculation, we just consider the color-singlet sub-process

$$b\bar{b}[^3P_J,\underline{1}]\to c\bar{c}[^3S_1,\underline{1}]+c+\bar{c}, \tag{3}$$

and three color-octet sub-processes

$$b\bar{b}[^3P_J,\underline{1}]\to c\bar{c}[^3S_1,\underline{8}]+g, \tag{4a}$$

$$b\bar{b}[^3S_1,\underline{8}]\to c\bar{c}[^3S_1,\underline{1}]+g+g, \tag{4b}$$

$$b\bar{b}[^3S_1,\underline{8}]\to c\bar{c}[^3S_1,\underline{1}]+c+c. \tag{4c}$$

Using the standard spinor project method, the Feynman amplitude for $b\bar{b}[m]\to c\bar{c}[n]+X$ can be expressed as

$$\mathcal{M}(b\bar{b}[^{2S+1}L_J,\underline{1}(\underline{8})](p_1)\to c\bar{c}[^{2S+1}L_J,\underline{1}(\underline{8})](p_2)+X(p_3))=$$

$$\sum_{s_1,s_2}\sum_{L_z,S_z}\sum_{i,j}\sum_{s_3,s_4}\sum_{L_z,S_z}\sum_{k,l}\langle s_1;s_2\mid SS_z\rangle_{b\bar{b}}\langle LL_z;SS_z\mid JJ_z\rangle_{b\bar{b}}\langle 3i;\bar{3}j\mid \underline{1}(\underline{8})\rangle_{b\bar{b}}\times$$

$$\langle s_3;s_4\mid SS_z\rangle_{c\bar{c}}\langle LL_z;SS_z\mid JJ_z\rangle_{c\bar{c}}\langle 3k;\bar{3}l\mid \underline{1}(\underline{8})\rangle_{c\bar{c}}\times$$

$$\begin{cases}\mathcal{M}(b_i(p_b,s_1)\bar{b}_j(p_{\bar{b}},s_2)\to c_k(p_c,s_3)\bar{c}_l(p_{\bar{c}},s_4)+X(p_3)) & (L=S),\\ \epsilon_\alpha(L_Z)\mathcal{M}^\alpha(b_i(p_b,s_1)\bar{b}_j(p_{\bar{b}},s_2)\to c_k(p_c,s_3)\bar{c}_l(p_{\bar{c}},s_4)+X(p_3)) & (L=P),\end{cases}\tag{5}$$

where $\langle 3i;\bar{3}j\mid \underline{1}\rangle=\delta_{ij}/\sqrt{N_c}$, $\langle 3i;\bar{3}j\mid \underline{8}\rangle=\sqrt{2}T^a$, $\langle s_1;s_2\mid SS_z\rangle$ and $\langle LL_z;SS_z\mid JJ_z\rangle$ are the $SU(3)$-color, $SU(2)$-spin and angular momentum Clebsch-Gordan (C-G) coefficients. $p_b=p_1/2+q_b$, $p_b=p_1/2-q_b$, $p_c=p_2/2+q_c$ and $p_c=p_2/2-q_c$. $2q_b$ and $2q_c$ are the relative momenta of $b\bar{b}$ and $c\bar{c}$, which will be zero in the non-relativistic limit, and $\mathcal{M}^\alpha$ is the derivative of the amplitude with respect to q_b^α.

For the spin triplet states $S=1$, the projection operator of the Dirac spinor can be written in the form of a product of γ matrices as [28]:

$$\sum_{s_1s_2}\langle s_1;s_2\mid 1S_z\rangle u(p_b,s_1)\bar{v}(p_{\bar{b}},s_2)=\frac{-1}{2\sqrt{2}(E_b+m_b)}\times$$

$$(\not{p}_b+m_b)\frac{\not{p}_{\eta_b}+2E_b}{2E_b}\not{\epsilon}(S_z)(\not{p}_{\bar{b}}-m_b),\tag{6a}$$

$$\sum_{s_3s_4}\langle s_3;s_4\mid 1S_z\rangle v(p_c,s_3)\bar{u}(p_{\bar{c}},s_4)=\frac{-1}{2\sqrt{2}(E_c+m_c)}\times$$

$$(\not{p}_c-m_c)\not{\epsilon}^*(S_z)\frac{\not{p}_1+2E_c}{2E_c}(\not{p}_{\bar{c}}+m_c),\tag{6b}$$

where

$$E_b=\sqrt{p^2_{\chi_{bJ}}}/2\simeq m_b$$

and

$$E_c=\sqrt{p^2_{J/\psi}}/2\simeq m_c$$

at leading order in v_b and v_c respectively.

