CPC(HEP & NP), 2011, **35**(2): 203–208 Chinese Physics C Vol. 35, No. 2, Feb., 2011

Spectral distributions of the scattered photons within an acceptance angle in Thomson scattering*

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Abstract: In this paper, we present the analysis of the spectral distributions of the scattered photons within a certain acceptance angle in Thomson scattering, in which the beam divergence, energy spread and spatial distribution are all considered. The analytical results are compared with the simulation results, and good agreement between the two approaches is obtained.

Key words: spectral distributions, Thomson scattering, acceptance angle

PACS: 25.20.Dc, 41.75.Ht, 42.55.Vc **DOI:** 10.1088/1674-1137/35/2/019

1 Introduction

The X-ray source based on Thomson scattering has broad application prospects in biology, physics, materials, photochemical engineering and medicine, for the short time structure, high peak brightness, and the feature of quasi-monochromaticity within a certain direction. In recent years, a larger number of universities and research institutions have built Thomson scattering systems and conducted experiments on them. The Accelerator Laboratory of Tsinghua University has also carried out the relevant aspects of the work.

In the Thomson scattering process, one of the main features of the generated X-ray is the spectral distributions within a certain acceptance angle. Tomassini did theoretical analysis of the spectral distributions in his paper [1]. However, it is only under the case that all electrons move exactly along the z direction colliding with a plane-wave laser propagating along the $-z$ direction, and the electron beam divergence and energy spread are both ignored. When he compared the analytical results with the simulation results of Thomson scattering, the difference is very obvious.

In this paper, theoretical analysis of the spectral distributions of the scattered photons within a certain acceptance angle is presented, in which the beam divergence, energy spread and spatial distribution are all considered. Good agreement is obtained between the analytical approach and numerical approach.

2 Spectral distributions of the scattered photons within a certain acceptance angle for a single electron

For a single photon colliding with a laser beam, the number of scattered photons per unit solid angle, per unit time is given by [2]

$$\frac{\mathrm{d}N_s}{\mathrm{d}\Omega\mathrm{d}t}=c\left(1-\vec{\beta}\cdot\vec{k}\frac{c}{\omega}\right)n_\gamma(r,t)\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}. \tag{1}$$

We can get

$$\frac{\mathrm{d}N_s}{\mathrm{d}\omega_s\mathrm{d}t}=(1-\cos\theta_0)cn_\gamma(r,t)\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}, \tag{2}$$

where σ is the Thomson cross section, n_γ is the photon density at the position r of the electron, $\vec{k}$ is the incident wave vector, θ_0 is the angle of the scattered photon direction from the direction of the electron in the lab frame, and ω_s is the scattered photon frequency.

In the electron rest frame, according to the Klein-Nishina Formula, the cross section per unit solid angle

Received 17 April 2010

* Supported by National Natural Science Foundation of China (10735050) and National Basic Research Program of China (973 Program) (2007CB815102)

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is given by [3]

$$\frac{\mathrm{d}N_s}{\mathrm{d}\Omega'}=\frac{1}{2}r_0^2\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega'}{\omega}+\frac{\omega}{\omega'}-\sin^2\theta'\right), \tag{3}$$

where ω, ω' are the incident photon frequency and the scattered photon frequency, and θ' is the angle of the scattered photon direction from the incident photon in the electron rest frame.

In the Compton scattering process,

$$\frac{\omega'}{\omega}=\frac{1}{1+\frac{\omega}{m}(1-\cos\theta')}\approx 1. \tag{4}$$

We get

$$\frac{\mathrm{d}\sigma'}{\mathrm{d}\Omega'}\approx\frac{1}{2}r_0^2\left(\frac{\omega'}{\omega}\right)^2(1+\cos^2\theta'). \tag{5}$$

Now we suppose that in the lab frame the angle of the electron and the incident light is $\pi-\theta_{\mathrm{e}}$, and the electron incident direction is $\vec{z}_{\mathrm{e}}$ axis, and in the electron rest frame the angle of the incident light and $\vec{z}_{\mathrm{e}}$ axis is $\pi-\theta'_{\mathrm{e}}$. According to Lorentz transformation we have

$$\cos\theta'_{\mathrm{e}}=\frac{\cos\theta_{\mathrm{e}}+\beta}{1+\beta\cos\theta_{\mathrm{e}}}. \tag{6}$$

