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Jacobi polynomials and non-singlet structure function $F_2(x,Q^2)$ up to N^{3}LO

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Abstract In this paper we present the non-singlet QCD analysis to determine valence quark distribution up to four loop. We obtain the fractional difference between the 4-loop and the 1-, 2- and 3-loop presentations of $xu_v(x,Q^2)$ and $xd_v(x,Q^2)$.

Key words PDFs, DIS data, QCD fit

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1 Introduction

Parton distributions functions (PDF's) provide the essential link between the theoretically calculated partonic cross-sections, and the experimentally measured physical cross-sections involving hadrons. To predict the rates of the various processes, a set of universal PDF's is required. On the other hand all calculations of high energy processes with initial hadrons, whether within the standard model or exploring new physics, require PDF's as an essential input. These distribution functions can determine by QCD global fits to all the available DIS and related hard-scattering data.

The assessment of PDF's, their uncertainties and extrapolation to the kinematics relevant for future colliders such as the LHC is an important challenge to high energy physics in recent years. For quantitatively reliable predictions of DIS and hard hadronic scattering processes, perturbative QCD corrections at the N^{2}LO and the next-to-next-to-next-to-leading order (N^{3}LO) need to be taken into account.

Building on our experience obtained in a series of LO, NLO and N^{2}LO analysis [1] of the non-singlet parton distribution functions, we here extend our work to N^{3}LO accuracy in perturbative QCD. The results of this level is reported [2] very recently which is in good agreement with the available theoretical model [3]. The non-singlet QCD analysis up to N^{2}LO is also performed before [4].

2 Theoretical formalism of the QCD analysis

The non-singlet structure function $F_{2,\mathrm{NS}}(x,Q^2)$ up to N^{3}LO and for three active (light) flavors has the representation

$$x^{-1}F_{2,\mathrm{NS}}(x,Q^2)=\sum_{n=0}^{3}\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^n C^{(n)}_{2,\mathrm{NS}}(x) \otimes\left[\frac{1}{18}q_8^+ +\frac{1}{6}q_3^+\right](x,Q^2)\ . \quad (1)$$

In LO, $C^{(0)}_{2,\mathrm{NS}}(x)=\delta(x)$ and the coefficient functions $C^{(n)}_{2,\mathrm{i}}$ up to N^{3}LO have been calculated [5].

The combinations of parton densities in the non-singlet regime and the valence region $x\geqslant 0.3$ for F_2^p in LO is

$$\frac{1}{x}F_2^p(x,Q^2)=\left[\frac{1}{18}q^+_{\mathrm{NS},8}+\frac{1}{6}q^+_{\mathrm{NS},3}\right](x,Q^2)+\frac{2}{9}\Sigma(x,Q^2), \quad (2)$$

where $q^+_{\mathrm{NS},3}=u_v-d_v$, $q^+_{\mathrm{NS},8}=u_v+d_v$ and $\Sigma=u_v+d_v$, since sea quarks can be neglected in the region $x\geqslant$

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0.3. So in the x-space we have

$$F_2^{\mathrm{p}}(x,Q^2) = \left(\frac{5}{18}\,x\,q^+_{\mathrm{NS},8}+\frac{1}{6}\,x\,q^+_{\mathrm{NS},3}\right)(x,Q^2) = \frac{4}{9}\,x\,\mathrm{u_v}(x,Q^2)+\frac{1}{9}\,x\,\mathrm{d_v}(x,Q^2)\ . \quad (3)$$

In the above region the combinations of parton densities for F_2^{d} are also given by

$$F_2^{\mathrm{d}}(x,Q^2) = \left(\frac{5}{18}\,x\,q^+_{\mathrm{NS},8}\right)(x,Q^2) = \frac{5}{18}\,x(\mathrm{u_v}+\mathrm{d_v})(x,Q^2)\ , \quad (4)$$

where $\mathrm{d}=(\mathrm{p}+\mathrm{n})/2$ and $q^+_{\mathrm{NS},3}=\mathrm{u_v}-\mathrm{d_v}$.

