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Latest lattice results for baryon spectroscopy

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Abstract Theoretical and computational advances have enabled not only the masses of the ground states, but also some of the low-lying excited states to be calculated using Lattice Gauge Theory. In this talk, I look at recent progress aimed at understanding the spectrum of baryon excited states, including both baryons composed of the light u and d quarks, and of the heavier quarks. I then describe recent work aimed at understanding the radiative transitions between baryons, and in particular the N-Roper transition. I conclude with the prospects for future calculations.

Key words Lattice QCD, spectroscopy, hadronic physics.

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1 Introduction

To claim a thorough understanding of QCD, and hence test whether it is the complete theory of the strong interaction, we must know the spectrum of mesons and baryons that it implies and test those spectra against high quality data. Baryons in particular, characterized by three valence quarks, are emblematic of the non-Abelian nature of QCD, and hence the subject of intense activity aimed at the experimental determination of their spectrum, not only for baryons composed of the light u/d quarks, but also of the heavier quarks. The interest in hadrons containing the c and b quarks was initially motivated by their use as a tool for determining the CKM matrix elements, but there is now a wealth of studies focused on their spectroscopy, notably here in Beijing. Given these intense experimental efforts in baryon spectroscopy, the need to predict and understand the baryon spectrum from first principles is clear, and indeed recognized as an important component of future experimental spectroscopy programs [1]. Furthermore, by considering baryons containing both the light and heavier quarks, we can aim to understand how the effective degrees of freedom describing the spectrum evolve as we change the masses of the component quarks.

The layout of the rest of this talk is as follows. In the next section, I will review the methodology for determining the spectrum from a lattice QCD calculation, and some of the issues that arise extracting the masses of the excited states. I will then review recent lattice results for the spectrum of baryons composed both of the light (u, d, s) and heavier charm quarks, and describe recent progress at understanding the electromagnetic properties of, and transitions between, excited states. Finally, I will describe prospects for future calculations.

2 Spectroscopy in Lattice QCD

The calculation of the ground-state spectrum has been a benchmark calculation of lattice QCD since its inception; calculations of the masses of the lightest states, with control over the systematic uncertainties such as the finite lattice spacing, finite volume and the need to approach the physical quark masses, can now confront experiment with high precision [2].

A comprehensive understanding of QCD requires that we go beyond a knowledge of the ground state mass in each channel, and obtain the masses of the lowest few states of a given quantum number. This we accomplish through the use of the variational method [3, 4]. Rather than measuring a single correlator $C(t)$, we determine a matrix of correlators

$$C_{ij}(t)=\sum_{\vec{x}}\langle O_i(\vec{x},t)O_j^{\dagger}(\vec{0},0)\rangle,$$

where $\{O_i; i=1,\cdots,N\}$ are a basis of interpolating

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operators with given quantum numbers. We then solve the generalized eigenvalue equation

$$C(t)u(t,t_0) = \lambda(t,t_0)C(t_0)u(t,t_0)$$

to obtain a set of real (ordered) eigenvalues $\lambda_n(t,t_0)$, where $\lambda_0 \geqslant \lambda_1 \geqslant \cdots \geqslant \lambda_{N-1}$. At large Euclidean times, these eigenvalues then delineate between the different masses

$$\lambda_n(t,t_0) \longrightarrow \mathrm{e}^{-M_n(t-t_0)}\{1+\mathcal{O}(\mathrm{e}^{-\Delta M_n(t-t_0)})\},$$

where $\Delta M_n = \min\{\mid M_n - M_i \mid : i \neq n\}$. The eigenvectors u are orthogonal with metric $C(t_0)$, and a knowledge of the eigenvectors can yield information about the partonic structure of the states.

Crucial to the application of the variational method is a use of a basis of interpolating operators that have a good overlap with the low-lying states of interest. The cubic lattice employed in our calculations does not admit the full rotational symmetry of the continuum, but rather the more restricted symmetry of the octahedral group. Thus states at rest are classified according to the irreducible representation (irreps) of the cubic group, and for spectroscopy calculations, interpolating operators must be constructed that transform irreducibly under the lattice group; this task has been the prerequisite for our study both of mesons and of baryons.