From Eq. (6) we get

$$\sin\theta'_{\mathrm{e}}=\frac{1}{\gamma}\frac{\sin\theta_{\mathrm{e}}}{1+\beta\cos\theta_{\mathrm{e}}}. \tag{7}$$

As $\gamma\gg 1$, obviously $\theta'_{\mathrm{e}}\to 0$. This means that, in the electron rest frame, the incident light direction is very close to the $\vec{z}_{\mathrm{e}}$ direction. Hence θ' in Eq. (3) can be approximately regarded as the angle of the scattered photon and $\vec{z}_{\mathrm{e}}$ axis. We get

$$\cos(\pi-\theta')=\frac{\cos\theta-\beta}{1-\beta\cos\theta}, \tag{8}$$

where θ is in the angle of the scattered photon and $\vec{z}_{\mathrm{e}}$ in the lab frame. According to Lorentz transformation we also have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{\mathrm{d}\sigma'}{\mathrm{d}\Omega}\frac{1-\beta^2}{1-\beta\cos\theta}. \tag{9}$$

We put Eq. (5) and Eq. (8) into Eq. (9) and get

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= r_0^2\left[1-\frac{\theta^2}{\gamma^2(1-\beta\cos\theta)^2}\right]\frac{1}{\gamma^2(1-\beta\cos\theta)^2}\\ &\approx r_0^2\frac{4\gamma^2}{(1+\theta^2\gamma^2)}\left[1-\frac{2\theta^2\gamma^2}{(1+\theta^2\gamma^2)^2}\right].\end{aligned} \tag{10}$$

According to Lorentz transformation we have

$$\frac{\omega_0}{\omega}=\frac{1}{\gamma(1-\beta\cos\theta_0)}, \tag{11}$$

$$\frac{\omega_s}{\omega'}=\frac{1}{\gamma(1-\beta\cos\theta)}. \tag{12}$$

Here ω_0, ω_s are the incident photon frequency and the scattered photon frequency in the lab frame.

We mark that $K_0=1-\beta\cos\theta_0$. From Eq. (4), Eq. (11) and Eq. (12) we get

$$\frac{\omega_s}{\omega_0}=\frac{K_0}{1-\beta\cos\theta}\approx\frac{2K_0\gamma^2}{1+\theta^2\gamma^2}. \tag{13}$$

We differentiate both sides of Eq. (13) and get

$$\frac{\mathrm{d}\omega_s}{\sin\theta\mathrm{d}\theta}=\frac{K_0\omega_0\beta}{(1-\beta\cos\theta)^2}\approx\frac{4K_0\gamma^2\omega_0}{(1+\theta^2\gamma^2)^2}. \tag{14}$$

From Eq. (10) and Eq. (14) we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s\mathrm{d}\phi}=\frac{r_0^2}{K_0\gamma^2\omega_0}\left[1-\frac{2\theta^2\gamma^2}{(1+\theta^2\gamma^2)}\right]. \tag{15}$$

According to Eq. (13) we get

$$\theta^2=\frac{2K_0\omega_0}{\omega_s}-\frac{1}{\gamma^2}. \tag{16}$$

We can put Eq. (13) into Eq. (15) to obtain the cross section per unit frequency per unit azimuthal ϕ, and have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s\mathrm{d}\phi}=\frac{r_0^2}{2K_0^3\gamma^6\omega_0^3}(\omega_s^2-2K_0\gamma^2\omega_0\omega_s+2K_0^2\gamma^4\omega_0^2). \tag{17}$$

Integrating over azimuthal angle ϕ, we can get $\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}$. Denoting the acceptance angle of the detector as θ_m, we can see that not all the scattered photons with direction angle θ are received by the detector. So here the scope of integration is not just $0\sim 2\pi$, but $0\sim 2\pi-2\Phi$, as shown in Fig. 1.