In the region $x\le 0.3$ for the difference of the proton and deuteron data we use

$$F_2^{\mathrm{NS}}(x,Q^2) \equiv 2(F_2^{\mathrm{p}}-F_2^{\mathrm{d}})(x,Q^2)=\frac{1}{3}\,x\,q^+_{\mathrm{NS},3}(x,Q^2) = \frac{1}{3}\,x(\mathrm{u_v}-\mathrm{d_v})(x,Q^2)+\frac{2}{3}\,x(\bar{\mathrm{u}}-\bar{\mathrm{d}})(x,Q^2), \quad (5)$$

where now $q^+_{\mathrm{NS},3}=\mathrm{u_v}-\mathrm{d_v}+2(\bar{\mathrm{u}}-\bar{\mathrm{d}})$ since sea quarks cannot be neglected for x smaller than about 0.3. In our calculation we supposed the $\bar{\mathrm{d}}-\bar{\mathrm{u}}$ distribution at $Q_0^2=4\ \mathrm{GeV}^2$ [1–4] which gives a good description of the Drell-Yan dimuon production data.

Now these results in the physical region $0<x\leqslant 1$ can transform to N-Mellin space by using Mellin-transform to obtain the moments of the structure function as $\frac{1}{x}F_2^{\mathrm{k}}$,

$$F_2^{\mathrm{k}}(N,Q^2)=\int_0^1 \mathrm{d}x\ x^{N-1}\frac{1}{x}F_2^{\mathrm{k}}(x,Q^2)\ , \quad (6)$$

here k denotes the three above cases, i.e. k=p, d, NS. Now by using the solution of the non-singlet evolution equation for the parton densities to 4− loop order, the non-singlet structure functions are available [3] in the moment space.

Now we choose the following parametrization for the valence quark densities in the input scale of $Q_0^2=4\ \mathrm{GeV}^2$

$$x\mathrm{u_v}(x,Q_0^2) = \mathcal{N}_{\mathrm{u}}\ x^{a_{\mathrm{u}}}(1-x)^{b_{\mathrm{u}}}(1+c_{\mathrm{u}}\sqrt{x}+d_{\mathrm{u}}\ x), \quad (7)$$

$$x\mathrm{d_v}(x,Q_0^2) = \mathcal{N}_{\mathrm{d}}\ x^{a_{\mathrm{d}}}(1-x)^{b_{\mathrm{d}}}(1+c_{\mathrm{d}}\sqrt{x}+d_{\mathrm{d}}\ x), \quad (8)$$

where the normalizations $\mathcal{N}_{\mathrm{u}}$ and $\mathcal{N}_{\mathrm{d}}$ being fixed by

$$\int_0^1 \mathrm{u_v}\mathrm{d}x=2$$

and

$$\int_0^1 \mathrm{d_v}\mathrm{d}x=1,$$

respectively. In this work we apply the method of the structure function reconstruction over their Mellin moments, which is based on the expansion of the structure function in terms of Jacobi polynomials. This method was developed and applied for QCD analysis to study the non-singlet structure function $\mathrm{x}F_3$ from their moments [6–10], for non-singlet structure function F_2 [11–15] and also for polarized structure function xg_1 [16–20].

By QCD fits [21] of the world data for $F_2^{\mathrm{p,d}}$, we can extract valence quark densities using the Jacobi polynomials method.

3 QCD Fits and results

For the non-singlet QCD analysis presented in this paper we used the structure function data measured in charged lepton proton and deuteron deep-inelastic scattering. The experiments contributing to the statistics are BCDMS [22], SLAC [23], NMC [24], H1 [25], and ZEUS [26]. In our QCD analysis we use three data samples: $F_2^{\mathrm{p}}(x,Q^2)$, $F_2^{\mathrm{d}}(x,Q^2)$ in the non-singlet regime and the valence quark region $x\geqslant 0.3$ and $F_2^{\mathrm{NS}}=2(F_2^{\mathrm{p}}-F_2^{\mathrm{d}})$ in the region $x<0.3$.