The procedure for constructing interpolating operators for baryons was described in two papers, the first of which employed a Clebsch-Gordon approach mirroring that of the continuum [5], and the second an automated method allowing a very general basis of operators to be constructed [6]. There are three double-valued irreducible representations of the cubic group, denoted $G_{1\mathrm{u/g}}(2)$, $H_{\mathrm{u/g}}(4)$ and $G_{2\mathrm{u/g}}(2)$, where g and u refer to positive and negative parity, respectively, and the brackets contain the dimension of the irrep. G_1 contains continuum spins $1/2,\ 7/2, \cdots$, H_{g} spins $3/2,\ 5/2, \cdots$ and G_2 spins $5/2,\ 7/2, \cdots$. Thus, at any fixed lattice spacing a, a state corresponding to spin-5/2 has four degrees of freedom in H, and two in G_2, with degeneracies between the energies in the two irreps. emerging in the continuum limit.

The variational method provides a means of delineating the different energy levels, but its successful application requires that the principle correlators be computed with sufficient precision that their exponential behavior is clear. For the case of higher mass states this poses additional challenges: the correlators decrease rapidly with increasing time, whilst the statistical noise decreases at the same rate as for the low-lying energies. Thus the correlators suffer from increasingly poor signal-to-noise ratios as the mass of the state increases. To circumvent this problem, it is necessary to examine the correlation functions at small temporal separations. A powerful method of accomplishing this is through the use of an anisotropic lattice, with smaller temporal that spatial lattice spacing; the efficacy of using the variational method on anisotropic lattices was first demonstrated in the calculation of the quenched glueball spectrum [7].

3 Results

The nucleon spectrum has been analysed in a calculation with two flavors of light Wilson fermions, at two values of the pion mass [8], building on an earlier calculation in the quenched approximation to QCD [9]. The data are shown in Fig. 1. The calculation employs interpolating operators in which the quarks can be displaced, and this enables, for the first time in a lattice calculation, a state of spin-5/2 to be identified.

At these relatively large values of the quark masses, the calculated nuclear spectrum appears remarkably quark-model like, with the ground state nucleon, then its negative parity partner heavier, with the next lightest $1/2^+$ state heavier still. This is in contrast to the ordering observed in nature, where the first excited state is the $1/2^+$ Roper (1440) which is lighter than the lowest-lying $1/2^-$.

This pattern above has long been a feature of most calculations in the quenched approximation to QCD. However, recent work using a basis of interpolating operators formed from quark fields smeared with different Gaussian widths [10, 11], sees a picture more in accord with experiment, and, at light quark masses, approaches the picture seen in a quenched analysis employing a Bayesian-statistics approach [12]. In contrast, a recent $N_{\mathrm{f}} = 2$ calculation, also employing smearings of different widths, finds the lightest $1/2^+$ excited state heavier than its negative parity partner [13]. Thus, despite many advances, the status of this state remains somewhat mysterious.

Finally, the multi-hadron states that should be seen in the spectrum, shown as the open boxes in 1, appear somewhat elusive. Each of the calculations above uses essentially three-quark baryon interpolating operators; naively, one might expect such operators to be insensitive to multi-hadron contributions, and the design of correlation functions that are sensitive to such contributions is an important goal for future work.