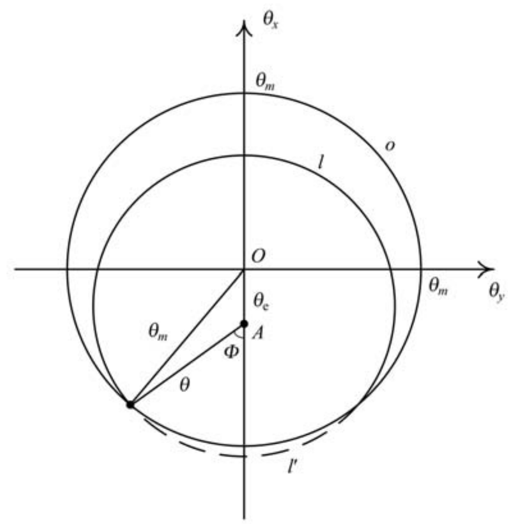

Fig. 1. Angular geometry of Thomson scattering. Circle O represents the acceptance range of the detector, and dot A represents an electron with an incident angle θ_{e}. Arc l represents the scattered photons with direction angle θ that are received by the detector, and arc l' represents those out of the acceptance range.

Finally we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}=\frac{\Delta\phi r_0^2}{2K_0^3\gamma^6\omega_0^3}(\omega_s^2-2K_0\gamma^2\omega_0\omega_s+2K_0^2\gamma^4\omega_0^2). \quad (18)$$

While $\theta_\mathrm{e}\leqslant\theta_m$,

$$\Delta\phi=\begin{cases}2\pi, & 0\leqslant\theta\leqslant\theta_m-\theta_\mathrm{e}\\ 2\pi-2\arccos\dfrac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}, & \theta_m-\theta_\mathrm{e}<\theta\leqslant\theta_m+\theta_\mathrm{e}\end{cases}, \quad (19)$$

and while $\theta_\mathrm{e}>\theta_m$,

$$\Delta\phi=\begin{cases}0, & 0\leqslant\theta\leqslant\theta_\mathrm{e}-\theta_m\\ 2\pi-2\arccos\dfrac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}, & \theta_\mathrm{e}-\theta_m<\theta\leqslant\theta_\mathrm{e}+\theta_m\end{cases}, \quad (20)$$

and here $\theta=\sqrt{\dfrac{2K_0\omega_0}{\omega_s}-\dfrac{1}{\gamma^2}}$.

When the incident angle is $\theta_\mathrm{e}=180°$, Eq. (18) becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}=\frac{\pi r_0^2}{8\gamma^6\omega_0^3}(\omega_s^2-4\gamma^2\omega_0\omega_s+8\gamma^4\omega_0^2). \quad (21)$$

In this situation we get the same result as in Tomassini's paper [1].

We can compare the simulation result with Eq. (18), using the CAIN code [4], and good agreement is obtained as in Fig. 2.

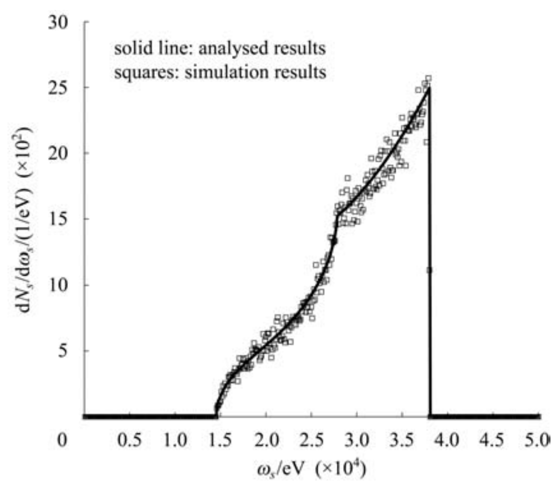

Fig. 2. Spectral distributions of the scattered photons obtained with acceptance angle $\theta_m=$ 4.2 mrad. Here the electron energy is 40 MeV, the laser wave length is 800 nm, the laser energy is 1 J, the laser radius is 20 μm, the laser length is 1ps, and the incident angle of the electron and the laser beam $\theta_\mathrm{e}=3$ mrad.

