Low emittance lattice optimization using a multi-objective evolutionary algorithm*

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Abstract: A low emittance lattice design and optimization procedure are systematically studied with a non-dominated sorting-based multi-objective evolutionary algorithm which not only globally searches the low emittance lattice, but also optimizes some beam quantities such as betatron tunes, momentum compaction factor and dispersion function simultaneously. In this paper the detailed algorithm and lattice design procedure are presented. The Hefei light source upgrade project storage ring lattice, with fixed magnet layout, is designed to illustrate this optimization procedure.

Key words: emittance, multi-objective evolutionary algorithm, lattice

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1 Introduction

Usually, a low emittance lattice is needed for high brightness synchrotron light source and damping ring. Thanks to our predecessors, the theoretical minimum emittance of different types of lattice structure has been well established. This is a guideline for realistic lattice design, which not only helps the designers to understand the limit of achievable low emittance, but also gives a criterion of the optimal solution which has been achieved. The minimum natural emittance of different types of lattice structure is given as the following [1]:

$$\epsilon_x = \mathscr{F}_{\min} C_{\rm q} \gamma^2 \theta^3, \tag{1}$$

where $\mathscr{F}_{\min}$ is a numerical factor based on different lattice types, γ is the Lorentz factor and θ is the total dipole bending angle in a bend section. For an electron storage ring lattice, it is clear that the weaker $\mathscr{F}_{\min}$ is, the smaller the emittance will be. However, designing a lattice to reach its theoretical minimum emittance is hard to be implemented, because additional criteria and constraint, in general, make it comprehensive. In particular, the theoretical minimum emittance requires different dipole lengths or magnet fields between the middle dipole and the outer dipole [2].

Thus, the numerical method is preferred to match the optical functions for attaining a minimum emittance lattice which satisfies appropriate beam qualities. Presently, the lattice design codes, such as MAD, OPA and ELEGANT are widely used. The critical problem of designing the lattice with these codes is that they will consume the designers' lots of time to adjust it repeatedly and what's more, it is based on the designers' intuition and experiences. Although the designers could get a satisfactory structure, it is impossible for them to judge whether the ultimate performance is reached. In order to achieve satisfactory beam qualities, both theoretical analysis and numerical calculation are needed and the design procedure is also very complicated.

Fortunately, MOEA (multi-objective evolutionary algorithm) is a simple, but effective method, which has received great attention regarding their potential as optimization method for complex problems. It is a randomly searching algorithm that globally searches a set of solutions over a domain without detailed understanding of the problem. This algorithm is extremely suitable for solving the questions which are nonlinear,

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discontinuous, with conflicting objectives, many local optimal and several decision variables. As it is known to all, these features coincide with the lattice design problem. Most importantly, it can optimize several objectives simultaneously, such as emittance, betatron tunes, momentum compaction factor, dispersion function, etc..

MOEA has been introduced into the damping ring [3] and storage ring lattice [4] optimization in recent years. It overcomes the difficulties of lattice design process by using theoretical minimum emittance or matching information. Simultaneously, this algorithm provides a database for lattice designers to choose a preferred solution, thus settles the high computational complexity.

In this paper, we use non-dominated sorting-based multi-objective evolutionary algorithm (called non-dominated sorting genetic algorithm-Ⅱ or NSGA-Ⅱ) [5] to optimize the linear optics of a lattice, while keeping the emittance as low as possible. The lattice optimization code developed by this paper is based on the concept of NSGA-Ⅱ.

The contents of this article are organized as follows. In Section 2, the detailed algorithm of this optimization code is given. For a better understanding of this methodology, two optimization examples, with different number of objectives, constraints and variables, are systematically studied in Section 3. The first example gives a low emittance lattice optimization problem with minimum vertical betatron function in the long insertion device (ID) section. The second example tries to designs a high-low betatron function mode for EPU (elliptical polarized undulator) installation.

2 Multi-objective evolutionary algorithm

In real life, the optimization objectives always conflict with each other. It is hard to get a reasonable solution by a single objective genetic algorithm, because a solution which is best on one objective will probably lead to an unacceptable result with respect to other objectives. Alternatively, MOEA gives a set of solutions which are superior to the rest of solutions when considering all of the objectives. They are called Pareto-optimal solutions or Pareto-optimal front.

