A note on the B*B̄, B*B̄*, D*D̄, D*D̄* molecular states [*]

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Abstract: In the framework of the one-boson-exchange model, we have performed an extensive study of the possible B*B̄, B*B̄*, D*D̄, D*D̄* molecular states with various quantum numbers after considering the S-wave and D-wave mixing. We also discuss the possible experimental research of these interesting states.

Key words: the one-boson-exchange model, molecular state

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1 Introduction

The study of a molecular state composed of heavy flavor mesons has been an interesting and important research topic. In 1976, Voloshin and Okun first studied molecular systems with charmed quarks [1]. In Ref. [2], De Rujula, Georgi and Glashow proposed $\psi(4040)$ as a D*D̄* molecular charmonium. By using the quark-pion interaction model, Tönqvist carried out the study of DD̄* and D*D̄* [3, 4]. The observations of X(3872), three charged charomonium-like states $Z^+(4350)$, $Z_1^+(4050)$, $Z^+(4250)$ and Y(4140), Y(4274) etc. again inspired theorists' interest in the molecular system composed of a charmed meson pair (see Refs. [5–35] for details).

The newly observed charged bottomonium-like structures $Z_b(10610)$ and $Z_b(10650)$ were reported by the Belle collaboration [36]. Before these observations of Z_b, the analysis presented in Refs. [31, 32] indicated that there probably exists a loosely bound S-wave BB̄* molecular state. The observed $Z_b(10610)$ gives direct support to the prediction in Refs. [31,

32] since $Z_b(10610)$ is near the BB̄* threshold and with $I^G(J^P) = 1^+(1^+)$. After the observations of $Z_b(10610)$ and $Z_b(10650)$, many theoretical groups carried out the study to reveal the underlying structure of $Z_b(10610)$ and $Z_b(10650)$ [37–44]. Among different explanations of $Z_b(10610)$ and $Z_b(10650)$, BB̄* and B*B̄* molecular state assignments to $Z_b(10610)$ and $Z_b(10650)$ respectively are popular. In our recent work [45], we have studied the interaction of the BB̄* and B*B̄* systems in the framework of the one-boson-exchange (OBE) model, where we consider the S-wave and D-wave mixing. Our result indicates that $Z_b(10610)$ and $Z_b(10650)$ can be explained as the BB̄* and B*B̄* molecular states quite naturally [45].

Besides the study of these BB̄* and B*B̄* systems directly corresponding to $Z_b(10610)$ and $Z_b(10650)$ [45], a more comprehensive and systematic study of BB̄* and B*B̄* systems is necessary to provide more valuable information for future experimental searches for these exotic states. Thus, by using the same technique as that proposed in Ref. [45], in this work we perform the systematical dynamical calculation of

BB̄* and B*B̄* systems. Additionally, we also extend the formulism to study the DD̄* and D*D̄* systems, which can be relevant to many charmonium-like states such as X(3872), Y(3930).

This paper is organized as follows. After the introduction, we present the formalism of the study of the BB̄* and B*B̄* systems, which includes the relevant effective Lagrangian and coupling constants, the derivation of the effective potential of the BB̄* and B*B̄* system, the numerical results, etc. Finally, the paper ends with the discussion and conclusion.

2 Deduction of effective potential

2.1 Flavor wave function

We list the flavor wave functions of the BB̄* and B*B̄* systems constructed in Refs. [34, 35]. The BB̄* systems can be categorized as the isovector and isoscalar states with the corresponding flavor wave functions

$$
\left\{
\begin{aligned}
|Z_{B\bar{B}^*}^{(T)\,+}\rangle &= \frac{1}{\sqrt{2}}\big(|B^{*+}\bar{B}^0\rangle + c|B^+\bar{B}^{*0}\rangle\big), \\
|Z_{B\bar{B}^*}^{(T)\,-}\rangle &= \frac{1}{\sqrt{2}}\big(|B^{*-}\bar{B}^0\rangle + c|B^-\bar{B}^{*0}\rangle\big), \\
|Z_{B\bar{B}^*}^{(T)\,0}\rangle &= \frac{1}{2}\big[\big(|B^{*+}B^-\rangle - |B^{*0}\bar{B}^0\rangle\big) \\
&\quad + c\big(|B^+B^{*-}\rangle - |B^0\bar{B}^{*0}\rangle\big)\big],
\end{aligned}
\right.
\tag{1}
$$