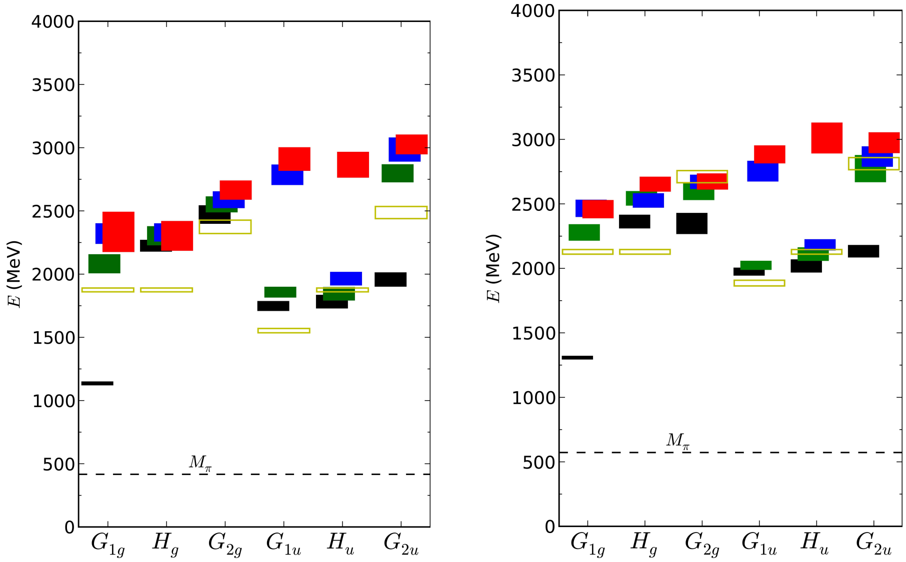

Fig. 1. (color online) The left- and right-hand panels show the spectrum of $I = 1/2$ baryon resonance, indicated by the solid boxes, obtained on $N_{\rm f} = 2$ Wilson fermion lattices at $m_\pi = 416$ and 578 MeV respectively [8]; the errors are indicated by the vertical width of the box. The open boxes show the expected thresholds for multiparticle states.

3.1 Baryons containing heavy quarks

The calculation of the spectrum of baryons containing the heavier charmed quark presents different challenges, notably the need to control discretisation uncertainties associated with the large value of the quark mass, rather than the excitation energy, demanding a variety of approaches: treating the heavy quark in the static approximation, valid for heavy-light systems, non-relativistic QCD, the use of an anisotropic lattice, and the so-called FNAL action. Fig. 2 shows a compendium of the spectrum of singly- and doubly-charmed baryons, taken from a recent paper [14] that employs the Fermilab action for the heavy quarks [15], but the Asqtad action for the light and sea quarks. The mass of the $\Omega_{\rm cc}$ in particular is a prediction $M_{\Omega_{\rm cc}} = 3763 \pm 19 \pm 26 (+13 - 79)$ MeV.

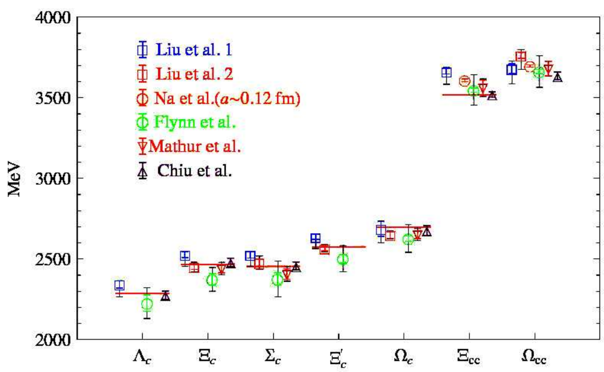

Fig. 2. (color online) Summary of spectrum of baryons containing one or two quarks calculated in lattice QCD, from a recent calculation [14] using the Fermilab action for the charm quark.

3.2 Electromagnetic properties

Once the mass of a state is established, we can go beyond the spectrum to look at the electromagnetic properties of the states, and the radiative transitions between them. The N-Δ transition form factors have long been studied, but recently the transition between the nucleon, and the lowest lying $1/2^+$ excitation has been investigated [16]. The transition between two $1/2^+$ states, N_1 and N_2, has two form factors, defined by

$$\langle N_2 \,|\, V_\mu \,|\, N_1 \rangle \;=\; \bar{u}_{N_2}(p') \left[F_1 q^2 \left(\gamma_\mu - \frac{q_\mu}{q^2} \gamma \cdot q \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) {\rm e}^{-{\rm i} q \cdot x}. \qquad (1)$$

The calculated form factors, in the quenched approximation to QCD on anisotropic lattices, are shown for the proton-to-Roper transition in Fig. 3, in a calculation with quark masses corresponding to a pion mass of 720 MeV. While the lattice calculation shows disagreement with the recent measurement by CLAS, the large quark masses used in the calculation should be noted, as should the fact that, in this calculation, the masses display the "quark-model" like ordering of masses: $M_N < M_{N(1/2^-)} < M_{N'(1/2^+)}$.

