# Application of the parametric formulae for electromagnetic showers in unconverted $\gamma / \pi^{0}$ discrimination ${ }^{*}$ 

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#### Abstract

In the LHC experiment，the neutral pions produced during jet fragmentation are the background sources for all physics channels with high－energy photons in their final state．In this paper，the application of the three－dimensional parametric formula for electromagnetic（EM）showers，which we developed in the Alpha Magnetic Spectrometer II experiment，is presented to distinguish the unconverted photons from the neutral pions．With the constructed electromagnetic calorimeter（ECAL）in a GEANT4 simulation，the parametric formulae were validated and the unconverted $\gamma / \pi^{0}$ discrimination was performed with the Toolkit for Mul－ tivariate Data Analysis（TMVA）package in ROOT for different transverse energies ranging from 15 GeV to 75 GeV ，which is the most sensitive region for light Higgs（with mass $\sim 120 \mathrm{GeV}$ ）searches with the channel $\mathrm{H} \rightarrow \gamma \gamma$ ．With this discrimination method and the selected transverse energy region，we can reject $\pi^{0}$ with the efficiency from $\sim 40 \%(65-75 \mathrm{GeV})$ to $\sim 90 \%(15-25 \mathrm{GeV})$ when keeping $90 \% \gamma$ efficiency．


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## 1 Introduction

In the LHC experiments，there are many neutral pions produced in jet fragmentation．If the photons from $\pi^{0}$ decay，$\pi^{0} \rightarrow \gamma \gamma$ ，are too close to each other， they can be detected as a single energy deposit and misinterpreted as a photon candidate．Therefore，the jets processed with neutral pions are an important background source for the analysis of high－energy photons in the final state，such as the Higgs search channel $\mathrm{H} \rightarrow \gamma \gamma$ ．The discrimination of photons and neutral pions is of great importance for many of the primary goals of the LHC physics program．

During the analysis of $\tau$ decay with one charged
particle and $\pi^{0}$＇s in the L3 experiment，the aver－ age transverse profile of hadronic and electromagnetic showers in the electromagnetic calorimeter as a func－ tion of energy and impact point were used to solve the problem of overlapping neutral and charged en－ ergy clusters to determine the number of photons and their energy［1］．For overlapping showers by two pho－ tons from $\pi^{0}$ decay，if we can find a standard formula to describe the EM shower，then we can split the overlapping clusters to get two standard EM showers． Alternatively，we can also use the difference，by com－ paring the values of the parameters using a standard EM shape formula to describe overlapping showers by two photons from $\pi^{0}$ and the ones for a shower by a

[^0]isolated photon with the same energy, to distinguish if the shower is produced by one photon or two photons for the purpose of $\gamma / \pi^{0}$ discrimination.

With the study of Alpha Magnetic Spectrometer II ECAL, we have developed an empirical formula to parameterize the 3-dimension (3D) distribution of electromagnetic showers [2]. In this paper, we apply the empirical formula in the discrimination of unconverted $\gamma / \pi^{0}$ with the help of the TMVA package.

In Section 2, we first give a general description of the parametric shower shape formulae. An ECAL is constructed and its geometry is described using the GEANT4 package in Section 3. The validation of the parametric shower shape formulae is described in Section 4 and its application to the discrimination of unconverted $\gamma / \pi^{0}$ in Section 5. Finally, the summary and conclusion is in Section 6.

## 2 Description of the parametric shower shape formulae

The spatial energy distribution of electromagnetic showers is given by a probability density function (PDF),

$$
\begin{equation*}
\mathrm{d} E(\vec{r})=E f(t, r, \phi) \mathrm{d} t \mathrm{~d} r \mathrm{~d} \phi \tag{1}
\end{equation*}
$$

to describe the shower developing in the longitudinal, radial, and azimuthal directions. Here, $E$ is the total energy of the particle, $t$ denotes the longitudinal shower depth in units of radiation length, $r$ measures the radial distance from the shower axis and $\phi$ is the azimuthal angle. In $\phi$, it is assumed that the energy is distributed uniformly. So the PDF in the azimuthal direction can be extracted as $f(\phi)=\frac{1}{2 \pi}$ from the spatial PDF $f(t, r, \phi)$.