For the S-wave spin-triplet state with momentum p_i, $(L=0,S=1)$, the sum over its spins is

$$\Pi_{\mu\nu}(p_i)=\sum_{S_z}\epsilon^*_\mu(S_z)\epsilon_\nu(S_z)=-g_{\mu\nu}+\frac{p_{i\mu}p_{i\nu}}{p_i^2},\tag{7}$$

where $p_i\cdot\epsilon=0$. For the spin-triplet P-wave state with momentum p_i we introduce $\epsilon^{J*}_{\alpha\beta}(J_z)$ to describe the L-S coupling which is defined as

$$\sum_{L_zS_z}\epsilon^*_\alpha(L_z)\epsilon^*_\beta(S_z)\langle 1L_z;1S_z|JJ_z\rangle=\epsilon^{J*}_{\alpha\beta}(J_z),\tag{8}$$

and the sums over their all possible polarization states for J=0,1,2 are

$$\sum_{J_z}\epsilon^{0*}_{\alpha\beta}(J_z)\epsilon^0_{\alpha1\beta1}(J_z)=\frac{1}{3}\Pi_{\alpha\beta}(p_i)\Pi_{\alpha1\beta1}(p_i),\tag{9a}$$

$$\sum_{J_z}\epsilon^{1*}_{\alpha\beta}(J_z)\epsilon^1_{\alpha1\beta1}(J_z)=\frac{1}{2}(\Pi_{\alpha\alpha1}(p_i)\Pi_{\beta\beta1}(p_i)-$$

$$\Pi_{\alpha\beta1}(p_i)\Pi_{\beta\alpha1}(p_i)),\tag{9b}$$

$$\sum_{J_z}\epsilon^{2*}_{\alpha\beta}(J_z)\epsilon^2_{\alpha1\beta1}(J_z)=\frac{1}{2}(\Pi_{\alpha\alpha1}(p_i)\Pi_{\beta\beta1}(p_i)+$$

$$\Pi_{\alpha\beta1}(p_i)\Pi_{\beta\alpha1}(p_i))-\frac{1}{3}\Pi_{\alpha\beta}(p_i)\Pi_{\alpha1\beta1}(p_i).\tag{9c}$$

The three body decay processes $b\bar{b}[^3L_J,\underline{1}(\underline{8})](p_0)\to p_1+p_2+p_3$ can be described in terms of the energy fraction x_i in the rest frame of the $b\bar{b}$

$$x_i=\frac{2p_0\cdot p_i}{p_0^2},\quad \sum_i x_i=2.\tag{10}$$

The three body phase space $d\Phi_3$ is then given by

$$d\Phi_3=\prod_{i=1,2,3}\frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}\cdot(2\pi)^4\delta^4(p_{\eta_b}-p_1-p_2-p_3)=$$

$$\prod_{i=1,2,3}\frac{2m_b^2}{(4\pi)^3}dx_i\delta(2-\sum x_i).\tag{11}$$

The variable x_3 can be integrated out by applying the delta function and then the phase space becomes

$$d\Phi_3=\frac{2m_b^2}{(4\pi)^3}\int_{x_1^{\min}}^{x_1^{\max}}dx_1\int_{x_2^{\min}}^{x_2^{\max}}dx_2.\tag{12}$$

For the J/ψ+$c\bar{c}$ production process the integral limits are $x_1^{\min}=2r$, $x_1^{\max}=1$,

$$x_2^{\min}=\frac{1}{2}\left(2-x_1-\sqrt{\frac{(1-x_1)(x_1-2r)(2r+x_1)}{1+r^2-x_1}}\right)$$

and

$$x_2^{\max}=\frac{1}{2}\left(2-x_1+\sqrt{\frac{(1-x_1)(x_1-2r)(2r+x_1)}{1+r^2-x_1}}\right),$$

where $r = m_{\rm c}/m_{\rm b}$. For the $\mathrm{J/\psi+g+g}$ process $x_1^{\min}=2r$, $x_1^{\max}=1+r^2$,

$$x_2^{\min}=\frac{1}{2}(2-x_1-\sqrt{x_1^2-4r^2})$$

and

$$x_2^{\max}=\frac{1}{2}(2-x_1+\sqrt{x_1^2-4r^2}).$$

3 Analytical and numerical results

3.1 Color-Singlet part

As we have mentioned above, at the leading order in $\alpha_{\rm s}$, $v_{\rm c}$ and $v_{\rm b}$, the color-singlet subprocess that contributes to the $\mathrm{J/\psi}$ inclusive production in $\chi_{\rm bJ}$ decay is Eq. (3). The two Feynman diagrams for these processes are shown in Fig. 1. Using the formulae given in the above section, the matrix elements for these processes are