During 1993–1995, a number of different multi-objective evolutionary algorithms were suggested, because multi-objective modeling is well fitted for many realistic problems. By comparing several tens of MOEA, the non-dominated sorting genetic algorithm (NSGA) aroused much attention. This algorithm was proposed by Srinivas and Deb in 1994. Later, the non-dominated sorting-based multi-objective evolutionary algorithm (NSGA-Ⅱ) [6], upgraded version of NSGA, was developed. It overcame the following difficulties of NSGA: high computational complexity, non-elitist and the need for specifying sharing parameters. In this paper we applied the idea of NSGA-Ⅱ to linear lattice optics optimization problem. The detailed procedures are outlined as follows:

Step 1: An initial population $\mathcal{P}_0$, with a population number N, is created and the individuals are sorted based on their non-domination.

Step 2: Generate a child population $\mathcal{Q}_0$ from initial population by using selection, crossover and mutation.

Step 3: If the stopping criterion is not satisfied, the following main loop is repeated.

1) Merge parent population $\mathcal{P}_{t-1}$ and child population $\mathcal{Q}_{t-1}$, $\mathcal{R}_t=\mathcal{P}_{t-1}\bigcup\mathcal{Q}_{t-1}$.

2) Sort the individuals of the merged population $\mathcal{R}_t$, based on their rank (fitness assignment) and crowding distance. In this step the individual of lower rank is better than the higher one. If two individuals have the same rank the one with larger crowding distance is better. The best N individuals of the merged population are chosen to make up new parent population $\mathcal{P}_t$.

3) Use tournament selection, crossing and mutating to generate a child population $\mathcal{Q}_t$ from parent population $\mathcal{P}_t$.

4) Increase the generation counter $t=t+1$.

As it is well known to all, lattice optimization is a complex problem with many local optimals, many constraints, many variables, especially when the betatron tunes are constrained to a fixed number, the area of feasible solutions will be divided into a large amount of discontinuous regions. However, such a complex problem can be optimized by NSGA-Ⅱ. In this algorithm, constraints are handled without any penalty functions. We define that all feasible solutions have a better rank than infeasible solutions, two feasible solutions are sorted based on their objective functions, for both infeasible solutions the better one is chosen according to the constraint violation (the lower the better), which is calculated from the sum of the every equality constraints and inequality constraints.

3 Low emittance lattice optimization

The HLSⅡ (Hefei light source upgrade project) storage ring, with a circumference of 66 m, is a separate focusing lattice. The operating energy is 800 MeV. There are four double bend cells and each cell is composed of eight quadrupoles. Since all magnet locations have been fixed, the focusing and defocusing quadrupole strengths can be viewed as variables during the optimization process. The following two optimization problems of this section are based on the HLS Ⅱ lattice.

3.1 Low emittance with low betatron function in ID section

Modern light sources expect to achieve high brightness for user requirements, in particular, achromatic mode for insertion devices. The brightness of a light source is defined as Eq. (2), which is the photon flux per unit solid angle and unit area emitted in a relative bandwidth. In this equation, $\mathrm{d}\Omega\mathrm{d}S$ is proportional to the transverse beam emittance $\epsilon_x\epsilon_y$ approximately. Thus the beam emittance must be minimized to achieve maximum spectral photon beam brightness [7]. In addition, smaller vertical betatron function at the ID section is expected to get optimum photon beam brightness from planar undulator.

$$B=\frac{\mathrm{d}^4N_{\mathrm{ph}}}{\mathrm{d}t\mathrm{d}\Omega\mathrm{d}S(\mathrm{d}\lambda/\lambda)}. \tag{2}$$