The mean longitudinal profile of the energy deposition in an electromagnetic cascade is well described by $\Gamma$-function [3]:

$$
\begin{equation*}
\frac{1}{E} \frac{\mathrm{~d} E}{\mathrm{~d} t}=b \frac{(b t)^{a-1} \mathrm{e}^{-b t}}{\Gamma(a)} \tag{2}
\end{equation*}
$$

where the shower depth $t$ is measured in units of radiation lengths $X_{0}$. Parameters $a$ and $b$ are dependent on the atomic number $Z$ of the absorber and the incident energy of an electron or a photon. Parameters $a, b$ and the maximum shower depth $t_{\text {max }}$ have the relation $t_{\max }=\frac{a-1}{b}$.

The average lateral shower shape can be parameterized as the following normalized formula, which is the function of the distance of the shower developing point to the shower center in each layer, as we
described in Ref. [2],

$$
\begin{equation*}
f(r) \mathrm{d} r=\frac{6 R^{2}}{(r+R)^{4}} \cdot r \mathrm{~d} r \tag{3}
\end{equation*}
$$

Parameter $R$ can be parameterized as a function of shower depth and can be written as $R=$ $A\left(\frac{t}{t_{\max }}\right)^{2}$, where parameter $A$ is related to the incident particle energy. This expression is the empirical formula that we obtained in the ECAL study of Alpha Magnetic Spectrometer II experiment.

## 3 ECAL description

The main goal of LHC experiments is to search for the Higgs particle. The $\gamma / \pi^{0}$ discrimination will be very important in order to suppress the backgrounds from jet fragmentation for Higgs searches with the channel $\mathrm{H} \rightarrow \gamma \gamma$. To do that, we constructed an ECAL geometry as the ECAL barrel region of the CMS detector at the LHC [4] with the GEANT4 package.

A Schematic view of the electromagnetic calorimeter with barrel only can be found in Fig. 1. The electromagnetic calorimeter is a hermetic homogeneous calorimeter made of 61200 scintillating lead tungstate $\left(\mathrm{PbWO}_{4}\right)$ crystals. The high density $\left(8.28 \mathrm{~g} / \mathrm{cm}^{3}\right)$, short radiation length $(0.89 \mathrm{~cm})$ and small Molière radius $(2.2 \mathrm{~cm})$ result in a fine granularity and a compact calorimeter. The ECAL consists of 36


Fig. 1. Schematic view of the constructed calorimeter used in this paper. (a) Longitudinal section of the constructed ECAL (one quadrant); (b) Transverse view of the constructed ECAL.
supermodules, half of them in $z$ positive side, half in $z$ negative side, seen from the global detector center point. The Supermodules extend to a pseudorapidity range of $|\eta|<1.479$ and all have the same structure, being composed of $20(\phi) \times 85(\eta)=1700 \mathrm{PbWO}_{4}$ crystals divided in four modules.

The 61200 barrel crystals are 230 mm long, corresponding to 25.8 X 0 and $0.0174 \times 0.0174$ wide in $\eta$ and $\phi$ (both in unit of radian) or $22 \mathrm{~mm} \times 22 \mathrm{~mm}$ at the front face, and $26 \mathrm{~mm} \times 26 \mathrm{~mm}$ at the rear face. They are assembled in a quasi projective geometry, so that the angle between the crystal axis and the direction from the detector center point is 3 degrees both in $\eta$ and $\phi$ projections. The centers of the front faces of the crystals are at a radius of 1.29 m from the global coordinate $z$-axis. Just as in the CMS ECAL, the crystals are contained in a thinwalled alveolar structure (submodule). The alveolar wall is 0.1 mm thick and is made of an aluminium layer, facing the crystal, and two layers of glass-fibre epoxy resin. To avoid oxidation, a special coating is applied to the aluminium surface. The nominal crystal to crystal distance is 0.35 mm inside a submodule, and 0.5 mm between submodules. The submodules are assembled into modules of different types, according to the position in $\eta$, each containing 400 (module 2 , module 3 and module 4) or 500 (module 1) crystals. Four modules, separated by aluminium conical webs $4-\mathrm{mm}$ thick, are assembled in a supermodule, which contains 1700 crystals.

## 4 Validation of the parametric formulae of the electromagnetic shower

In this section, we will give the results of the val-
idation of our 3D-parameterized formula for the electromagnetic shower, which is a combination of formula (2) and formula (3) described in the second section.