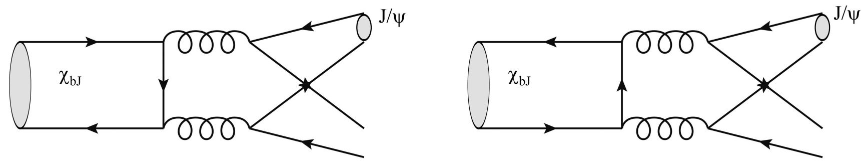

Fig. 1. The Feynman diagrams for the color-singlet process $\chi_{\rm bJ}\rightarrow {\rm c\bar{c}}[^3S_1,\underline{1}]+{\rm c}+{\rm \bar{c}}$.

$$\begin{aligned}|\mathcal{M}^{\chi_{\rm b0}}_{\mathrm{J/\psi+c\bar{c}}}|^2 = &-\frac{32\alpha_{\rm s}^4 C_{\rm A}C_{\rm F}^2\pi^2}{9m_{\rm b}^{11}m_{\rm c}(x_1-2)^4(x_2-1)^2(x_1+x_2-1)^2}((x_2-1)(9x_1^5+(x_2-125)x_1^4+\\&(600-76x_2)x_1^3+4(10x_2^3-6x_2^2+113x_2-285)x_1^2+48(x_2^4-3x_2^3+3x_2^2-15x_2+20)x_1+\\&16(x_2^5-5x_2^4+10x_2^3-10x_2^2+23x_2-19))m_{\rm b}^4+m_{\rm c}^2(-2x_1^5+(8x_2+25)x_1^4+\\&8(9x_2^2-32x_2-1)x_1^3+8(16x_2^3-88x_2^2+188x_2-57)x_1^2+\\&32(2x_2^4-21x_2^3+66x_2^2-95x_2+33)x_1-16(13x_2^4-52x_2^3+114x_2^2-124x_2+40))m_{\rm b}^2-\\&3m_{\rm c}^4(x_1^2+4(x_2-3)x_1+4(x_2^2-2x_2+4))^2),\end{aligned}\tag{13}$$

$$\begin{aligned}|\mathcal{M}^{\chi_{\rm b1}}_{\mathrm{J/\psi+c\bar{c}}}|^2 = &-\frac{64\alpha_{\rm s}^4 C_{\rm A}C_{\rm F}^2\pi^2}{9m_{\rm b}^{9}m_{\rm c}(x_1-2)^4(x_2-1)^2(x_1+x_2-1)^2}((-2x_1^5+(17-8x_2)x_1^4+\\&(-8x_2^2+88x_2-92)x_1^3+4(22x_2^2-72x_2+51)x_1^2+16(2x_2^3-13x_2^2+23x_2-12)x_1+\\&16(x_2-1)^2(x_2^2-2x_2+4))m_{\rm c}^2+m_{\rm b}^2x_1^2(x_2-1)(9x_1^3+(13x_2-29)x_1^2+\\&8(x_2-2)^2x_1+4(x_2^3-3x_2^2+5x_2-3))),\end{aligned}\tag{14}$$

$$\begin{aligned}|\mathcal{M}^{\chi_{\rm b2}}_{\mathrm{J/\psi+c\bar{c}}}|^2 = &-\frac{64\alpha_{\rm s}^4 C_{\rm A}C_{\rm F}^2\pi^2}{45m_{\rm b}^{11}m_{\rm c}(x_1-2)^4(x_2-1)^2(x_1+x_2-1)^2}((x_2-1)(9x_1^5+(x_2-17)x_1^4+\\&8(16x_2+3)x_1^3+8(5x_2^3+21x_2^2-56x_2+6)x_1^2+48(x_2^4-x_2^3-9x_2^2+13x_2-2)x_1+\\&16(x_2^5-5x_2^4+x_2^3+17x_2^2-16x_2+2))m_{\rm b}^4+m_{\rm c}^2(-2x_1^5+(8x_2+1)x_1^4+\\&4(18x_2^2-49x_2+22)x_1^3+4(32x_2^3-149x_2^2+262x_2-126)x_1^2+16(4x_2^4-\\&36x_2^3+99x_2^2-118x_2+51)x_1-32(5x_2^4-20x_2^3+36x_2^2-32x_2+14))m_{\rm b}^2-\\&3m_{\rm c}^4(x_1^4+4(2x_2-3)x_1^3+8(3x_2^2-8x_2+7)x_1^2+16(2x_2^3-7x_2^2+8x_2-6)x_1+\\&16(x_2^4-4x_2^3+6x_2^2-4x_2+4))),\end{aligned}\tag{15}$$

where C_A=3, C_F=4/3.