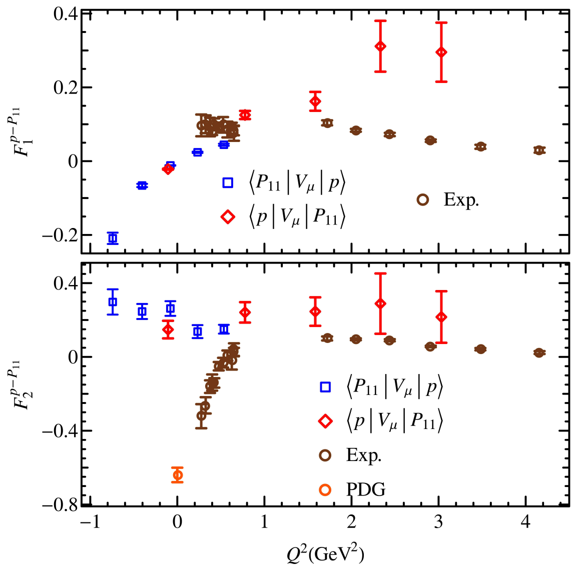

Fig. 3. (color online) Proton-Roper form factors $F^*_{1,2}$ obtained from CLAS experiments and PDG number (circles) and the lattice calculation (squares, diamonds)

4 Future prospects

The calculations outlined above have demonstrated our ability to determine the baryon spectrum, and establish the properties of some of the low-lying excitations, from lattice QCD. Precise calculations that can truly confront the experimental program require that we generate lattice with the correct number of light-quark flavors. The use of anisotropic lattices has proved essential in the reliable determination of the energy levels necessary to extract the resonance spectrum. Thus a crucial activity of the Hadron Spectrum collaboration has been the generation of lattices with two flavors of fully dynamical light and a dynamical strange quark, using the clover fermion action, designed both for studies of the spectrum, and for the calculation of the scattering lengths important for understanding the nucleon-nucleon interaction. An important milestone was the tuning of the parameters of the action, beginning with the three-flavor theory [17].

A major challenge in calculations in lattice QCD is a procedure for specifying the values of the quark masses, and in particular those of the light u/d and s quarks, in a way that enables lattice calculations to be extrapolated to the physical values of these masses. This was accomplished through the introduction of a novel pair of dimensionless coordinates l_Ω and s_ω that are primarily sensitive to the light and strange quark masses, respectively [18]:

$$l_\Omega=\frac{9m_\pi^2}{4m_\Omega^2},\quad s_\Omega=\frac{9(2m_K^2-m_\pi^2)}{4m_\Omega^2}. \tag{2}$$

The use of these coordinates is illustrated in Fig. 4, showing the approach at fixed strange-quark mass to the physical values of the light (u/d) quark masses.

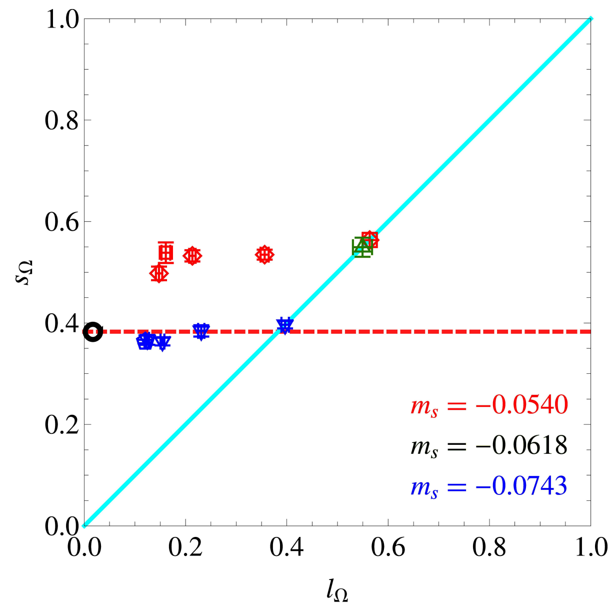

Fig. 4. (color online) The figure shows the dynamical ensembles generated in [18] plotted in terms of the dimensionless coordinates l_Ω and s_Ω, as described in the text. The green line corresponds to the theory with three degenerate quark flavors, whilst the red horizontal line shows the approach to the physical quark masses; the black circle is the physical value of the coordinates.

A comprehensive determination of the spectrum requires a means of efficiently computing a sufficiently wide basis of interpolating operators, including multi-particle operators. Recently, a new method, known as "distillation", has been proposed [19], and, for baryons composed of quarks with a mass around that of the strange quark, yields the lowest energies with very high precision.

A further requirement if lattice calculations are to confront experiment is to identify the spins of the states. With increasing excitation, the observed spectrum of states becomes increasingly dense. For a lattice calculation performed at a single value of the lattice spacing, these states, together with the two- and higher-particle scattering states, have to be assigned to the relatively small number of irreducible representations admitted by the symmetries of the lattice. To identify the energies, and then watch the appearance of degeneracies in the approach to the continuum limit, would be a formidable undertaking, and computationally highly demanding indeed.