3 Spectral distributions of the scattered photons for an electron beam within a certain acceptance angle

In the previous section we obtain the equation for the spectral distributions of the scattered photons for a single electron, as shown in Eq. (18). Based on this equation, we consider the influence of the beam divergence, energy spread and spatial distribution. Here we consider the head-on collision (the colliding angle between the electron beam and laser is 180°). Then for each electron we have that $\theta_0\rightarrow180°$, and so in Eq. (18), $K_0\rightarrow2$, and it becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}=\frac{\Delta\phi r_0^2}{16\gamma^6\omega_0^3}(\omega_s^2-4\gamma^2\omega_0\omega_s+8\gamma^4\omega_0^2), \quad (22)$$

where $\Delta\phi$ is shown as in Eq. (18).

3.1 Spatial and time distribution of laser and beam

According to the Thomson Scattering equation [2], we have

$$\begin{aligned}N_s&=\frac{\overline{\sigma}}{ce}c\int J^\mu\Phi^\mu\mathrm{d}^4x\\&=c\overline{\sigma}\int\left(1-\vec{\beta}\cdot\vec{k}\frac{c}{\omega}\right)n_\mathrm{e}(r,t)n_\gamma(r,t)\mathrm{d}^3r\mathrm{d}t, \quad (23)\end{aligned}$$

where $\overline{\sigma}$ is the average Thomson cross section of all the electrons.

Here $\vec{\beta}\cdot\vec{k}\dfrac{c}{\omega}\rightarrow-1$, as we consider the head-on collision, and we get

$$\frac{\mathrm{d}N_s}{\mathrm{d}\omega_s}=c\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\omega_s}\cdot2\int n_\mathrm{e}(r,t)n_\gamma(r,t)2\pi r\mathrm{d}r\mathrm{d}z\mathrm{d}t. \quad (24)$$

The electron beam density can be modeled by the Gaussian radial and temporal distributions

$$n_\mathrm{e}=\frac{N_\mathrm{e}}{\pi^{3/2}c\Delta\tau r_b^2}\exp\left[-\left(\frac{t-\dfrac{z}{c}}{\Delta\tau}\right)^2-\frac{r^2}{r_b^2}\right], \quad (25)$$

where $N_\mathrm{e}=q/e$ is the number of electrons in the bunch, $\Delta\tau$ is the bunch duration, and r_b is the radius at focus.

The photon density of laser pulse can be described by the Gaussian radial and temporal distributions [5]

$$n_\gamma=\frac{N_\gamma}{(\pi/2)^{3/2}c\Delta tw_0^2}\frac{1}{1+\left(\dfrac{z}{z_0}\right)^2}\times\exp\left\{-2\left(\frac{t+\dfrac{z}{c}}{\Delta t}\right)^2-\frac{r^2}{w_0^2\left[1+\left(\dfrac{z}{z_0}\right)^2\right]}\right\}, \quad (26)$$

where $N_\gamma=\dfrac{W}{\hbar\omega_0}$ is the total number of photons in the laser pulse, Δt is the pulse duration, and is related to the bandwidth as $\Delta t\Delta\omega=\sqrt{2}$, w_0 is the $\dfrac{1}{e^2}$ focal radius, and $z_0=\dfrac{\pi w_0^2}{\lambda_0}$ is the Rayleigh length.

In the linear regime for high brightness X-ray operation, the laser pulse duration should be short compared with the Rayleigh length. So during the interaction of Thomson scattering, the focusing and diffracting of the pulse could be ignored and the photon density could be approximated as follows:

$$n_\gamma\approx\frac{N_\gamma}{(\pi/2)^{3/2}c\Delta t w_0^2}\exp\left[-2\left(\frac{t+\frac{z}{c}}{\Delta t}\right)^2-\frac{r^2}{w_0^2}\right].\quad(27)$$

Putting Eq. (25) and Eq. (27) into Eq. (24) we have

$$\frac{\mathrm{d}N_s}{\mathrm{d}\omega_s}=\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\omega_s}\frac{N_\gamma N_\mathrm{e}}{\pi\left(r_b^2+\frac{w_0^2}{2}\right)}.\quad(28)$$

In the section below, we will obtain the equation of $\mathrm{d}\overline{\sigma}/\mathrm{d}\omega_s$ through the known $\mathrm{d}\sigma/\mathrm{d}\omega_s$ as shown in Eq. (18).