To obtain the numerical results, we take m_b = 4.65 GeV, m_c = 1.5 GeV and $\alpha_s(m_b)$=0.22. The values of the wave function at the origin for J/ψ and the first derivative of the wave function for χ_{bJ} are chosen as $|R^{J/\psi}(0)|^2 = 0.81$ GeV^3, $|R_{1P}^{(\chi_{bJ})\prime}(0)|^2 = 1.417$ GeV^3, which are calculated with Buchmüller-Tye potential [29]. The LDMEs are expressed as:

$$\langle \mathcal{O}_1^{J/\psi}(^3S_1)\rangle = \frac{6N_c|R^{J/\psi}(0)|^2}{4\pi},$$

$$\langle \chi_{bJ}(nP)|\mathcal{O}_1(^3P_J)|\chi_{bJ}(nP)\rangle = \frac{6N_c|R'_{nP}(0)|^2}{4\pi}. \quad (16)$$

In Fig. 2 we show the scaled energy distributions of J/ψ in the color-singlet decay channels for $\chi_{bJ}(1P,2P)$. It can be seen that the decays of χ_{b0} and χ_{b1} give a relatively flat scaled energy distribution for J/ψ, and that of χ_{b2} shows a maximum value at $x_1 \approx 0.93$. The decay widths of the color-singlet channel for $\chi_{bJ}(1P,2P)$ are presented in Table 1. The width of $\chi_{b2}(1P,2P)$ is larger than that of the others, which is consistent with the result shown in Fig. 2.

Table 1. The decay width for the J/ψ inclusive production from Eq. (3). (unit: keV).

	J=0	J=1	J=2
$\chi_{bJ}(1P)$	6.33×10^{-2}	2.30×10^{-3}	1.18×10^{-1}
$\chi_{bJ}(2P)$	7.39×10^{-2}	2.68×10^{-3}	1.37×10^{-1}

3.2 Color-Octet parts

For the J/ψ inclusive production in χ_{bJ} decays through the color-octet mechanism, there are three processes shown in Eq. (4), which have 2, 6 and 5 Feynman diagrams respectively. We plot the typical ones in Fig. 3. Using the method that is presented above, we can easily obtain the $|M|^2$ for the processes as follows

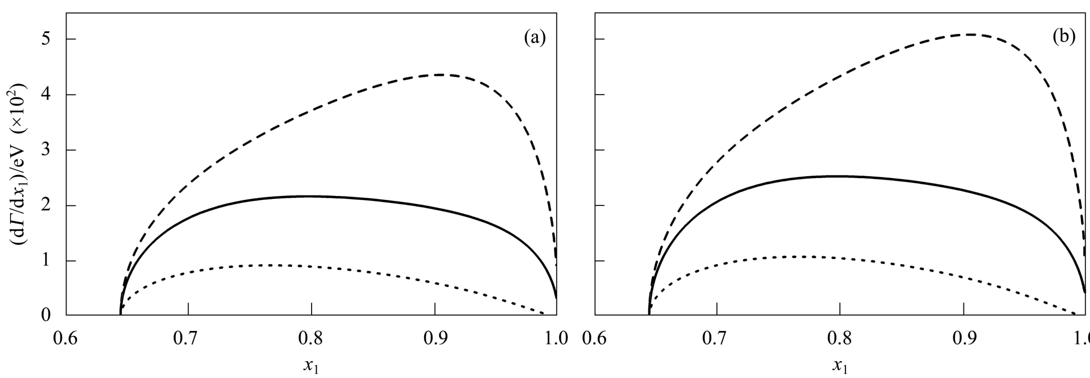

Fig. 2. The J/ψ scaled energy distribution for the processes (a) $\chi_{bJ}(1P)\to J/\psi+c\bar{c}$; (b) $\chi_{bJ}(2P)\to J/\psi+c\bar{c}$. The solid, dotted and dashed lines correspond to the results of χ_{b0}, χ_{b1} and χ_{b2} respectively. The result of χ_{b1} (dotted line) is multiplied by a scale factor of 10.