To give a clear explanation of the low emittance lattice design procedure, the expression of horizontal equilibrium emittance for an electron storage ring is also given in the following Eq. (3):

$$\epsilon_x=\frac{C_{\mathrm{q}}\gamma^2\langle\mathcal{H}\rangle_{\mathrm{dipole}}}{\mathcal{J}_x\rho}, \tag{3}$$

where ρ is the bending radius, γ is the total energy in mc^2 units, $\mathcal{J}_x$ is the horizontal damping partition number, $C_{\mathrm{q}}=3.83\times10^{-13}$ m and

$$\langle\mathcal{H}\rangle=\frac{1}{2\pi\rho}\oint_{\mathrm{diploe}}(\gamma_x\eta_x^2+2\alpha_x\eta_x\eta_x'+\beta_x\eta_x'^2)\mathrm{d}s.$$

After having the two equations, the issue of low emittance lattice design procedure becomes evident. It is clear that the horizontal emittance is determined by $\langle\mathcal{H}\rangle$ function, thus a possible emittance suppression method by varying the quadrupole strengths to optimize twiss parameters in dipole magnets is straightforward.

The HLS Ⅱ storage ring has eight quadrupoles in each cell, which are symmetric about midpoint and compose four families of variables. The emittance can be minimized by NSGA-Ⅱ and the four families of quadrupole strength are the independent variables of this algorithm. From both the cost and physical points of view, each quadrupole strength is limited in the following range: $-5<Q_{\mathrm{i}}[\mathrm{K1}]<5$.

There are two objectives, the emittance and β_y (at the ID section), to be minimized in the chromatic mode. In achromatic mode, there are three objectives to be minimized, who are the emittance, β_y (at the ID section) and the absolute value of dispersion function (at the ID section). Some interesting optical parameters should meet the designing constraints. They are listed as follows:

1. $\beta_{x,y}(\max)<35$ m.
2. $\eta_x(\max)<1.5$ m.
3. Horizontal tune $Q_x=4.4$ and permissible error is ±0.05.
4. Vertical tune $Q_y=3.2$ and permissible error is ±0.05.

During the optimization procedure, the evolving individuals of every generation are subjected to the stable region of both planes, which means $|\mathrm{trace}(M_{x,y})|<2$. Because the unstable point is of no use to the lattice design, clearly, the initial population can be viewed as stable points of the lattice.

An appropriate crossover rate and a mutational rate are critical for the convergence. A large crossover rate will speed up the convergence but probably a local optimal, while too large mutational rates will lead to non convergence. Nevertheless, the quality of the optimal front is mainly determined by the population size. In this problem we use a population with 1500 individuals and a maximum generation is 50. The crossover rate 0.60 and mutational rate 0.01 are selected. Pareto-optimal solutions of the two modes are plotted in the objectives space, as shown in Figs. 1 and Fig. 2.

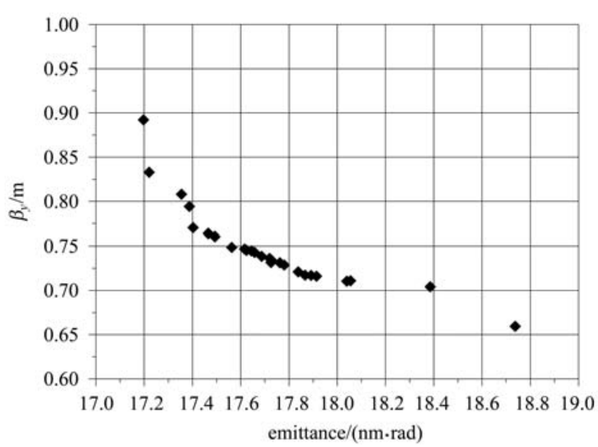

Fig. 1. The distribution of Pareto-optimal solutions in objective space (chromatic mode), the two objectives are non-dominated for each other. The horizontal axis is emittance and the vertical axis is another objective β_y (in the middle of the ID section).

Actually the quadrupole strengths of achromatic mode are roughly a subset of chromatic mode, because the achromatic mode has three objectives and two of them are the same as the chromatic mode. If one objective (dispersion function at the ID section) of the achromatic mode is eliminated, the optimization problem becomes the chromatic mode. Fig. 2 shows the Pareto-optimal front for the achromatic mode.