$$
\begin{aligned}
|Z_{B\bar{B}^*}^{(S)\,0}\rangle &= \frac{1}{2}[(|B^{*+}B^-\rangle + |B^{*0}\bar{B}^0\rangle) \\
&\quad + c|(B^+B^{*-}\rangle + |B^0\bar{B}^{*0}\rangle)],
\end{aligned}
\tag{2}
$$

where $c = \pm$ corresponds to C-parity $C = \mp$ respectively [34, 35]. The flavor wave functions of the B*B̄* systems can be constructed as

$$
\left\{
\begin{aligned}
|Z_{B^*\bar{B}^*}^{(T)}[J]^+\rangle &= |B^{*+}\bar{B}^{*0}\rangle, \\
|Z_{B^*\bar{B}^*}^{(T)}[J]^-\rangle &= |B^{*-}\bar{B}^{*0}\rangle, \\
|Z_{B^*\bar{B}^*}^{(T)}[J]^0\rangle &= \frac{1}{\sqrt{2}}\big(|B^{*+}B^{*-}\rangle - |B^{*0}\bar{B}^{*0}\rangle\big)
\end{aligned}
\right.
\tag{3}
$$

for the isovector states, and

$$
\left|Z_{B^*\bar{B}^*}^{(S)}[J]^0\right\rangle = \frac{1}{\sqrt{2}}\big(|B^{*+}B^{*-}\rangle + |B^{*0}\bar{B}^{*0}\rangle\big),
\tag{4}
$$

for the isoscalar state. In the above expressions, the superscripts T and S in Eqs. (1)–(4) are applied to distinguish the isovector and isoscalar states, respectively. The total angular momentum of the S-wave B*B̄* systems is J=0, 1, 2. Thus, we use the extra notation $[J]$ in Eqs. (3)–(4) to distinguish the B*B̄* systems with different total angular momentum J.

Belle indicates that both $Z_b(10610)$ and $Z_b(10650)$ belong to the isotriplet states. If $Z_b(10610)$ and $Z_b(10650)$ are the BB̄* or B*B̄* molecular states respectively, they should correspond to $Z_{B\bar{B}^*}^{(T)}$ and $Z_{B^*\bar{B}^*}^{(T)}$ [1] in Eqs. (1) and (3), respectively. Since $Z_b(10610)^0$ is of C-odd parity, i.e., $C = -1$, thus the coefficient $c = +1$ is taken in Eq. (1). The choice of the coefficient $c = -1$ and $C = +1$ leads to X(3872) and its partners, where X(3872) corresponds to $Z_{D\bar{D}^*}^{(S)\,'}$ listed in Table. 1.

In Table 1, we summarize the quantum numbers of the states when we discuss whether there exist the BB̄* and B*B̄* molecular states. Moreover, we extend the same formalism to study the DD̄* and D*D̄* systems, where the flavor wave function of the DD̄* and D*D̄* systems can be obtained with replacement $B^{(*)} \to \bar{D}^{(*)}$ and $\bar{B}^{(*)} \to D^{(*)}$.

Table 1. A summary of the BB̄*, B*B̄*, DD̄*, D*D̄* systems. If taking $c = -1$ in Eqs. (1) and (2), we obtain the flavor wave functions of $Z_{B\bar{B}^*}^{(T)\,'}$ and $Z_{B\bar{B}^*}^{(S)\,'}$, which are the partners of $Z_{B\bar{B}^*}^{(T)}$ and $Z_{B\bar{B}^*}^{(S)}$ respectively.

BB̄*/B*B̄*	DD̄*/D*D̄*	$I^G(J^{PC})$
$Z_{B\bar{B}^*}^{(T)}$	$Z_{D\bar{D}^*}^{(T)}$	$1^+(1^+)$
$Z_{B\bar{B}^*}^{(S)}$	$Z_{D\bar{D}^*}^{(S)}$	$0^-(1^{+-})$
$Z_{B^*\bar{B}^*}^{(T)}[J]$	$Z_{D^*\bar{D}^*}^{(T)}[J]$	$1^-(0^+),1^-(2^+),1^+(1^+)$
$Z_{B^*\bar{B}^*}^{(S)}[J]$	$Z_{D^*\bar{D}^*}^{(S)}[J]$	$0^+(0^{++}),0^+(2^{++}),0^-(1^{+-})$
$Z_{B\bar{B}^*}^{(T)\,'}$	$Z_{D\bar{D}^*}^{(T)\,'}$	$1^-(1^+)$
$Z_{B\bar{B}^*}^{(S)\,'}$	$Z_{D\bar{D}^*}^{(S)\,'}$	$0^+(1^{++})$