### 4.1 Verification and parameter determination of the longitudinal profile

For the validation of the longitudinal profile of EM shower in the ECAL, as is described in the third section, the data from the GEANT4 simulation were used. Considering the ECAL geometry, we select a crystal close to the ECAL center $(\eta=0)$ in $\eta$ direction as the incident point. The incident particle hits almost the center of the front face of the crystal, which is the third crystal along the positive $\eta$ direction. The nearby crystals around the incident direction are almost perpendicular to the global coordinate $z$-axis. Along the ECAL longitudinal direction ( $R$ direction with $R=\sqrt{x^{2}+y^{2}}$ ), starting from the front face, the constructed ECAL is divided into 26 layers with about $1 \mathrm{X}_{0}$ each.

We simulated 9 energy points, $10,20,30,50,80$, $120,150,200$ and 250 GeV , for the single $\gamma$ samples. Each sample contains 10000 events. Fig. 2(a) shows that the $\Gamma$-function can well describe the longitudinal distributions of photon showers in ECAL. Fig. 2(b) and (c) show the distribution of the parameters $a$ and $b$ changing with different energy, $\lg y=\lg \left(E / E_{\mathrm{c}}\right)$, where $E$ is the incident energy and $E_{\mathrm{c}}$ is the critical energy with $E_{\mathrm{c}} \approx 8.74 \mathrm{MeV}$ for the constructed calorimeter. A second order polynomial function was used for the fitting in these 2 plots respectively. From the fitting results, we can obtain the values of parameters $a$ and $b$ for different energy points, then the values of the maximum shower depth $t_{\text {max }}$.


Fig. 2. Based on the GEANT4 simulated data, (a) The average longitudinal profile of the energy deposition can be well fitted using the $\Gamma$-function, taking the $30 \mathrm{GeV} \gamma$ sample as example. The relations of the parameter $a$ and parameter $b$ with the incident energy $\lg y=\lg \left(E / E_{\mathrm{c}}\right)$ are shown in (b) and (c), respectively, for $\gamma$ samples.

### 4.2 Data samples in lateral profile validation and shower position measurement

Considering the one-dimensional readout of each crystal, the validation of the lateral profile means the validation of the PDF of the whole spatial energy distribution. The data from the GEANT4 simulation were also used here. We simulated $\gamma$ samples with the incident point $(\eta=0.743351, \phi=0.0463416)$, which is the center of a crystal a little far away from the ECAL center, with 9 energy points from 10 GeV to 200 GeV and 20000 events for each sample.

A $5 \times 5$ crystals array was used to reconstruct an EM shower produced by a $\gamma$, because the $5 \times 5$ crystals array contain almost all of the energy deposit. For a $5 \times 5$ cluster, a simple position measurement of the shower center can be obtained by calculating the energy-weighted mean position of the crystals in the cluster [5]

$$
\begin{equation*}
x=\frac{\sum x_{i} \cdot W_{i}}{\sum W_{i}} \tag{4}
\end{equation*}
$$

where $x_{i}$ is the position of crystal $i$ and $W_{i}$ is the weight of the crystal defined as

$$
\begin{equation*}
W_{i}=W_{0}+\lg \frac{E_{i}}{\sum E_{j}} \tag{5}
\end{equation*}
$$

with the $\sum$ to sum up all in $5 \times 5$ crystals.
The position of the crystal $i$ with the length of shower maximum depth $t_{\text {max }}$ from the front face was used and the weight $W_{0}=4.61$ was used to exclude the lower energy hits with $E<0.01 \mathrm{GeV}$ and increase the relative weights of the central crystals in the calculation of the reconstructed position of a $5 \times 5$ cluster. Fig. 3 shows the difference between the reconstructed position and the incident position, $\Delta \eta=\eta_{\text {reconstructed }}-\eta_{\text {incident }}$ in $\eta$ direction and $\Delta \phi=\phi_{\text {reconstructed }}-\phi_{\text {incident }}$ in $\phi$ direction, for 50 GeV $\gamma$ samples. From the plots, a reconstructed position of a shower can be obtained with high precision using this method.