The standard optical functions of the pareto-optimal front, obtained by this code, are plotted in Fig. 3 and Fig. 4. The betatron function and dispersion function of the chromatic mode are shown in Fig. 3 and the optics of achromatic mode are shown in Fig. 4. The emittance of chromatic mode is about 17 nm·rad and 35 nm·rad for the achromatic mode, which is 1.7 and 1.18 times the theoretical minimum emittance of each structure. If the constraints of tunes are eliminated, the emittance can be further minimized, probably 14 nm·rad for the chromatic mode.

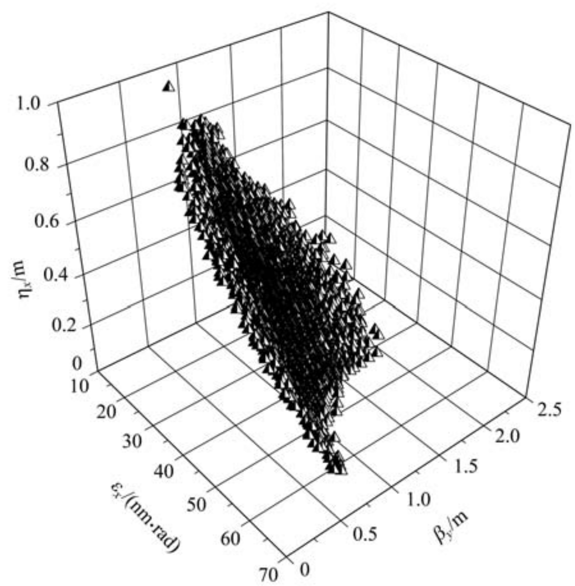

Fig. 2. The distribution of Pareto-optimal solutions in objective space (achromatic mode), the three objectives are the emittance, the vertical betatron function (at the ID section) and the dispersion function (at the ID section).

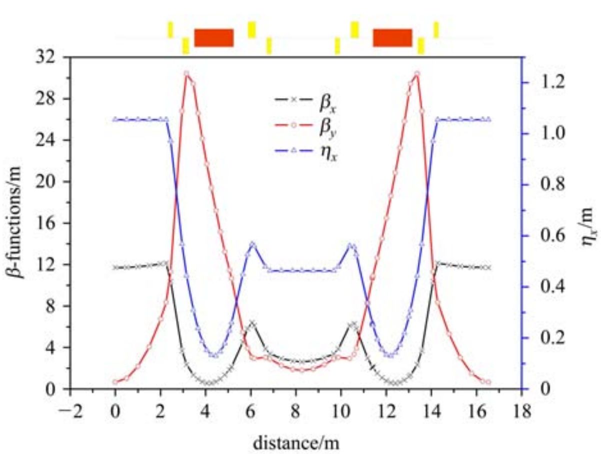

Fig. 3. (color online) The standard betatron functions of the Pareto-optimal solutions (chromatic mode). The emittance of this lattice is 17.52 nm·rad.

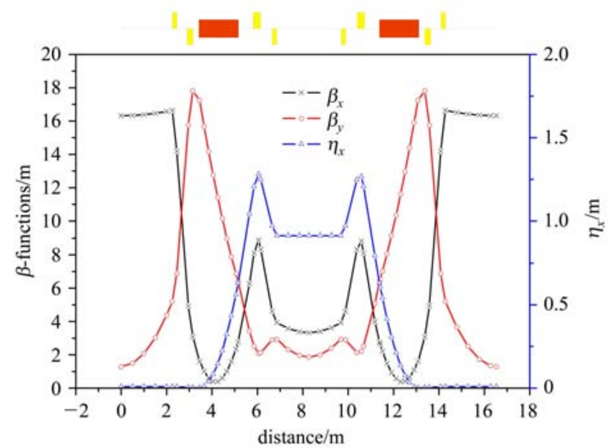

Fig. 4. (color online) The standard betatron functions of the Pareto-optimal solutions (achromatic mode). The emittance of this lattice is 35.78 nm·rad.