2.2 Effective Lagrangian and coupling constant

In the frame work of the OBE model, we study the interaction of BB̄* or B*B̄* system. Thus, we adopt the effective Lagrangian approach. Thus, the effective Lagrangians relevant to our calculation include [24, 46–51]

$$
\begin{aligned}
\mathcal{L}_{HH\mathbb{P}} &= \mathrm{i}g\langle H_b^{(Q)}\gamma_\mu A_{ba}^\mu \gamma_5 \bar{H}_a^{(Q)}\rangle \\
&\quad + \mathrm{i}g\langle \bar{H}_a^{(\bar{Q})}\gamma_\mu A_{ab}^\mu \gamma_5 H_b^{(\bar{Q})}\rangle,
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\mathcal{L}_{HHV} &= \mathrm{i}\beta\langle H_b^{(Q)}v_\mu(\mathcal{V}_{ba}^\mu - \rho_{ba}^\mu)\bar{H}_a^{(Q)}\rangle \\
&\quad + \mathrm{i}\lambda\langle H_b^{(Q)}\sigma_{\mu\nu}F^{\mu\nu}(\rho)\bar{H}_a^{(Q)}\rangle \\
&\quad - \mathrm{i}\beta\langle \bar{H}_a^{(\bar{Q})}v_\mu(\mathcal{V}_{ab}^\mu - \rho_{ab}^\mu)H_b^{(\bar{Q})}\rangle \\
&\quad + \mathrm{i}\lambda\langle H_b^{(\bar{Q})}\sigma_{\mu\nu}F'^{\mu\nu}(\rho)\bar{H}_a^{(\bar{Q})}\rangle,
\end{aligned}
\tag{6}
$$

$$
\mathcal{L}_{HH\sigma} = g_s\langle H_a^{(Q)}\sigma\bar{H}_a^{(Q)}\rangle + g_s\langle \bar{H}_a^{(\bar{Q})}\sigma H_a^{(\bar{Q})}\rangle,
\tag{7}
$$

where the multiplet field $H^{(Q)}$ is composed of the pseudoscalar $\mathcal{P}$ and vector $\mathcal{P}^*$ with $\mathcal{P}^{(*)T} = (D^{(*)0}, D^{(*)+})$ or $(B^{(*)-}, \bar{B}^{(*)0})$. And $H^{(Q)}$ and $\bar{H}^{(Q)}$ are defined by

$$H_a^{(Q)} = \frac{1+\slashed{v}}{2}[\mathcal{P}_{a\mu}^*\gamma^\mu - \mathcal{P}_a\gamma_5], \tag{8}$$

$$\bar{H}_a^{(Q)} = [\mathcal{P}_{a\mu}^{*\dagger}\gamma^\mu + \mathcal{P}_a^\dagger\gamma_5]\frac{1+\slashed{v}}{2}. \tag{9}$$

Here, $\bar{H} = \gamma_0 H^\dagger\gamma_0$ and $v = (1,0)$.

As given in Refs. [24, 52], the anti-charmed or bottom meson fields $\widetilde{\mathcal{P}}^{(*)T} = (\bar{D}^{(*)0}, D^{(*)-})$ or $(B^{(*)+}, B^{(*)0})$ satisfy

$$\widetilde{P}_{a\mu}^* = -\mathcal{C}P_{a\mu}^*\mathcal{C}^{-1}, \quad \widetilde{P}_a = \mathcal{C}P_a\mathcal{C}^{-1}. \tag{10}$$

The multiplet field $H^{(\bar{Q})}$ with the heavy antiquark can be defined as

$$H_a^{(\bar{Q})} = C(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1})^T C^{-1}$$

$$= [\widetilde{P}_a^{*\mu}\gamma_\mu - \widetilde{P}_a\gamma_5]\frac{1-\slashed{v}}{2}, \tag{11}$$

$$\bar{H}_a^{(\bar{Q})} = \frac{1-\slashed{v}}{2}[\widetilde{P}_a^{*\mu}\gamma_\mu + \widetilde{P}_a\gamma_5]. \tag{12}$$