Recently, an alternative method has been proposed which employs a continuum operator construction subduced to the lattice irreps.. The overlap of the subduced operators between a state of definite continuum quantum numbers and the vacuum in the approach to the continuum limit is know, and this feature is thereby used to identify the continuum states. The method has been successfully applied in charmonium [20], and subsequently, with further refinements, to the identification of the excited meson spectrum, including those with exotic quantum numbers, in "strangonium" [21]; further details are in the talk of Christopher Thomas [22].

Finally, a significant impediment to understanding the excited baryon spectrum is the lack of expectation for the quark mass dependence of excited states. Chiral corrections to the Roper mass have been studied [23], and the finite-volume and quark mass dependence of the Δ investigated with the aim of determining the resonance parameters [24], but further work will be necessary to fully understand the spectrum of QCD.

5 Conclusions

Recent advances have demonstrated the ability of lattice gauge calculations both to calculate the spectrum of baryons, and to probe some of their properties. Recent computational and theoretical developments, together with the advent of petascale computing, promise calculations at the physical values of the light-quark masses in the near future. There remain challenges that still need to be overcome, notably the identification of multi-particle states and an understanding of the quark-mass dependence of the masses, but lattice gauge calculations are now integral to our understanding of spectroscopy.

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References

1 Aznauryan I et al. 2009 (Preprint 0907.1901)

2 Durr S et al. Science, 2008, **332**: 1224–1227 (Preprint 0906.3599)

3 Michael C. Nucl. Phys. B, 1985, **259**: 58

4 Luscher M, Wolff U. Nucl. Phys. B, 1990, **339**: 222–252

5 Basak S et al. (Lattice Hadron Physics (LHPC)). Phys. Rev. D, 2005, **72**: 074501 (Preprint hep-lat/0508018)

6 Basak S et al. Phys. Rev. D, 2005, **72**: 094506 (Preprint hep-lat/0506029)

7 Morningstar C J, Peardon M J. Phys. Rev. D, 1997, **56**: 4043–4061 (Preprint hep-lat/9704011)

8 Bulava J M et al. Phys. Rev. D, 2009, **79**: 034505 (Preprint 0901.0027)

9 Basak S et al. Phys. Rev. D, 2007, **76**: 074504 (Preprint 0709.0008)

10 Mahbub M S et al. Phys. Lett. B, 2009, **679**: 418–422 (Preprint 0906.5433)

11 Mahbub M S et al. Phys. Rev. D, 2009, **80**: 054507 (Preprint 0905.3616)

12 Mathur N et al. Phys. Lett. B, 2005, **605**: 137–143 (Preprint hep-ph/0306199)

13 Engel G et al. 2009 (Preprint 0910.2802)

14 LIU L, LIN H W, Orginos K, Walker-Loud A. 2009 (Preprint 0909.3294)

15 El-Khadra A X, Kronfeld A S, Mackenzie P B. Phys. Rev. D, 1997, **55**: 3933–3957 (Preprint hep-lat/9604004)

16 LIN H W, Cohen S D, Edwards R G, Richards D G. Phys. Rev. D, 2008, **78**: 114508 (Preprint 0803.3020)

17 Edwards R G, Joo B, LIN H W. Phys. Rev. D, 2008, **78**: 054501 (Preprint 0803.3960)

18 LIN H W et al. (Hadron Spectrum). Phys. Rev. D, 2009, **79**: 034502 (Preprint 0810.3588)

19 Peardon M et al. (Hadron Spectrum). Phys. Rev. D, 2009, **80**: 054506 (Preprint 0905.2160)

20 Dudek J J, Edwards R G, Mathur N, Richards D G. Phys. Rev. D, 2008, **77**: 034501 (Preprint 0707.4162)

21 Dudek J J, Edwards R G, Peardon M J, Richards D G, Thomas C E. 2009 (Preprint 0909.0200)

22 Thomas C E. These proceedings

23 Borasoy B, Bruns P C, Meissner U G, Lewis R. Phys. Lett. B, 2006, **641**: 294–300 (Preprint hep-lat/0608001)

24 Bernard V, Meissner U G, Rusetsky A. Nucl. Phys. B, 2008, **788**: 1–20 (Preprint hep-lat/0702012)