### 4.3 Validation of the parametric formulae

In order to see the average effect of the validation, we use the $\gamma$ samples which hit the same point in ECAL and are described in the previous subsection. The validation is based on the comparison between the deposit energy and the prediction by the formula in which the parameters are determined by MINUIT fitting [6]. The detailed procedure will be described as follows.

1) In the global detector system, we selected the $5 \times 5$ crystals array around the most energic crystal. Their position and energy deposit will be used in the


Fig. 3. The shower position calculation with energy-weighted logarithm. (a) The difference between the reconstructed position and the true one in $\eta$ direction; (b) The difference between the reconstructed position and the true one in $\phi$ direction.
analysis. For convenience, we numbered the crystals first along the $\phi$ direction and then along the $\eta$ direction, as cna be seen from Fig. 4(a).
2) As shown in Fig. 4(b), along the shower developing direction, starting from the front face, the ECAL is divided into 26 layers. The showerdeveloping direction can be obtained from the produced point of photons and the center of gravity (COG) of the shower, which can be calculated using the energy-weighted method described in the previous subsection. During the fitting procedure, the center of gravity of the shower can be changed and the calculated values from the energy-weighted method were taken as the initial values. Then we can obtain the energy fraction in each nominal layer, marked as $f_{\text {ilayer }}$, from the longitudinal formula (2). The parameters $a$
and $b$ in this formula can be calculated by their relations to the energy as described in the subsection 4.1. For EM showers, there is almost no longitudinal energy leakage with GeV scale. However, from the calculation of formula (2), the sum of the energy fraction in the total 26 layers should be less than 1, for the limited region used in the integral of the formula. Therefore, during the analysis, the energy fraction in each layer should multiply a factor, which is equal to 1 over the sum of the energy fraction in the total 26 layers calculated using the longitudinal formula.
3) The center of the shower in each layer is the crossing point of the shower developing direction with the medium plane of this layer. As shown in Fig. 4(b), the position of the center in each layer is $\left(x_{\mathrm{c}}, y_{\mathrm{c}}, z_{\mathrm{c}}\right)$. In the local system of each layer, the energy fraction can be calculated from the lateral formula (3) with the parameter $A$ changed during the fitting process. For different layers, the formula (3) has different values of the medium parameter $R$ but keeps the same value of parameter $A$. The method for the energy fraction calculation of each crystal in a layer is the same as that used in the L3 experiment. Starting from the center in a layer, we know that the energy distribution is isotropic, i.e., the same value for the same distance $r$ from the center. As shown in Fig. 4(c), the factor of the area the crystal intercepted with two adjacent circles $(\Delta S[j])$ over the area between the same two adjacent circles $\left(S[i]=\pi\left(r_{i+1}^{2}-r_{i}^{2}\right)\right.$ or $\pi r_{0}^{2}$ for the innermost circle), times the integral of the lateral formula (3) in the area between the same two adjacent circles (with radiuses $r_{i}$ and $r_{i+1}$ ), will give the energy fraction in the area the crystal intercepted with these two adjacent circles,

$$
f_{j}=\frac{\Delta S[j]}{S[i]} \times \int_{r_{i}}^{r_{i+1}} f(r) \mathrm{d} r .
$$

For each crystal, from the first crossing circle to the last crossing circle, the sum of the energy fraction in each area interacted with two adjacent circles will give the energy fraction of the crystal in each layer, which can be expressed as

$$
\begin{align*}
f_{\text {ilayer }}^{j \text { crystal }}= & \sum_{\text {FirstCrossingCircle }}^{\text {LastCrossingCircle }} \\
f_{j}= & \sum_{\text {FirstCrossingCircle }}^{\text {LastCrossingCircle }}\left(\frac{\Delta S[j]}{S[i]} \times \int_{r_{i}}^{r_{i+1}} f(r) \mathrm{d} r\right) \tag{6}
\end{align*}
$$