If considering the following charge conjugation transformation,

$$\mathcal{C}\xi\mathcal{C}^{-1} = \xi^T, \mathcal{C}\mathcal{V}_\mu\mathcal{C}^{-1} = -\mathcal{V}_\mu^T,$$

$$\mathcal{C}\mathcal{A}_\mu\mathcal{C}^{-1} = \mathcal{A}_\mu^T, \mathcal{C}\rho^\mu\mathcal{C}^{-1} = -\rho^{\mu T}, \tag{13}$$

one obtains the Lagrangian relevant to the mesons with heavy antiquark $\bar{Q}$, which is converted from the one related to the meson with heavy quark Q, where the Lagrangians are given in Eqs. (5)–(7) [24, 52]. In the above expressions, the $\mathcal{P}(\widetilde{\mathcal{P}})$ and $\mathcal{P}^*(\widetilde{\mathcal{P}}^*)$ satisfy the normalization relations $\langle 0|\mathcal{P}|Q\bar{q}(0^-)\rangle = \langle 0|\widetilde{\mathcal{P}}|\bar{Q}q(0^-)\rangle = \sqrt{M_\mathcal{P}}$ and $\langle 0|\mathcal{P}_\mu^*|Q\bar{q}(1^-)\rangle = \langle 0|\widetilde{\mathcal{P}}_\mu^*|\bar{Q}q(1^-)\rangle = \epsilon_\mu\sqrt{M_{\mathcal{P}^*}}$. The axial current is $A^\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) = \frac{i}{f_\pi}\partial_\mu\mathbb{P} + \cdots$ with $\xi = \exp(i\mathbb{P}/f_\pi)$ and $f_\pi = 132$ MeV. $\rho_{ba}^\mu = ig_V\mathbb{V}_{ba}^\mu/\sqrt{2}$, $F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + [\rho_\mu, \rho_\nu]$, $F'_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu - [\rho_\mu, \rho_\nu]$ and $g_V = m_\rho/f_\pi$, with $g_V = 5.8$. Here, $\mathbb{P}$ and $\mathbb{V}$ are two by two pseudoscalar and vector matrices

$$\mathbb{P} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{\eta}{\sqrt{6}} & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{\eta}{\sqrt{6}} \end{pmatrix}, \tag{14}$$

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \end{pmatrix}. \tag{15}$$

By expanding Eqs. (5)–(7), one further obtains the effective Lagrangian of the light pseudoscalar mesons $\mathbb{P}$ with the heavy flavor mesons

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{P}} = -i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\lambda\dagger}\partial^\nu\mathbb{P}_{ba}$$

$$+i\frac{2g}{f_\pi}\varepsilon_{\alpha\mu\nu\lambda}v^\alpha\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\lambda}\partial^\nu\mathbb{P}_{ab}, \tag{16}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{P}} = -\frac{2g}{f_\pi}(\mathcal{P}_b\mathcal{P}_{a\lambda}^{*\dagger} + \mathcal{P}_{b\lambda}^*\mathcal{P}_a^\dagger)\partial^\lambda\mathbb{P}_{ba}$$

$$+\frac{2g}{f_\pi}(\widetilde{\mathcal{P}}_{a\lambda}^{*\dagger}\widetilde{\mathcal{P}}_b + \widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_{b\lambda}^*)\partial^\lambda\mathbb{P}_{ab}. \tag{17}$$

The effective Lagrangian depicting the coupling of the light vector mesons $\mathbb{V}$ and heavy flavor mesons reads as

