Study of Robinson instabilities with a higher-harmonic cavity for the HLS phase Ⅱ project*

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Abstract: In the Phase Ⅱ Project at the Hefei Light Source, a fourth-harmonic "Landau" cavity will be operated in order to suppress the coupled-bunch instabilities and increase the beam lifetime of the Hefei storage ring. Instabilities limit the utility of the higher-harmonic cavity when the storage ring is operated with a small momentum compaction. Analytical modeling and simulations show that the instabilities result from Robinson mode coupling. In the analytic modeling, we operate an algorithm to consider the Robinson instabilities. To study the evolution of unstable behavior, simulations have been performed in which macroparticles are distributed among the buckets. Both the analytic modeling and simulations agree for passive operation of the harmonic cavity.

Key words: Robinson instability, higher-harmonic cavity, analytic modeling, Landau damping rate, simulation, energy spread

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1 Introduction

To increase the beam lifetime, a fourth-harmonic cavity will be operated in the Phase Ⅱ Project at the Hefei Light Source (HLS). In a low or medium energy storage ring, the lifetime is dominated by Touschek scattering. Theory and experience have indicated that adding a higher harmonic RF system can effectively increase the Touschek lifetime and meanwhile will not compromise the transverse beam brightness. For this reason, a higher harmonic cavity will be used to increase the beam lifetime and suppress coupled-bunch instabilities in the HLS Ⅱ.

We will use a fourth-harmonic cavity and it will be operated in passive mode, where its voltage is induced by the beam current. In this case, some negative effects such as Robinson instabilities, which are coupled-bunch instabilities where all bunches oscillate in unison, should be avoided. We have predicted the instabilities by analytical modeling and studied the evolution of unstable behaviour by performing simulations.

In this paper, we firstly operate an algorithm in the analytic modeling to consider Robinson instabilities [1–3] and we also discuss the procedures in detail [2, 4]. Secondly, simulations are performed in which macroparticles are distributed among the buckets to study the evolution of unstable behaviour. Comparing these two results, we obtain good agreement between analytic modeling and simulations.

2 Analytic modeling

For HLS Ⅱ, we use the parameters [5] shown in Table 1. The harmonic cavity impedance and Q factor are estimated respectively for the 4th harmonic cavities. We have modified an algorithm to consider Robinson instabilities for a given fundamental rf peak voltage V_{T1}, ring current I, and harmonic cavity tuning angle ϕ_2.

We initially set the bunch form factors $F_1=1$ and $F_2=0.1$, and iterate until they are consistent with the computed bunch length. Let Cavity 1 be the fundamental rf cavity and Cavity 2 be the higher harmonic cavity. We operate the algorithm as follows.

1) Calculate the synchronous phase of the funda-

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mental rf cavity Ψ_1, using the equation

$$V_S = V_{T1}\cos\Psi_1 + V_{T2}\cos\Psi_2 = V_{T1}\cos\Psi_1 - 2IR_2F_2\cos^2\phi_2. \quad (1)$$

R_2 is the resonant impedance for Cavity 2.

2) Calculate the tuning angle of Cavity 1 ϕ_1, using the equation

$$\phi_1 = \tan^{-1}(2F_1IR_1\sin\Psi_1/V_{T1}). \quad (2)$$

Table 1. The machine parameters for the Hefei Light Source Ⅱ Project.

parameter	value
beam energy/GeV	0.8
beam revolution frequency/MHz	4.533
number of bunches	45
synchronous voltage/kV	16.73
natural relative energy spread	0.00047
fundamental rf angular frequency/MHz	204
fundamental cavity shunt impedance/MΩ	3.3
fundamental quality factor	28000
fundamental cavity coupling coefficient	2
harmonic-cavity harmonic number	4
harmonic-cavity shunt impedance/MΩ	2.5
harmonic-cavity quality factor	18000
harmonic-cavity coupling coefficient	0
momentum compaction	0.02
fundamental rf peak voltage/kV	250
harmonic frequency/MHz	816
radiation-damping time constant/ms	10
HOM resonant angular frequency/MHz	1000
HOM resonant impedance/kΩ	10
HOM quality factor	3000

3) Calculate the coefficients of the Taylor expansion of the synchrotron potential

$$U(\tau) = a\tau^2 + b\tau^3 + c\tau^4, \quad (3)$$

using Eqs. (4)–(6) and τ is the arrival time of a single electron relative to the synchronous time.

$$a = \frac{\alpha e\omega_g}{2ET_0}(V_{T1}\sin\Psi_1 + \nu V_{T2}\sin\Psi_2). \quad (4)$$

$$b = \frac{\alpha e\omega_g^2}{6ET_0}(V_{T1}\cos\Psi_1 + \nu^2 V_{T2}\cos\Psi_2). \quad (5)$$

$$c = \frac{\alpha e\omega_g^3}{6ET_0}(V_{T1}\sin\Psi_1 + \nu^3 V_{T2}\sin\Psi_2). \quad (6)$$