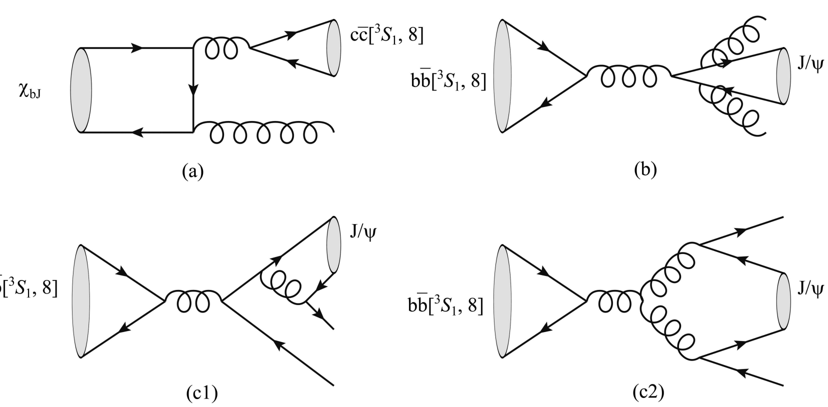

Fig. 3. The typical Feynman diagrams for the color-octet processes: (a) for $\chi_{bJ}\to c\bar{c}[^3S_1,\underline{8}]+g$; (b) for $b\bar{b}[^3S_1,\underline{8}]\to J/\psi+g+g$; (c1) and (c2) for $b\bar{b}[^3S_1,\underline{8}]\to J/\psi+c+\bar{c}$.

$$|\mathcal{M}^{\chi_{b0}}_{c\bar{c}[^3S_1,\underline{8}]+g}|^2 = \frac{4\alpha_s^3 C_A C_F \pi^2}{9m_b^3 m_c^3 (m_b^2-m_c^2)^2}(m_c^2-3m_b^2)^2, \tag{17}$$

$$|\mathcal{M}^{\chi_{b1}}_{c\bar{c}[^3S_1,\underline{8}]+g}|^2 = \frac{8\alpha_s^3 C_A C_F \pi^2}{9m_b m_c (m_b^3-m_b m_c^2)^2}(m_b^2+m_c^2), \tag{18}$$

$$|\mathcal{M}^{\chi_{b2}}_{c\bar{c}[^3S_1,\underline{8}]+g}|^2 = \frac{8\alpha_s^3 C_A C_F \pi^2}{45m_b^3 m_c^3 (m_b^2-m_c^2)^2}(6m_b^4+3m_c^2m_b^2+m_c^4), \tag{19}$$

$$\begin{aligned}|\mathcal{M}^{8}_{J/\psi+gg}|^2 = {} & \frac{8\alpha_s^4 (C_A^2-4) C_F \pi^3}{9m_b^5 m_c (x_1-2)^2((x_2-1)m_b^2+m_c^2)^2(m_c^2-m_b^2(x_1+x_2-1))^2}((x_2-1)(x_2^3- \\ & 3x_2^2+x_2+x_1^2(x_2+1)+2x_1(x_2^2-x_2-1)+1)m_b^8+2m_c^2(x_1^4+2(x_2-3)x_1^3+ \\ & (3x_2^2-12x_2+16)x_1^2+(2x_2^3-12x_2^2+22x_2-17)x_1+x_2^4-4x_2^3+10x_2^2- \\ & 12x_2+6)m_b^6+m_c^4(-4x_1^3+(21-6x_2)x_1^2+(-6x_2^2+26x_2-46)x_1+14(x_2^2- \\ & 2x_2+2))m_b^4+2m_c^6(x_1^2+(x_2-5)x_1+x_2^2-2x_2+8)m_b^2+m_c^8).\end{aligned} \tag{20}$$

The $|M|^2$ for $b\bar{b}[^3S_1,\underline{8}]\rightarrow J/\psi+c+\bar{c}$ is too tedious to be presented here. Compared with the color-singlet processes, there are another two parameters needed to calculate the numerical results. One of them is the color-octet matrix element $\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle$ that describes the transition from $c\bar{c}[^3S_1,\underline{8}]$ to a J/ψ. We set it as 1.06×10^{-2} GeV^3 that is extracted from the hadroproduction of J/ψ [30]. The other one is $\langle\chi_{bJ}|\mathcal{O}_8(^3S_1)|\chi_{bJ}\rangle$. As in Ref. [24] we use the relation $\rho_8=m_b^2\langle\chi_{bJ}|\mathcal{O}_8(^3S_1)|\chi_{bJ}\rangle^{m_b}/\langle\chi_{bJ}|\mathcal{O}_8(^3P_J)|\chi_{bJ}\rangle$ and choose ρ_8 =0.1. So the decay widths can be obtained and are listed in Table 2.

Table 2. The contributions to the J/ψ+X production in the decay of $\chi_{bJ}(1P,2P)$ from color-octet processes are listed.(unit: keV)

	$\chi_{b0}(1P)$	$\chi_{b1}(1P)$	$\chi_{b2}(1P)$	$\chi_{b0}(2P)$	$\chi_{b1}(2P)$	$\chi_{b2}(2P)$
Eq. (4a)	1.66×10^{-1}	4.54×10^{-3}	4.99×10^{-2}	1.93×10^{-1}	5.29×10^{-3}	5.82×10^{-2}
Eq. (4b)		1.32×10^{-3}			1.54×10^{-3}	
Eq. (4c)		2.57×10^{-3}			3.00×10^{-3}	