3.2 Beam divergence

Here we assume that the electron beam divergences in x and y directions are the same, and then the beam divergences distribution could be described as

$$f(\theta_\mathrm{e})=\frac{2\theta_\mathrm{e}}{\Delta\theta_\mathrm{e}^2}\exp\left(-\frac{\theta_\mathrm{e}^2}{\Delta\theta_\mathrm{e}^2}\right).\quad(29)$$

Then we have

$$\frac{\mathrm{d}\sigma_\mathrm{e}}{\mathrm{d}\omega_s}=\int\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}f(\theta_\mathrm{e})\mathrm{d}\theta_\mathrm{e}.\quad(30)$$

From Eq. (17), Eq. (28) and Eq. (29) we get

$$\frac{\mathrm{d}\sigma_\mathrm{e}}{\mathrm{d}\omega_s}=\frac{\Delta\phi_\mathrm{e}r_0^2}{16\gamma^6\omega_0^3}(\omega_s^2-4\gamma^2\omega_0\omega_s+8\gamma^4\omega_0^2),\quad(31)$$

where

$$\Delta\phi_\mathrm{e}=\begin{cases}\displaystyle\int_{\theta_m-\theta}^{\theta_m+\theta}2\left(\pi-\arccos\frac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}\right)f(\theta_\mathrm{e})\,\mathrm{d}\theta_\mathrm{e}+\int_0^{\theta_m-\theta}2\pi f(\theta_\mathrm{e})\,\mathrm{d}\theta_\mathrm{e}, & \theta\leqslant\theta_m\\ \displaystyle\int_{\theta-\theta_m}^{\theta+\theta_m}2\left(\pi-\arccos\frac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}\right)f(\theta_\mathrm{e})\,\mathrm{d}\theta_\mathrm{e}, & \theta>\theta_m\end{cases},\quad(32)$$

where

$$\theta=\sqrt{\frac{4\omega_0}{\omega_s}-\frac{1}{\gamma^2}}.$$

To calculate the above integral equation, here we carry out the Taylor expansion to the inverse trigonometric functions and take the first-order term for approximation. Later we will see that this approximation is suitable for the final analysed result.

Then we get

$$\arccos\frac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}\approx\frac{\pi}{2}-\frac{\theta_m^2-\theta^2-\theta_\mathrm{e}^2}{2\theta\theta_\mathrm{e}}.\quad(33)$$

From Eq. (32) and Eq. (33) we get

$$\Delta\phi_\mathrm{e}=\int\Delta\phi_1 f(\theta_\mathrm{e})\mathrm{d}\theta_\mathrm{e}=\begin{cases}A, & \theta\leqslant\theta_m\\ C, & \theta>\theta_m\end{cases},\quad(34)$$

where

$$\begin{aligned}A\ =\ &2\pi\left\{1-\exp\left[-\frac{(\theta_m-\theta)^2}{\Delta\theta_\mathrm{e}^2}\right]\right\}+\left(\pi-\frac{\theta_m-\theta}{\theta}\right)\exp\left[-\frac{(\theta_m-\theta)^2}{\Delta\theta_\mathrm{e}^2}\right]\\ &-\left(\pi-\frac{\theta_m+\theta}{\theta}\right)\exp\left[-\frac{(\theta_m+\theta)^2}{\Delta\theta_\mathrm{e}^2}\right]+\sqrt{\pi}\frac{\theta_m^2-\theta^2-\frac{\Delta\theta_\mathrm{e}^2}{2}}{\theta\Delta\theta_\mathrm{e}}\left[\mathrm{erf}\left(\frac{\theta_m+\theta}{\Delta\theta_\mathrm{e}}\right)-\mathrm{erf}\left(\frac{\theta_m-\theta}{\Delta\theta_\mathrm{e}}\right)\right],\end{aligned}\quad(35)$$

$$C = \left(\pi-\frac{\theta-\theta_m}{\theta}\right)\exp\left[-\frac{(\theta-\theta_m)^2}{\Delta\theta_{\rm e}^2}\right]-\left(\pi-\frac{\theta+\theta_m}{\theta}\right)\exp\left[-\frac{(\theta+\theta_m)^2}{\Delta\theta_{\rm e}^2}\right]$$

$$+\sqrt{\pi}\frac{\theta_m^2-\theta^2-\frac{\Delta\theta_{\rm e}^2}{2}}{\theta\Delta\theta_{\rm e}}\left[\mathrm{erf}\left(\frac{\theta+\theta_m}{\Delta\theta_{\rm e}}\right)-\mathrm{erf}\left(\frac{\theta-\theta_m}{\Delta\theta_{\rm e}}\right)\right]. \tag{36}$$