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\mathbb{V}} = -\sqrt{2}\beta g_V\mathcal{P}_b\mathcal{P}_a^\dagger v\cdot\mathbb{V}_{ba}$$

$$+\sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b v\cdot\mathbb{V}_{ab}, \tag{18}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}\mathbb{V}} = -2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\mathcal{P}_b\mathcal{P}_a^{*\mu\dagger}$$

$$+\mathcal{P}_b^{*\mu}\mathcal{P}_a^\dagger)(\partial^\alpha\mathbb{V}^\beta)_{ba}$$

$$-2\sqrt{2}\lambda g_V v^\lambda\varepsilon_{\lambda\mu\alpha\beta}(\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b$$

$$+\widetilde{\mathcal{P}}_a^\dagger\widetilde{\mathcal{P}}_b^{*\mu})(\partial^\alpha\mathbb{V}^\beta)_{ab}, \tag{19}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\mathbb{V}} = \sqrt{2}\beta g_V\mathcal{P}_b^*\cdot\mathcal{P}_a^{*\dagger}v\cdot\mathbb{V}_{ba}$$

$$-i2\sqrt{2}\lambda g_V\mathcal{P}_b^{*\mu}\mathcal{P}_a^{*\nu\dagger}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ba}$$

$$-\sqrt{2}\beta g_V\widetilde{\mathcal{P}}_a^{*\dagger}\cdot\widetilde{\mathcal{P}}_b^*v\cdot\mathbb{V}_{ab}$$

$$-i2\sqrt{2}\lambda g_V\widetilde{\mathcal{P}}_a^{*\mu\dagger}\widetilde{\mathcal{P}}_b^{*\nu}(\partial_\mu\mathbb{V}_\nu - \partial_\nu\mathbb{V}_\mu)_{ab}. \tag{20}$$

The effective Lagrangian of the scalar meson σ interacting with the heavy flavor mesons can be expressed as

$$\mathcal{L}_{\mathcal{P}\mathcal{P}\sigma} = -2g_s\mathcal{P}_b\mathcal{P}_b^\dagger\sigma - 2g_s\widetilde{\mathcal{P}}_b\widetilde{\mathcal{P}}_b^\dagger\sigma, \tag{21}$$

$$\mathcal{L}_{\mathcal{P}^*\mathcal{P}^*\sigma} = 2g_s\mathcal{P}_b^*\cdot\mathcal{P}_b^{*\dagger}\sigma + 2g_s\widetilde{\mathcal{P}}_b^*\cdot\widetilde{\mathcal{P}}_b^{*\dagger}\sigma. \tag{22}$$

As shown in Eqs. (16)–(20), the terms for the interactions between the anti-heavy flavor mesons and light mesons can be obtained by taking the following replacements in the corresponding terms for the interactions between the heavy flavor mesons and light mesons:

$$v \to -v, a \to b, b \to a,$$
$$\mathcal{P}_\mu^* \to \widetilde{\mathcal{P}}_\mu^{*\dagger}, \mathcal{P} \to -\widetilde{\mathcal{P}}^\dagger,$$
$$\mathcal{P}_\mu^{*\dagger} \to \widetilde{\mathcal{P}}_\mu^*, \mathcal{P}^\dagger \to -\widetilde{\mathcal{P}}.$$

$g = 0.59$ is extracted from the experimental width of D^{*+} [53]. The parameter β relevant to the vector meson can be fixed as $\beta = 0.9$ by the vector meson dominance mechanism while $\lambda = 0.56$ GeV^{-1} was obtained by comparing the form factor calculated by the light cone sum rule with the one obtained by lattice QCD. As the coupling constant related to the scalar meson σ, $g_s = g_\pi/(2\sqrt{6})$ with $g_\pi = 3.73$ was given in Refs. [35, 51].

2.3   Effective potential

With the above preparation, we deduce the effective potentials of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems in the following. Generally, the scattering amplitude $i\mathcal{M}(J, J_z)$ is related to the interaction potential in the momentum space in terms of the Breit approximation

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\boldsymbol{q}) = -\frac{\mathcal{M}(B^{(*)}\bar{B}^{(*)} \to B^{(*)}\bar{B}^{(*)})}{\sqrt{\prod_i 2M_i \prod_f 2M_f}},$$

where M_i and M_j denote the masses of the initial and final states respectively. The potential in the coordinate space $\mathcal{V}(\boldsymbol{r})$ is obtained after performing the Fourier transformation

$$\mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\boldsymbol{r}) = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} e^{i\boldsymbol{p}\cdot\boldsymbol{r}} \mathcal{V}_E^{B^{(*)}\bar{B}^{(*)}}(\boldsymbol{q})\mathcal{F}^2(q^2, m_E^2), \tag{23}$$

where we need to introduce the monopole form factor (FF) $\mathcal{F}(q^2, m_E^2) = (\Lambda^2 - m_E^2)/(\Lambda^2 - q^2)$ to reflect the structural effect of the vertex of the heavy mesons interacting with the light mesons. m_E denotes the exchange meson mass. For $q^2 \to 0$ we can treat FF as a constant while for $\Lambda \gg m$ FF approaches unity. The behavior of FF indicates [34] (1) when the distance becomes infinitely large, the interaction vertex looks like a perfect point corresponding to the constant FF; (2) when the distance is very small, the inner structure will manifest itself. In reality, the phenomenological cutoff Λ is around one to several GeV, which also plays the role of regulating the effective potential.