3.2 Low emittance for EPU installation

The influence of betatron motion over undulator brightness is an important issue for high brightness light source lattice designers. Since the harmonic spectrum of undulator depends on the betatron function matching condition, smaller betatron function at the ID section is preferred [8]. Especially, installation of elliptical polarized undulator (EPU), low betatron functions on both the horizontal and the vertical planes are desirable to minimize its effects on tune shifts and Beta-beating.

In the following, we use the NSGA-Ⅱ to optimize a high-low betatron function mode for installing EPU. In this example, the four cells of double bend structure lattice form two super-period cells, with 8 families quadrupoles whose locations are retained the same as the above example. Consider the injection procedure need for large horizontal betatron function and the EPU need for small betatron function in two directions, the lattice is designed to obtain high horizontal betatron function for injection in one straight section and the other straight section is designed to get low $\beta_{x,y}$ for installing EPU. The 8 families quadrupole strengths are the variables and the polarity is kept the same as Example 1, while minimizing the emittance and dispersion function (at one of the ID sections) are two objectives. The six inequality constrains are listed as follows:

1. $\beta_{x,y}(\text{max}) < 45$ m.
2. $\eta_x(\text{max}) < 3.0$ m.
3. $\beta_{x,y}(\text{EPU}) < 6$ m.
4. $|\eta_x(\text{EPU})| < 0.1$ m.
5. $\beta_x(\text{injection}) > 10$ m.
6. $\beta_y(\text{injection}) < 6$ m.

The feasible solutions of last generation are plotted on objective space, see Fig. 5.

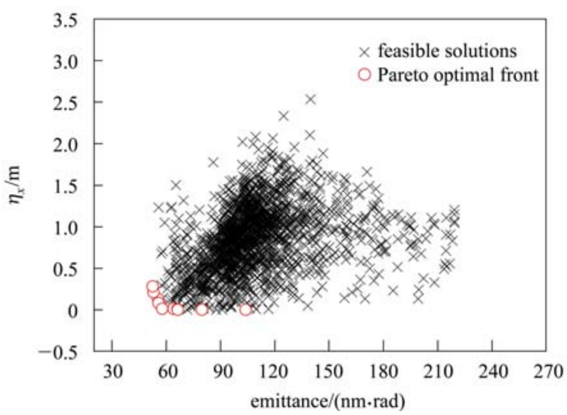

Fig. 5. (color online) The feasible solutions and Pareto-optimal front obtained by a population with 8000 individuals, plotted on the objective space. The variable number is 8 and the constraint number is 6. The points with prod symbols are the feasible solutions and the circle dots are the optimal solutions.

With the variables and constraints increased, we choose a population with 8000 individuals to optimize this two super periods lattice. After 50 evolving generations, we obtained the Pareto-optimal front, as shown in the above Fig. 5. The prod points are the feasible solutions which satisfy the above 6 constraints and the circle dots are the Pareto-optimal solutions, which are traded off between the two objectives. All of the Pareto-optimal solutions are no worse than the feasible solutions and each one of the Pareto-solutions are non dominated by each other. As can be seen from the figure, the Pareto-optimal solutions are uniformly distributed in the boundary of the feasible region.

The standard horizontal betatron function and dispersion function of the Pareto-optimal solutions with different emittance value are given in Fig. 6. To give a comparison of twofold symmetric and fourfold symmetric lattice properties, the betatron function of four super periods is also drawn in Fig. 6 (the line with solid square symbol) and the others are those solutions obtained from two super periods. The different symbol line corresponds to different emittance value, which ranges from 50 nm·rad to 100 nm·rad.