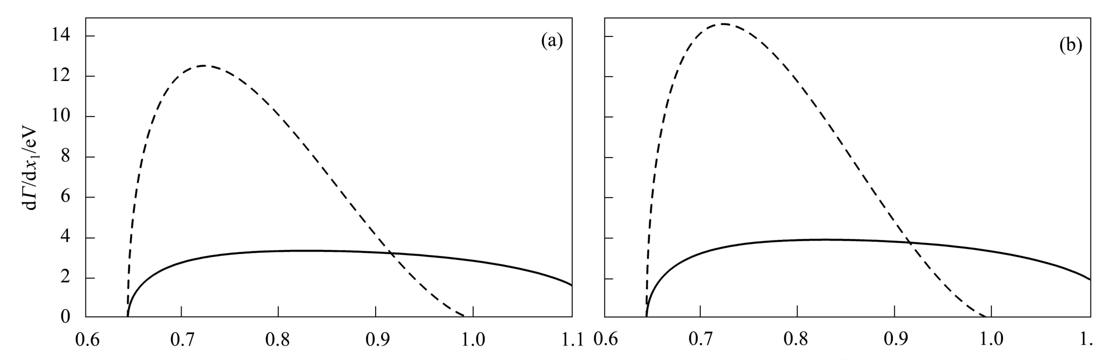

Fig. 4. The J/ψ scaled energy distribution for the processes $b\bar{b}[^3S_1,\underline{8}]\rightarrow J/\psi$+gg (solid lines) and $b\bar{b}[^3S_1,\underline{8}]\rightarrow J/\psi+c\bar{c}$ (dashed lines). (a) The results on $\chi_{bJ}(1P)$ decay; (b) The results on $\chi_{bJ}(2P)$ decay.

In Fig. 4 the J/ψ scaled energy distributions for $\chi_{bJ}(^3S_1,8) \to J/\psi + c\bar{c}$ and $\chi_{bJ}(^3S_1,8) \to J/\psi + gg$ are shown. It can be seen that the energy of J/ψ coming from $\chi_{bJ}(^3S_1,8) \to J/\psi + c\bar{c}$ is mainly distributed over the middle of the region with a peak at $x_1 \approx 0.72$. The scaled energy distribution of J/ψ from $\chi_{bJ}(^3S_1,8) \to J/\psi + gg$ is relatively flat almost in the whole region and it does not go to zero at the endpoint that is different from the other processes with three final states. However, the decay widths for these two color-octet processes are too small compared with that of χ_{b0} and χ_{b2} except for the decay of χ_{b1}. For the gluon fragmentation processes, the momentum of J/ψ is fixed as a δ function at $|\boldsymbol{p}| = (M^2_{\chi_{bJ}} - M^2_{J/\psi})/2M_{\chi_{bJ}}$.

To obtain the branch ratio of these decay channels, the total width of $\chi_{bJ}(1P,2P)$ should be given. Also from the NRQCD factorization formula, the leading-order cross section for $\chi_{bJ}\to$light hadron(LH) can be presented as

$$\Gamma[\chi_{bJ} \to \mathrm{LH}] = A_J(\Lambda)\frac{\langle\chi_{bJ}|\mathcal{O}_1(^3P_J)|\chi_{bJ}\rangle}{m_b^4} + A_8(\Lambda)\frac{\langle\chi_{bJ}|\mathcal{O}_8(^3S_1)|\chi_{bJ}\rangle^{\Lambda}}{m_b^2}\ . \quad (21)$$

The coefficients A_J and A_8 are calculated in Refs. [2, 5, 31] as

$$A_0 = \frac{3C_F}{N_c}\pi\alpha_s^2,\quad A_2 = \frac{4C_F}{5N_c}\pi\alpha_s^2,\quad A_8 = \frac{1}{3}n_f\pi\alpha_s^2,$$
$$A_1(\Lambda) = \frac{C_F\alpha_s^3}{N_c}\left[\left(\frac{587}{54} - \frac{317}{288}\pi^2\right)C_A + \left(-\frac{16}{27} - \frac{4}{9}\lg\frac{\Lambda}{2m_b}\right)n_f\right], \quad (22)$$

where $N_c = 3$, $n_f = 4$, and $\Lambda = m_b$. Because $\chi_{bJ}(1P,2P)$ has a large branching ratio for the transition into the lower bottomonium states, we use the following expression to estimate their total width

$$\Gamma_{\mathrm{total}} = \frac{\Gamma(\chi_{bJ} \to \mathrm{LH})}{1 - Br_{\Sigma\mathrm{transition}}}. \quad (23)$$

The branching ratios are taken from PDG08 [32]. Here we list the total width for $\chi_{bJ}(1P, 2P)$ in Table 3.