In this work, we consider both S-wave and D-wave interactions between $B^{(*)}$ and $\bar{B}^{(*)}$ mesons. In general, the $B\bar{B}^*$ and $B^*\bar{B}^*$ states can be expressed as

$$|Z_{B\bar{B}^*}^{(\alpha)(\prime)}\rangle = \begin{pmatrix} |B\bar{B}^*(^3S_1)\rangle \\ |B\bar{B}^*(^3D_1)\rangle \end{pmatrix}, \tag{24}$$

$$|Z_{B^*\bar{B}^*}^{(\alpha)}[0]\rangle = \begin{pmatrix} |B^*\bar{B}^*(^1S_0)\rangle \\ |B^*\bar{B}^*(^5D_0)\rangle \end{pmatrix}, \tag{25}$$

$$|Z_{B^*\bar{B}^*}^{(\alpha)}[1]\rangle = \begin{pmatrix} |B^*\bar{B}^*(^3S_1)\rangle \\ |B^*\bar{B}^*(^3D_1)\rangle \\ |B^*\bar{B}^*(^5D_1)\rangle \end{pmatrix}, \tag{26}$$

$$|Z_{B^*\bar{B}^*}^{(\alpha)}[2]\rangle = \begin{pmatrix} |B^*\bar{B}^*(^5S_2)\rangle \\ |B^*\bar{B}^*(^1D_2)\rangle \\ |B^*\bar{B}^*(^3D_2)\rangle \\ |B^*\bar{B}^*(^5D_2)\rangle \end{pmatrix} \tag{27}$$

with α=S, T, where we use the notation $^{2S+1}L_J$ to denote the total spin S, angular momentum L, total angular momentum J of the $B\bar{B}^*$ or $B^*\bar{B}^*$ system. Indices S and D indicate that the couplings between B^* and $\bar{B}^*$ occur via the S-wave and D-wave interactions, respectively.

Thus, the total effective potentials of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems are

$$V_{\text{Total}}^{Z_{B\bar{B}^*}^{(\alpha)(\prime)}}$$
$$= \Big\langle Z_{B\bar{B}^*}^{(\alpha)(\prime)}\Big| \sum_{E=\pi,\eta,\sigma,\rho,\omega} \mathcal{V}_E^{B\bar{B}^*}(r)\Big|Z_{B\bar{B}^*}^{(\alpha)(\prime)}\Big\rangle, \tag{28}$$

$$V_{\text{Total}}^{Z_{B^*\bar{B}^*}^{(\alpha)}[J]}$$
$$= \Big\langle Z_{B^*\bar{B}^*}^{(\alpha)}[J]\Big| \sum_{E=\pi,\eta,\sigma,\rho,\omega} \mathcal{V}_E^{B^*\bar{B}^*}(r)\Big|Z_{B^*\bar{B}^*}^{(\alpha)}[J]\Big\rangle, \tag{29}$$

which are 2×2 and $(J+2)\times(J+2)$ matrices respectively. We impose the following constraint

$$\Big|B\bar{B}^*\big(^{2S+1}L_J\big)\Big\rangle = \sum_{m,m_L,m_S} C_{1m,Lm_L}^{JM}\epsilon_n^m Y_{Lm_L}, \tag{30}$$

$$\Big|B^*\bar{B}^*\big(^{2S+1}L_J\big)\Big\rangle$$
$$= \sum_{m,m',m_L,m_S} C_{Sm_S,Lm_L}^{JM} C_{1m,1m'}^{Sm_S}\epsilon_{n'}^{m'}\epsilon_n^m Y_{Lm_L}, \tag{31}$$

to the effective potential obtained from the scattering amplitude. C_{1m,Lm_L}^{JM}, C_{Sm_S,Lm_L}^{JM} and $C_{1m,1m'}^{Sm_S}$ are the Clebsch-Gordan coefficients. Y_{Lm_L} is the spherical harmonics function. The polarization vector for the vector heavy flavor meson is defined as $\epsilon_\pm^m = \mp\frac{1}{\sqrt{2}}(\epsilon_x^m \pm i\epsilon_y^m)$ and $\epsilon_0^m = \epsilon_z^m$. Here, the polarization vector in Eqs. (30)–(31) is just the one appearing in the effective potentials, which will be presented later.