The more circles there are, the more accurate the results. In order to save computer CPU time, considering the energy concentrated on the central several
crystals, the radius or the difference on the radiuses between two adjacent circles is much too small for the inner circles, 0.2 mm was used. The value was changed to several mm for the outer circles. Enough number of circles were used in order to contain all crystals in the $5 \times 5$ array.
4) Then, for each crystal, the sum of the energy fraction in each layer multiplies the energy weight in each layer which is equal to the longitudinal energy fraction calculated in Step 2) $\left(\sum_{\text {ilayer=1 }}^{26} f_{i \text { layer }}^{j \text { crystal }} \times\right.$ $f_{i \text { layer }}$ ), then multiplies the total shower energy $(E)$ will give the predicted energy if the parameters are well tuned during the fitting procedure. The predicted energy for each crystal can be expressed as

$$
\begin{align*}
E_{j \text { crystal }}= & E \times \sum_{i \text { layer }=1}^{26}\left(f_{i \text { layer }} \times f_{\text {ilayer }}^{j \text { crystal }}\right)=E \\
& \times \sum_{i \text { layer }=1}^{26}\left(f_{i \text { layer }} \times \sum_{\text {FirstCrossingCircle }}^{\text {LastCrossingCircle }}\left(\frac{\Delta S[j]}{S[i]}\right.\right. \\
& \left.\left.\times \int_{r_{i}}^{r_{i+1}} f(r) \mathrm{d} r\right)\right) \tag{7}
\end{align*}
$$

5) Considering the gaps between crystals and the integral of the lateral formula on the $5 \times 5$ crystals array in each layer less than 1 , the total energy was treated as a variable parameter during the procedure. So, in total, there are 4 parameters that are needed to be tuned during the fitting procedure, two from

| crystal (or cell) number |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 10 15 20 25 <br> 4 9 14 19 24 <br> 3 8 13 18 23 <br> 2 7 12 17 22 <br> 1 6 11 16 21 <br> $\eta$     |  |  |  |  |

$5 \times 5$ crystals array
(a)


Fig. 4. (a) The numbering of the $5 \times 5$ crystals array; (b) One nominal layer of ECAL along the shower developing direction; (c) Schematic view of the area calculation. All the circles have the same center $r=0$. The quadrangle represents a crystal in the $5 \times 5$ crystals array. $\Delta S[j]$ is the area the crystal intercepted with two adjacent circles and $S[i]$ is the total area between the same two adjacent circles.
the center of gravity $\left(\eta_{c}, \phi_{c}\right)$ (in fact the bias from the calculated values of COG, $\Delta \eta$ and $\Delta \phi$ ), parameter $A$ in the lateral formula and the total fitted energy $E_{0}$. The MINUIT fitting procedure is to minimize the un-weighted $\chi^{2}$ with the expression $\chi^{2}=\sum_{i \text { crystal }=1}^{25}\left(E_{\text {deposit }}-E_{\text {predicted }}\right)^{2}$ as used in Ref. [2], where $E_{\text {deposit }}$ is the deposit energy in crystal, $E_{\text {predicted }}$ is the predicted energy in this crystal and $\sum_{i \text { crystal }=1}^{25}$ means the sum of all 25 crystals in the $5 \times 5$ crystals array.

With the $\gamma$ samples described in the previous subsection, the fitting results are shown in Fig. 5. In
these plots, the $y$-axis represents the averaged energy in each crystal with the whole statistic and $x$-axis is the crystal number described in Fig. 4(a). From Fig. 5(a), we can see that the predicted energy in each crystal is consistent with the original deposit energy. For the same energy sample, the distribution of parameter $A$ in the lateral formula is well Gaussian, seen from Fig. 5(b) for $50 \mathrm{GeV} \gamma$ samples. The fitting result for equally weighted $\chi^{2}$ is also shown in Fig. 5(c). The relation between parameter $A$ and the total deposit energy in the array is shown in Fig. 5(d), and the fitting function is $A=p_{0} \lg y+p_{1}$.


Fig. 5. (a) Comparisons of the average predicted energy with the average original deposit energy in each crystal of the $5 \times 5$ crystals array for $50 \mathrm{GeV} \gamma$ samples. The dots represent the average original deposit energy in each crystal and the histogram represents the predicted one; (b) Fitting results of parameter $A$ and (c) $\chi^{2}$ for $50 \mathrm{GeV} \gamma$ samples, and (d) The relation between parameter $A$ and the total deposit energy $\lg y=\lg \left(E / E_{\mathrm{c}}\right)$ in the array, are also shown here.