The branching ratios for the color-singlet and color-octet channels are also presented in Table 3. From the results it can be seen that the contributions from the color-octet channel are about 2–3 times that of the color-singlet ones for J/ψ production in the decays of χ_{b0} and χ_{b1}. In the case of χ_{b2} decay, the color-singlet contribution is about 2 times that of the color-octet one. Anyway, the branching ratios show that the J/ψ inclusive production is a detectable process at B-factories.

Table 3. The total decay width of $\chi_{bJ}(1P, 2P)$ estimated by Eq. (21) and Eq. (23) and the branch ratios for the color-singlet and color-octet processes.(unit of Γ_{total}: keV)

	$\chi_{b0}(1P)$	$\chi_{b1}(1P)$	$\chi_{b2}(1P)$	$\chi_{b0}(2P)$	$\chi_{b1}(2P)$	$\chi_{b2}(2P)$
Γ_{total}	250	130	163	290	144	198
Br_{singlet}	2.53×10^{-4}	1.77×10^{-5}	7.24×10^{-4}	2.55×10^{-4}	1.86×10^{-5}	6.92×10^{-4}
Br_{octet}	6.80×10^{-4}	6.42×10^{-5}	3.30×10^{-4}	6.81×10^{-4}	6.83×10^{-5}	3.17×10^{-4}

4 Conclusion

In this work we have investigated the J/ψ inclusive production in the decays of $\chi_{bJ}(1P,2P)$ within the NRQCD factorization approach. It includes four sub-processes. The decay widths, branching ratio and scaled energy distribution of the J/ψ have been studied. The numerical results indicate that the color-octet processes give an important contribution to the decays of χ_{b0} and χ_{b1}, especially the gluon fragmentation process. For the decays of χ_{b0} and χ_{b1}, the contribution coming from the gluon fragmentation process is about 2 times that of the color-singlet ones. But the contrary is true for the decay of the χ_{b2}. The color-octet process $b\bar{b}(^3S_1,8) \to J/\psi + c + \bar{c}$ mainly contributes to the decay of χ_{b1} and can almost be ignored in the decays of the χ_{b0} and χ_{b2}. Furthermore, the scaled energy distributions for the gluon fragmentation process give a δ function at $|\boldsymbol{p}| = (M^2_{\chi_{bJ}} - M^2_{J/\psi})/2M_{\chi_{bJ}}$. In view of the large decay widths of this fragmentation process, the experimental measurements on the J/ψ scaled energy distribution will help us to study the color-octet mechanism in NRQCD.

References

1 Einhorn M B, Ellis S D. Phys. Rev. D, 1975, **12**: 2007; Ellis S D, Einhorn M B, Quigg C. Phys. Rev. Lett., 1976, **36**: 1263; CHANG C H. Nucl. Phys. B, 1980, **172**: 425; Berger E L, Jones D L. Phys. Rev. D, 1981, **23**: 1521; Baier R, Ruckl R. Nucl. Phys. B, 1982, **201**: 1

2 Barbieri R, Gatto R, Kogerler R. Phys. Lett. B, 1976, **60**: 183; Barbieri R, Gatto R, Remiddi E. Phys. Lett. B, 1976, **61**: 465; Barbieri R, Caffo M, Gatto R, Remiddi E. Phys. Lett. B, 1980, **95**: 93; Nucl. Phys. B, 1981, **192**: 61

3 Barbieri R, Gatto R, Remiddi E. Phys. Lett. B, 1976, **61**: 465

4 Bodwin G T, Braaten E, Lepage G P. Phys. Rev. D, 1992, **46**: 1914

5 Bodwin G T, Braaten E, Lepage G P. Phys. Rev. D, 1995, **51**: 1125; 1997, **55**: 5853

6 Abe F et al. (CDF Collaboration). Phys. Rev. Lett., 1992, **69**: 3704; Abe F et al. (CDF Collaboration). Phys. Rev. Lett., 1993, **71**: 2537

7 Braaten E, YUAN T C. Phys. Rev. D, 1994, **50**: 3176; Braaten E, Fleming S. Phys. Rev. Lett., 1995, **74**: 3327; Cho P L, Leibovich A K. Phys. Rev. D, 1996, **53**: 150; Cho P L, Leibovich A K. Phys. Rev. D, 1996, **53**: 6203

8 Klasen M, Kniehl B A, Mihaila L N, Steinhauser M. Phys. Rev. Lett., 2002, **89**: 032001

9 Brambilla N et al. (Quarkonium Working Group). arXiv:hep-ph/0412158; Lansberg J P. Int. J. Mod. Phys. A, 2006, **21**: 3857 ; Lansberg J P et al.. AIP Conf. Proc., 2008, **1038**: 15; Chung H S, Lee J, Yu C. Nucl. Phys. Proc. Suppl., 2009, **187**: 180