ω_g is the fundamental rf angular frequency, ν is the harmonic number, α is the momentum compaction, E is the beam energy and T_0 is the storage ring recirculation time. For a passive Landau cavity,

$$V_{T2}\cos\Psi_2 = -2IF_2R_2\cos^2\phi_2, \quad (7)$$

$$V_{T2}\sin\Psi_2 = IF_2R_2\sin 2\phi_2. \quad (8)$$

4) Calculate the bunch length σ_t, which obeys $\sigma_t^2 = \langle\tau^2\rangle - \langle\tau\rangle^2 \approx \langle\tau^2\rangle$, where

$$\langle\tau^n\rangle = \frac{\int \tau^n \exp[-U(\tau)/2U_0]d\tau}{\int \exp[-U(\tau)/2U_0]d\tau}, \quad (9)$$

and $U_0 = \alpha^2(\sigma_E/E)^2/2$, σ_E is the natural electron energy spread.

5) Calculate the form factors, using $F_1 = \exp(-\omega_g^2\sigma_t^2/2)$ and $F_2 = \exp(-\nu^2\omega_g^2\sigma_t^2/2)$. We repeat steps 1) –5) until the form factors are consistent with the bunch length.

6) Calculate the frequency of collective dipole oscillations ω_R, which approximately obeys

$$\omega_R^2 = \frac{\alpha e\omega_g}{ET_0}(F_1V_{T1}\sin\Psi_1 + \nu F_2V_{T2}\sin\Psi_2). \quad (10)$$

Then we estimate whether the dipole longitudinal coupled-bunch instability is damped for resonant interaction with a parasitic longitudinal cavity mode of impedance $Z(\omega_{CB})$ at approximate frequency ω_{CB}, which is the angular frequency of the parasitic mode. If $\Delta\Omega_{CB} - \tau_L^{-1} > |\Delta\Omega|_{thresh}$, we consider that Landau damping is not sufficient to prevent instability. $\Delta\Omega_{CB}$ is the complex frequency shift, given by

$$\Delta\Omega_{CB} = \frac{eI\alpha\omega_{CB}Z(\omega_{CB})F_{\omega_{CB}}^2}{2ET_0\omega_R}, \quad (11)$$

where $F_{\omega_{CB}}$ is the bunch form factor at frequency ω_{CB}. τ_L^{-1} is the radiation damping rate and $|\Delta\Omega|_{thresh}$ is the dipole Landau damping rate which is given by

$$|\Delta\Omega|_{thresh} = 0.78\frac{\alpha^2(\sigma_E/E)^2}{\omega_R}\left|\frac{3c}{\omega_R^2} - \left(\frac{3b}{\omega_R^2}\right)^2\right|. \quad (12)$$

7) Now we consider the Robinson instabilities. Firstly, we analyze the Robinson instability without considering mode coupling. If the zero-frequency instability is not predicted, we calculate the complex frequency shift and the Robinson damping rate. When the Robinson damping rate is negative and the complex frequency shift is greater than the Landau damping rate, the Robinson instability will happen.

8) Next, to include the effects of dipole-quadrupole mode coupling, we compute the coupled-dipole and the coupled-quadrupole Robinson frequency and complex frequency shift, which include mode coupling. If the zero-frequency coupled instability is not predicted, we use a coupled mode criterion in which we compare the complex frequency shift with the Landau damping rate to estimate whether the Landau damping is overcome. When the Robinson damping rate is negative and the complex frequency shift is

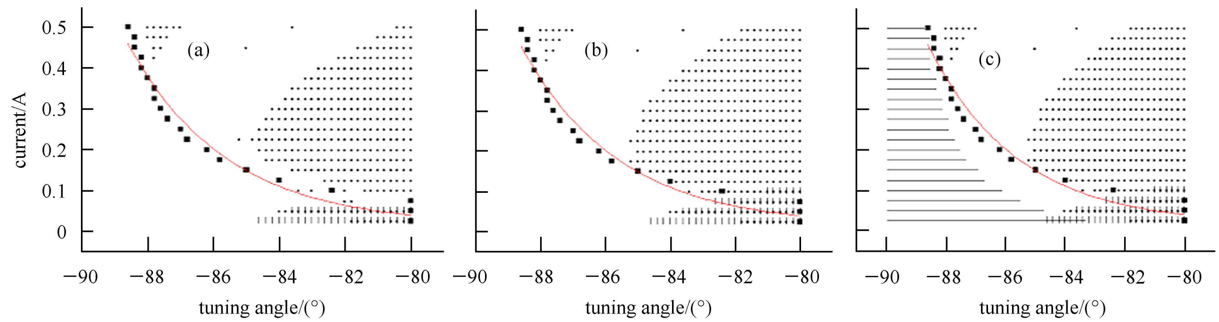

Fig. 1. (a) Robinson instabilities are predicted without consideration of the mode coupling. The solid curve shows the parameters for optimal bunch lengthening, in which case the linear synchrotron frequency is zero. Vertical line: dipole instability; spot: quadrupole instability. (b) Dipole-quadrupole mode coupling is included. Vertical line: coupled-dipole instability; spot: coupled-quadrupole instability. (c) Coupled bunch instability is included. Horizontal line: coupled bunch instability.

greater than the Landau damping rate, coupled mode Robinson instability will happen.