## 5 Application in the unconverted $\gamma / \pi^{0}$ discrimination

In this section, we will give one of the applications of the parametric shower shape method: its application in the $\gamma / \pi^{0}$ discrimination. The overlapping showers in the ECAL from $\pi^{0} \rightarrow \gamma \gamma$ should be wider
than the shower of single photon with the same energy. In this section, we will use the fitting parameters in the formulae, which can describe the shower produced by one real photon well, to the discrimination. During the analysis, the TMVA package in ROOT will be used [7].

We simulated single $\gamma$ and $\pi^{0}$ samples uniformly
in the whole constructed ECAL with different transverse momentum. $6 E_{\mathrm{T}}$ bins were used, $15-25 \mathrm{GeV}$, $25-35 \mathrm{GeV}, 35-45 \mathrm{GeV}, 45-55 \mathrm{GeV}, 55-65 \mathrm{GeV}$ and $65-75 \mathrm{GeV}$, which are the most interesting $E_{\mathrm{T}}$ ranges for standard model Higgs analysis with $\mathrm{H} \rightarrow \gamma \gamma$. Each sample contains 200000 single particle events. After the simulation, we selected the most energic cluster as the reconstructed $\gamma$ or $\pi^{0}$ candidates. The particles can hit the gaps between crystals. So we required that the energy should be compatible with the $E_{\mathrm{T}}$ ranges. For $\gamma$ samples with $E_{\mathrm{T}} 65-75 \mathrm{GeV}, 2$ of the total 100 subsamples were excluded. For $\pi^{0}$ samples with $E_{\mathrm{T}} 65-75 \mathrm{GeV}, 5$ of the total 100 subsamples were excluded. Finally, the total number of selected candidates are listed in Table 1.

Then, with these samples, we performed our fitting procedure as described step by step in the above section on each $5 \times 5$ cluster, to obtain the fitted values of the parameters and also the $\chi^{2}$. With the parameters, we performed the training and testing analysis with the TMVA package, half of the events
used for training and the rest for testing. With the fitting results using the formulae, 5 variables were considered as inputs for TMVA, including the parameter $A$ in the lateral formula, bias on the COG ( $\Delta \eta$ and $\Delta \phi$ ), the fraction of the difference between the total fitted energy $E_{0}$ and the original deposit one $\left(E_{0}^{\text {fitted }}-E_{0}^{\text {deposit }}\right) / E_{0}^{\text {deposit }}$, and the normalized $\chi^{2}$, $\sqrt{\chi^{2}} / E_{0}^{\text {deposit }}$.

Table 1. Number of reconstructed $\gamma / \pi^{0}$ candidates after selections in each $E_{\mathrm{T}}$ bin.

| $E_{\mathrm{T}} / \mathrm{GeV}$ | $\gamma$ events | $\pi^{0}$ events |
| :---: | :---: | :---: |
| $15-25$ | 196140 | 196679 |
| $25-35$ | 195457 | 196056 |
| $35-45$ | 194934 | 195519 |
| $45-55$ | 194577 | 195140 |
| $55-65$ | 194291 | 194839 |
| $65-75$ | 190227 | 184794 |

Figure 6 shows the comparisons of the inputs for TMVA of the single $\gamma$ and $\pi^{0}$ samples with $E_{\mathrm{T}}$ ranging from 35 GeV to 45 GeV .


Fig. 6. Comparisons of the inputs for TMVA of the single $\gamma$ and $\pi^{0}$ samples with $E_{\mathrm{T}}$ ranging from 35 GeV to 45 GeV . (a) Parameter $A$; (b) $\Delta \eta$ and (c) $\Delta \phi$ from the COG; (d) The fraction of the difference between the total fitted energy $E_{0}$ and the original deposit one, $\left(E_{0}^{\text {fitted }}-E_{0}^{\text {deposit }}\right) / E_{0}^{\text {deposit }}$, and (e) The normalized $\chi^{2}, \sqrt{\chi^{2}} / E_{0}^{\text {deposit }}$.

During the TMVA, 2 methods or classifiers were used at the same time, including the multilayer perceptrons (MLP) and boosted decision tree (BDT). MLP is a newly developed neural network that is faster and more flexible than the traditional artificial neural network and is the recommended neural network to use for TMVA. A decision (regression) tree is a binary tree structured classifier. Repeated left/right (yes/no) decisions are taken on one single variable at a time until a stop criterion is fulfilled. The phase space is split in this way into many regions that are eventually classified as signal or background, depending on the majority of training events that end up in the final leaf node. The boosting of a decision (regression) tree extends this concept from one tree to several trees, which then form a forest. Boosting stabilizes the response of the decision trees with respect to fluctuations in the training sample and is able to considerably enhance the performance w.r.t. a single tree. More detailed descriptions about these classifiers can be found in Ref. [7].