10 ZHANG Y J, GAO Y J, CHAO K T. Phys. Rev. Lett., 2006, **96**: 092001; ZHANG Y J, CHAO K T. Phys. Rev. Lett., 2007, **98**: 092003; ZHANG Y J, MA Y Q, CHAO K T. Phys. Rev. D, 2008, **78**: 054006; MA Y Q, ZHANG Y J, CHAO K T. Phys. Rev. Lett., 2009, **102**: 162002; GONG B, WANG J X. Phys. Rev. Lett., 2009, **102**: 162003; Phys. Rev. D, 2009, **80**: 054015

11 Campbell J M, Maltoni F, Tramontano F. Phys. Rev. Lett., 2007, **98**: 252002

12 GONG B, WANG J X. Phys. Rev. Lett., 2008, **100**: 232001; GONG B, WANG J X. Phys. Rev. D, 2008, **78**: 074011

13 GONG B, LI X Q, WANG J X. Phys. Lett. B, 2009, **673**: 197

14 Kramer M. Nucl. Phys. B, 1996, **459**: 3; Artoisenet P, Campbell J M, Maltoni F, Tramontano F. Phys. Rev. Lett., 2009, **102**: 142001; CHANG C H, LI R, WANG J X. Phys. Rev. D, 2009, **80**: 034020

15 Fritzsch H, Streng K H. Phys. Lett. B, 1978, **77**: 299

16 Barbieri R, Caffo M, Remiddi E. Phys. Lett. B, 1979, **83**: 345

17 Guberina B, Kuhn J H. Lett. Nuovo Cim. 1981, **32**: 295

18 Trottier H D. Phys. Lett. B, 1994, **320**: 145; Ko P. Phys. Rev. D, 1995, **52**: 3108; Cheung K m, Keung W Y, YUAN T C. Phys. Rev. D, 1996, **54**: 929; Napsuciale M. Phys. Rev. D, 1998, **57**: 5711; LI S Y, XIE Q b, WANG Q. Phys. Lett. B, 2000, **482**: 65; HAN W, LI S Y. Phys. Rev. D, 2006, **74**: 117502

19 Braaten E, Fleming S, Leibovich A K. Phys. Rev. D, 2001, **63**: 094006; JIA Y. Phys. Rev. D, 2008, **78**: 054003; HAO G, JIA Y, QIAO C F, SUN P. JHEP, 2007, **0702**: 057; GAO Y J, ZHANG Y J, CHAO K T. arXiv:hep-ph/0701009. Santorelli P. Phys. Rev. D, 2008, **77**: 074012; GONG B, JIA Y, WANG J X. Phys. Lett. B, 2009, **670**: 350

20 Maltoni F, Polosa A D. Phys. Rev. D, 2004, **70**: 054014

21 HAO G, QIAO C F, SUN P. Phys. Rev. D, 2007, **76**: 125013

22 JIA Y. Phys. Rev. D, 2007, **76**: 074007

23 Braguta V V, Likhoded A K, Luchinsky A V. Phys. Rev. D, 2009, **80**: 094008

24 Bodwin G T, Braaten E, KANG D, Lee J. Phys. Rev. D, 2007, **76**: 054001

25 Kang D, Kim T, Lee J, Yu C. Phys. Rev. D, 2007, **76**: 114018; Chung H S, Kim T, Lee J. Phys. Rev. D, 2008, **78**: 114027

26 ZHANG Y J, CHAO K T. Phys. Rev. D, 2008, **78**: 094017

27 Aubert B et al. (BABAR Collaboration). arXiv:0902.2176 [hep-ex]; Phys. Rev. Lett., 2009, **103**: 161801

28 Kuhn J H, Kaplan J, Safiani E G O. Nucl. Phys. B, 1979, **157**: 125; Guberina B, Kuhn J H, Peccei R D, Ruckl R. Nucl. Phys. B, 1980, **174**: 317; Bodwin G T, Petrelli A. Phys. Rev. D, 2002, **66**: 094011

29 Buchmuller W, Tye S H H. Phys. Rev. D, 1981, **24**: 132; Eichten E J, Quigg C. Phys. Rev. D, 1995, **52**: 1726

30 Beneke M, Kramer M 1. Phys. Rev. D, 1997, **55**: 5269

31 HUANG H W, CHAO K T. Phys. Rev. D, 1996, **54**: 6850; 1997, **56**: 1821; Petrelli A, Cacciari M, Greco M, Maltoni F, Mangano M L. Nucl. Phys. B, 1998, **514**: 245

32 Amsler C et al. (Particle Data Group). Phys. Lett. B, 2008, **667**: 1