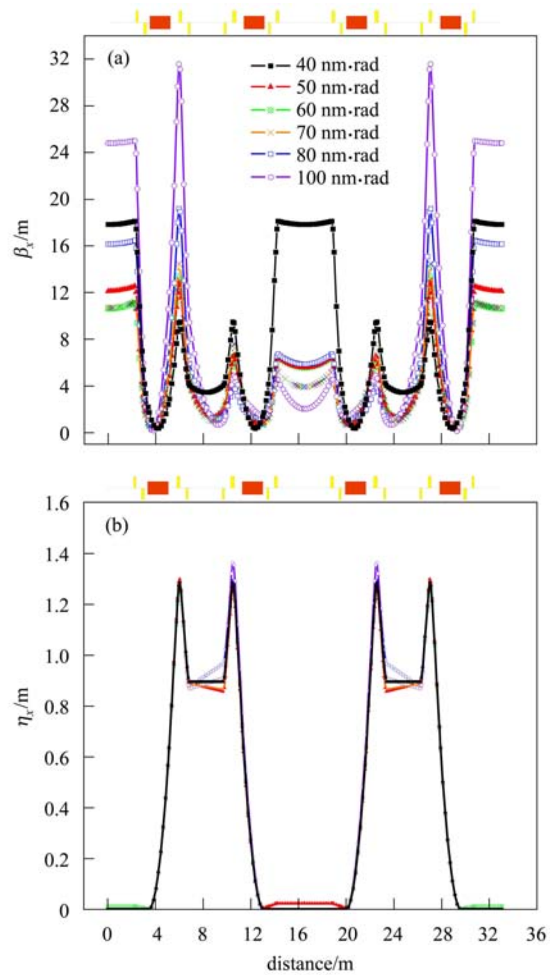

Fig. 6. (color online) The optical functions of HLS II lattice with two super periods and 8 families of quadrupoles in each cell. (a) is the horizontal betatron function and (b) is the dispersion function. The the line with solid square symbol is the twiss function of four super periods (obtained by Example 1) and the other lines are the twiss function of two super periods obtained by this example.

From Fig. 6(a), it is evident that, for attaining low betatron functions of both planes at the EPU straight section, the horizontal betatron function at the second and third dipole magnets is increased, while maintaining a small value at the two outer dipoles. In Fig. 6(b), the maximum dispersion functions at dipoles are very similar to the four super period cases. It is possible that the emittance increase is mainly caused by the increase of horizontal betatron function at the second and third dipoles. These data give a guidance for lattice designers where to place additional quadrupoles for further minimizing the emittance. The result of this code not only gives satisfactory beam qualities with a fixed magnet structure, but also provides a message for designers to arrange magnet location by analyzing the solutions in detail.

In this algorithm, the objectives and constraint should be changed for each other. For example, the betatron functions at EPU can be considered as objectives and also the objectives can be changed to constraints. This is the flexibility aspect of MOEA.

4 Conclusion

This paper presents an optimization code, which is based on the idea of NSGA-Ⅱ to optimize the low emittance lattice optics. First of all, the detailed algorithm is given. Then, two examples, with four and eight variables, lots of constraints and some objectives, are optimized. The result given by this algorithm is a set of solutions, globally searched from variable spaces, which satisfy all of the constraints and trade off between each of the optimizing objectives. Therefore we get a small emittance lattice which satisfies all of the beam performances. Due to the excellent property of this algorithm, such as high computational capability, no need for understanding the solving problem, robust and inherently parallel, we can extend its application to the very complicated nonlinear optimization problem in an accelerator design.

References

1 Lee S Y, Teng L. Theroretical Minimum Emittance Lattice for an Electron Storage Ring. In: Proceedings of 1991 Particle Accelerator Conference. San Francisco, California, 1991. 2679–2681

2 Lee S Y. Phys. Rev. E, 1996, **54**: 1940–1944

3 Emery L. Global Optimization of Damping Ring Using a Muti-objective Evolutionary Algorithm. In: Proceedings of 2005 Particle Accelerator Conference. Knoxville, Tennessee, 2005. 2962–2964

4 YANG L, Robin D, Sannibale F, Steier C. Global Optimization of the Magnetic Lattice Using Genetic Algorithm. In: Proceedings of the 2008 European Particle Accelerator Conference. Genoa, Italy, 2008. 3050–3052

5 Kalyanmoy Deb. IEEE Trans Evol Comput, 2002, **6**: 182–197

6 Konak A, Coit D W, Smith A E. Reliability Engineering and System Safety, 2006, **91**: 992–1007

7 Lee S Y. Accelerator Physics. Second Edition. Singapore: World Scientific, 2004. 466–488

8 Dattoli G, Voykov G K, Carpanese M. Phys. Rev. E, 1995, **52**: 6809–6817