The analytical results are shown in Fig. 1.

In Fig. 1(a), the uncoupled dipole and quadrupole Robinson instabilities are predicted. When tuning in the cavity with currents below 80 mA, the onset of dipole Robinson instability is predicted before optimal bunch lengthening is attained. For currents exceeding 100 mA, the onset of a quadrupole Robinson instability is predicted occur after optimal bunch lengthening is obtained. For currents below 100 mA, a dipole Robinson instability is predicted to occur before optimal bunch lengthening is obtained.

In Fig. 1(b), the dipole-quadrupole mode coupling is included in the analysis. Compared with Fig. 1(a), we conclude that when the cavity is tuned with currents near 100 mA and optimal bunch lengthening is obtained, the onset of coupled-dipole instability and coupled-quadrupole instability are predicted.

In Fig. 1(c), the coupled bunch instability is included. We see that when the tuning angle is from $-88.5°$ to $-89.5°$ and the current is below the current value of the optimal bunch lengthening, coupled bunch instability is predicted to be excited intensely by the harmonic cavity.

3 Simulations

We have performed 500000-turn simulations for 20 values of the ring current and 50 values of the harmonic-cavity tuning angle to study the evolution of unstable behavior. In our simulations, 450 macroparticles are evenly distributed among the 45 buckets. Fig. 2 shows the simulation results in which ■ is plotted for mild instability, ● is plotted for moderate instability, and ▲ is plotted for strong instability.

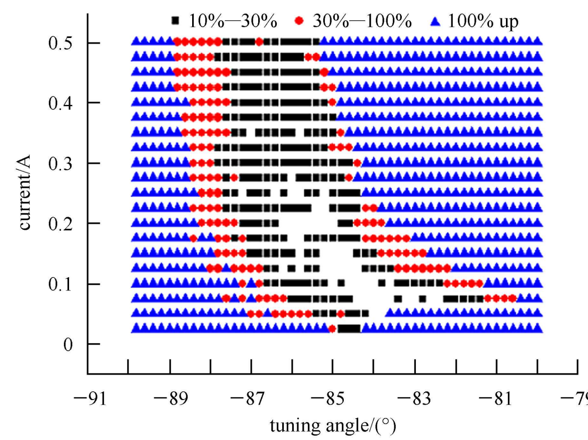

Fig. 2. Results of 500000-turn simulations of 450 macroparticles. ■: mild instability, where the energy spread exceeds its natural value by (10–30)%; ●: moderate instability, where the energy spread has increased by (30–100)%; ▲: strong instability, where the energy spread has increased more than 100%.

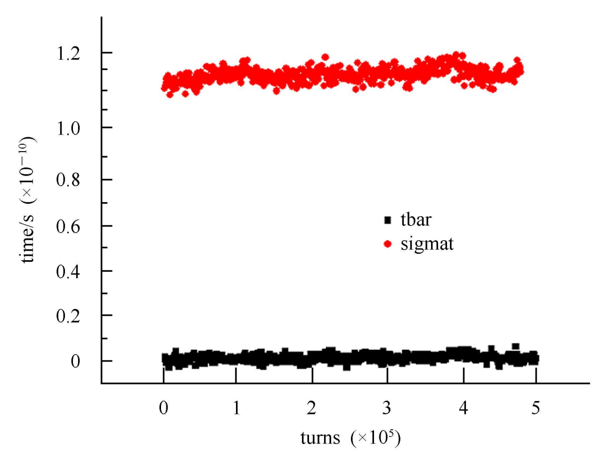

Fig. 3. Simulation of coupled-quadrupole Robinson instability for a ring current of 125 mA and a Landau-cavity tuning angle of $-79.8°$. ● : bunch length σ_t, ■ : bunch centroid $\langle t\rangle$.

There is good agreement between the analytic predictions of Fig. 1(b) and the simulated instabilities observed in Fig. 2. According to the simulations, stable optimally lengthened bunches are obtained for ring currents of 80–500 mA. The results show that tuning in the harmonic cavity strongly suppresses the parasitic coupled-bunch instability.

The simulated instability growth and saturation for a current of 125 mA and harmonic-cavity tuning angle $-79.8°$ are shown in Fig. 3. The amplitude of σ_{t} oscillations is much greater than that of the beam centroid, which is consistent with the analytic prediction of a coupled-quadrupole instability.

4 Conclusion and discussion

We have studied the Robinson instabilities with a higher harmonic cavity using analytic modelling and simulations. In our analytic model, we estimate the currents and tuning angles at the onset of instability and the parameters for optimal bunch lengthening are also obtained. In our simulations, the results confirm that tuning in the harmonic cavity strongly suppresses the parasitic coupled-bunch instability. There is good agreement between the analytic predictions and the simulated instabilities.

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