3.3 Beam energy spread

The electron beam energy distribution function could be described as

$$f(\gamma)=\frac{1}{\sqrt{2\pi}\sigma_\gamma}\exp\left[-\frac{(\gamma-\gamma_0)^2}{2\sigma_\gamma^2}\right]. \tag{37}$$

Then we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega_s}=\int\frac{\mathrm{d}\sigma_{\rm e}}{\mathrm{d}\omega_s}f(\gamma)\mathrm{d}\gamma. \tag{38}$$

As the beam energy spread is very small ($\sigma_\gamma/\gamma_0\ll 1$), so $\gamma\approx\gamma_0$. And from Eq. (28) and Eq. (36) we have

$$\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\omega_s}=\frac{r_0^2}{16\gamma_0^6\omega_0^3}(\omega_s^2-4\gamma_0^2\omega_0\omega_s+8\gamma_0^4\omega_0^2)\int\Delta\phi_{\rm e}f(\gamma)\mathrm{d}\gamma, \tag{39}$$

where

$$\int\Delta\phi_{\rm e}f(\gamma)\mathrm{d}\gamma=\int\limits_{\theta\leqslant\theta_m}Af(\gamma)\,\mathrm{d}\gamma+\int\limits_{\theta\geqslant\theta_m}Cf(\gamma)\,\mathrm{d}\gamma. \tag{40}$$

As mentioned in the previous section, we know that

$$\theta=\sqrt{\frac{4\omega_0}{\omega_s}-\frac{1}{\gamma^2}}\approx\sqrt{\frac{4\omega_0}{\omega_s}-\frac{1}{\gamma_0^2}+\frac{2\Delta\gamma}{\gamma_0^3}}, \tag{41}$$

where $\Delta\gamma=\gamma-\gamma_0$, and from Eq. (38) and Eq. (39) we get

$$\int\Delta\phi_{\rm e}f(\gamma)\mathrm{d}\gamma=\int\limits_{\frac{\gamma_0^3}{2}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}\right)}^{\frac{\gamma_0^3}{2}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}+\theta_m^2\right)}\frac{A\cdot\exp\left(-\frac{\Delta\gamma^2}{2\sigma_\gamma^2}\right)}{2\sqrt{2\pi}\sigma_\gamma}\mathrm{d}\gamma+\int\limits_{\frac{\gamma_0^3}{2}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}+\theta_m^2\right)}^{\infty}\frac{C\cdot\exp\left(-\frac{\Delta\gamma^2}{2\sigma_\gamma^2}\right)}{2\sqrt{2\pi}\sigma_\gamma}\mathrm{d}\gamma. \tag{42}$$

As the beam energy spread is very small $\left(\frac{\sigma_\gamma}{\gamma_0}\ll 1\right)$, we can see that $A(\gamma)\approx A(\gamma_0)$, $C(\gamma)\approx C(\gamma_0)$, so we get

$$\Delta\Phi = A(\gamma_0)\cdot\frac{1}{2}\left\{\mathrm{erf}\left[\frac{\gamma_0^3}{2\sqrt{2}\sigma_\gamma}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}+\theta_m^2\right)\right]-\mathrm{erf}\left[\frac{\gamma_0^3}{2\sqrt{2}\sigma_\gamma}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}\right)\right]\right\}$$

$$+C(\gamma_0)\cdot\frac{1}{2}\left\{1-\mathrm{erf}\left[\frac{\gamma_0^3}{2\sqrt{2}\sigma_\gamma}\left(\frac{1}{\gamma_0^2}-\frac{4\omega_0}{\omega_s}+\theta_m^2\right)\right]\right\}, \tag{43}$$

where A and C are described in Eq. (35) and Eq. (36).