For data used in the global analysis, most experiments combine various systematic errors into one effective error for each data point, along with the statistical error. Then, in addition, the fully correlated normalization error of the experiment is usually specified separately. For this reason, it is natural to adopt the following definition for the effective χ^2 [1]

$$\chi^2_{\mathrm{global}}=\sum_n w_n\chi^2_n$$

$$\chi^2_n=\left(\frac{1-\mathcal{N}_n}{\Delta\mathcal{N}_n}\right)^2+\sum_{\mathrm{i}}\left(\frac{\mathcal{N}_nF_{2,\mathrm{i}}^{\mathrm{data}}-F_{2,\mathrm{i}}^{\mathrm{theor}}}{\mathcal{N}_n\Delta F_{2,\mathrm{i}}^{\mathrm{data}}}\right)^2. \quad (9)$$

For the n^{th} experiment, $F_{2,\mathrm{i}}^{\mathrm{data}}$, $\Delta F_{2,\mathrm{i}}^{\mathrm{data}}$, and $F_{2,\mathrm{i}}^{\mathrm{theor}}$ denote the data value, measurement uncertainty (statistical and systematic combined), and theoretical value for the i^{th} data point. $\Delta\mathcal{N}_n$ is the experimental normalization uncertainty and $\mathcal{N}_n$ is an overall normalization factor for the data of experiment n. The factor w_n is a possible weighting factor(with default value−1). However, we allowed for a relative normalization shift $\mathcal{N}_n$ between the different data sets within the normalization uncertainties $\Delta\mathcal{N}_n$ quoted by the experiments. For example the normalization uncertainty of the NMC(combined) data is estimated to be 2.5%. The normalization shifts $\mathcal{N}_n$ were fitted once and then kept fixed.

Now the sums in χ^2_{global} run over all data setsand in each data set over all data points. The minimization of the above χ^2 value to determine the best parametrization of the unpolarized parton distributions is done using the program MINUIT [21]. In Fig. 1 we show the ration of $[x\mathrm{u}_\mathrm{v}^{N^3\mathrm{LO}} - x\mathrm{u}_\mathrm{v}^{N^i\mathrm{LO}}]/x\mathrm{u}_\mathrm{v}^{\mathrm{N^3LO}}$ and $[x\mathrm{d}_\mathrm{v}^{\mathrm{N^3LO}} - x\mathrm{d}_\mathrm{v}^{\mathrm{N}^i\mathrm{LO}}]/x\mathrm{d}_\mathrm{v}^{\mathrm{N^3LO}}$, for $i=0,1,2$ at $Q^2 = 4$ GeV^2 in based on the Jacobi polynomials method [2].

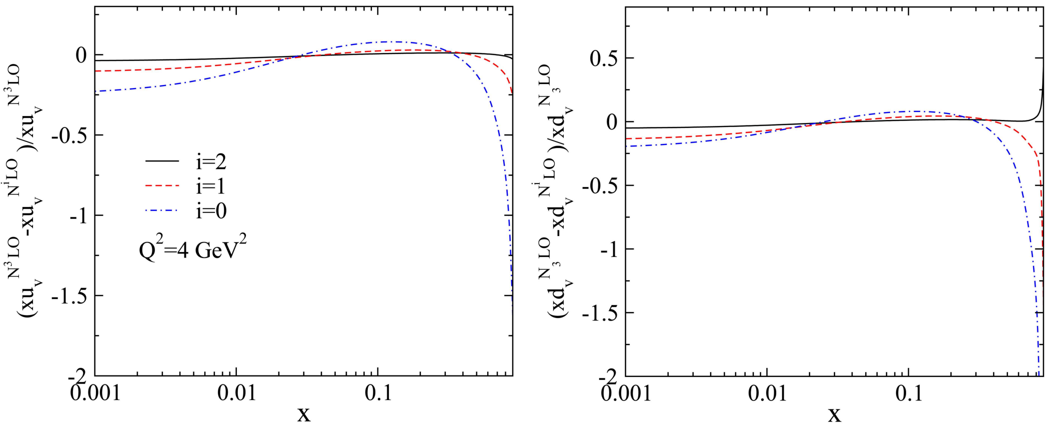

Fig. 1. Fractional difference between the 4-loop and the 1-, 2- and 3-loop presentations of $x\mathrm{u}_\mathrm{v}(x,Q^2)$ and $x\mathrm{d}_\mathrm{v}(x,Q^2)$.

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