Figure 7 shows the results of TMVA of the single $\gamma$ and $\pi^{0}$ samples with $E_{\mathrm{T}}$ ranging from 35 GeV to

45 GeV . Fig. 7(a) shows the responses for both the training sample and testing sample from the MLP method. Fig. 7(b) shows the responses from the BDT method. We also checked that there is no overflow or underflow problem for both methods. For both methods, the response from the testing sample is consistent with the one from the training sample. The background rejection changes with the signal efficiency from the testing samples and is shown in Fig. 7(c). If $90 \%$ signal $(\gamma)$ efficiency is kept, the rejection of background $\left(\pi^{0}\right)$ is about $71.7 \%$ for the MLP method and $74.4 \%$ for the BDT method. We also checked the convergence of the MLP method, the result is shown in Fig. 7(d). For both methods, the signal efficiency and purity are shown in Fig. 7(e) and Fig. 7(f) respectively. The background efficiency and the significance, $S / \sqrt{S+B}$, are also shown in the plots.

With the TMVA analysis for various $E_{\mathrm{T}}$ bins, the background rejections from the test samples are listed in Table 2, if $90 \% \gamma$ efficiency is kept. The two classifiers have the similar $\pi^{0}$ rejection for the same $E_{\mathrm{T}}$ bin. However, the BDT method can give more robust results.


Fig. 7. Results from TMVA of the single $\gamma$ and $\pi^{0}$ samples with $E_{\mathrm{T}}$ ranging from 35 GeV to 45 GeV . (a) The response from the MLP classifier; (b) The response from the BDT classifier; (c) The background rejection versus the signal efficiency; (d) Convergence check for MLP classifier; (e) Cut efficiency versus the output of the MLP classifier and (f) Cut efficiency versus the output of the BDT classifier, are shown here.

Table 2. $\quad \pi^{0}$ rejection if $90 \% \gamma$ efficiency is kept in each $E_{\mathrm{T}}$ bin with the MLP and BDT methods.

| $E_{\mathrm{T}} / \mathrm{GeV}$ | BDT | MLP |
| :---: | :---: | :---: |
| $15-25$ | $92.8 \% \pm 0.1 \%$ | $92.8 \% \pm 0.1 \%$ |
| $25-35$ | $85.8 \% \pm 0.1 \%$ | $85.0 \% \pm 0.1 \%$ |
| $35-45$ | $74.4 \% \pm 0.1 \%$ | $71.1 \% \pm 0.1 \%$ |
| $45-55$ | $64.0 \% \pm 0.1 \%$ | $60.2 \% \pm 0.1 \%$ |
| $55-65$ | $53.4 \% \pm 0.1 \%$ | $49.5 \% \pm 0.1 \%$ |
| $65-75$ | $45.9 \% \pm 0.1 \%$ | $42.8 \% \pm 0.1 \%$ |

## 6 Summary and conclusion

In this paper, we provided an application of the three-dimensional parametric formulae of the EM shower in unconverted $\gamma / \pi^{0}$ discrimination. Firstly, we constructed an ECAL detector and described its
geometry. With the GEANT4 simulated data, we validated the formula both in the longitudinal and lateral directions. The results show that the combined formulae can describe the EM shower well in the constructed ECAL. With the fitting results from the formulae on the $5 \times 5$ cluster produced by one $\gamma$ and two $\gamma$ s from $\pi^{0}$ decay, and with the TMVA package in ROOT, we performed the analysis of unconverted $\gamma / \pi^{0}$ discrimination. Even with a transverse momentum higher to $65-75 \mathrm{GeV}$, we can also reject more than $40 \% \pi^{0}$ when keeping $90 \% \gamma$. From the analysis results, this method can help a lot in the photonrelated analysis, such as the light Higgs (with mass $\sim 120 \mathrm{GeV}$ ) searches with $\mathrm{H} \rightarrow \gamma \gamma$ in the LHC experiments, by suppressing the jet backgrounds which contain a lot of neutral pions, in a wider transverse momentum region.

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