Finally the spectral distributions of the scattered photons for electron beam within a certain acceptance angle could be described as

$$\frac{\mathrm{d}N_s}{\mathrm{d}\omega_s} = \Delta\Phi\frac{N_\gamma N_{\rm e}}{\pi\left(r_b^2+\frac{w_0^2}{2}\right)}\frac{r_0^2}{16\gamma_0^6\omega_0^3}(\omega_s^2-4\gamma_0^2\omega_0\omega_s+8\gamma_0^4\omega_0^2). \tag{44}$$

4 Comparison with simulation results

Here we use the typical parameters of the Thomson scattering system in many universities and research institutions.

We use the CAIN code and get the simulation result, as shown in Fig. 3. We can see that the analysed results show good agreement with the simulation results.

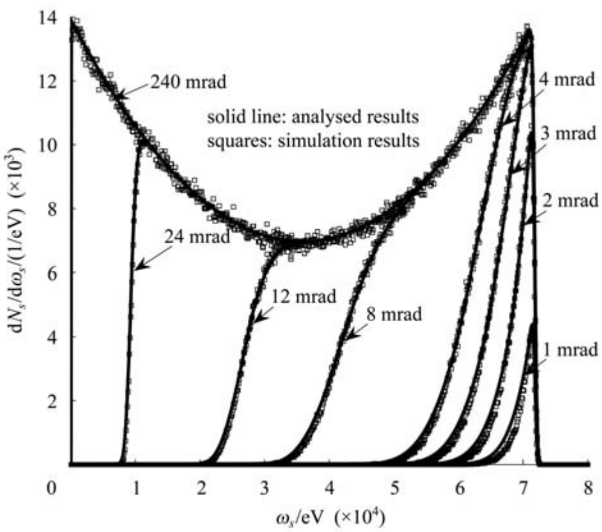

Fig. 3. Spectral distributions of the scattered photons for a Thomson scattering system obtained with different acceptance angles.

The left of each curve shows less coincidence, which is because of the approximation in Eq. (33). As for the left of each curve, we have that $\theta \approx \theta_{\rm e} \gg \theta_m$, and so $\dfrac{\theta_m^2-\theta^2-\theta_{\rm e}^2}{2\theta\theta_{\rm e}} \rightarrow -1$, which is just when the maximum error of the approximation is produced.

Table 1. The electron beam and laser beam parameters.

electron beam		laser beam	
energy	55 MeV	wave length	800 nm
radius	20 μm	energy	1 J
emittance	1.5 mm·mrad	length	3 ps
charge	1 nC	radius	20 μm
length	0.5 ps	quality	$M^2=1$
energy spread	0.2%		

5 Conclusion

In this paper, we have derived spectral distributions of the Thomson scattered photons within a certain acceptance angle considering the beam divergence, energy spread and spatial distribution. When compared with the simulation results, good agreement is obtained. Our results could be used for rapid estimation of the spectral distributions, wide range scanning of the spectral distributions with the proper parameters in Thomson scattering, and are also important in the optimal design of a tunable Thomson-scattering based X-ray source.

References

1 Tomassini P, Giulietti A, Giulietti D, Gizzi L A. Appl. Phys. B, 2005, **80**: 419–436

2 Brown W J, Hartemann F V. Physical Review Special Topics - Accelerators and Beams, 2004, **7**: 060703

3 Berestetskii V B, Lifshitz E M, Pitaevskii L P. Quantum Electrodynamics, Volume 4 of Course of theoretical Physics. Second Edition Translated. London: Pergamon Press, 1982. 356

4 Yokoya K. A Computer Simulation Code for the Beam-Beam Interaction in Linear Colliders. KEK Report. 1985: 85–9. Also Nucl. Instrum. Methods A, 1986, **251**(1): 1–16

5 Hartemann F V, Brown W J, Gibson D J, Anderson S G, Tremaine A M, Springer P T, Wootton A J, Hartouni E P, Barty C P J. Physical Review Special Topics - Accelerators and Beams, 2005, **8**: 100702