2.3.1   The $B\bar{B}^*$ system

We obtain the general expressions of the total effective potentials of the isoscalar and isovector $B\bar{B}^*$

systems [45], i.e.,

$$
\mathcal{V}^{Z_{B\bar{B}^*}^{(T)(\prime)}} = V_\sigma^{\text{Direct}} - \frac{1}{2}V_\rho^{\text{Direct}} + \frac{1}{2}V_\omega^{\text{Direct}} + \frac{c}{4}\bigg(-2V_\pi^{\text{Cross}}
$$
$$
+\frac{2}{3}V_\eta^{\text{Cross}} - 2V_\rho^{\text{Corss}} + 2V_\omega^{\text{Cross}}\bigg), \tag{32}
$$

$$
\mathcal{V}^{Z_{B\bar{B}^*}^{(S)(\prime)}} = V_\sigma^{\text{Direct}} + \frac{3}{2}V_\rho^{\text{Direct}} + \frac{1}{2}V_\omega^{\text{Direct}} + \frac{c}{4}\bigg(6V_\pi^{\text{Cross}}
$$
$$
+\frac{2}{3}V_\eta^{\text{Cross}} + 6V_\rho^{\text{Corss}} + 2V_\omega^{\text{Corss}}\bigg), \tag{33}
$$

where the subpotentials from the π, η, σ, ρ and ω meson exchanges are expressed as [45]

$$
V_\pi^{\text{Cross}} = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_2,m_2,r)
$$
$$
+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_2,m_2,r)\bigg], \tag{34}
$$

$$
V_\eta^{\text{Cross}} = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_3,m_3,r)
$$
$$
+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_3,m_3,r)], \tag{35}
$$

$$
V_\sigma^{\text{Direct}} = -g_s^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\sigma,r), \tag{36}
$$

$$
V_\rho^{\text{Direct}} = -\frac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\rho,r), \tag{37}
$$

$$
V_\rho^{\text{Cross}} = 2\lambda^2 g_V^2\bigg[\frac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_0,m_0,r)
$$
$$
-\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_0,m_0,r)\bigg], \tag{38}
$$

$$
V_\omega^{\text{Direct}} = -\frac{1}{2}\beta^2 g_V^2(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda,m_\omega,r), \tag{39}
$$

$$
V_\omega^{\text{Cross}} = 2\lambda^2 g_V^2\bigg[\frac{2}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z(\Lambda_1,m_1,r)
$$
$$
-\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T(\Lambda_1,m_1,r)\bigg]. \tag{40}
$$

In these obtained effective potentials, Λ_i^2 and m_i^2 are defined as

$$
\Lambda_2^2 = \Lambda^2 - (m_{B^*}-m_B)^2,\ m_2^2 = m_\pi^2 - (m_{B^*}-m_B)^2,
$$
$$
\Lambda_3^2 = \Lambda^2 - (m_{B^*}-m_B)^2,\ m_3^2 = m_\eta^2 - (m_{B^*}-m_B)^2,
$$
$$
\Lambda_0^2 = \Lambda^2 - (m_{B^*}-m_B)^2,\ m_0^2 = m_\rho^2 - (m_{B^*}-m_B)^2,
$$
$$
\Lambda_1^2 = \Lambda^2 - (m_{B^*}-m_B)^2,\ m_1^2 = m_\omega^2 - (m_{B^*}-m_B)^2,
$$

and $S(\hat{\boldsymbol{r}}, \boldsymbol{a}, \boldsymbol{b})$ is expressed as $S(\hat{\boldsymbol{r}}, \boldsymbol{a}, \boldsymbol{b}) = 3(\hat{\boldsymbol{r}}\cdot\boldsymbol{a})(\hat{\boldsymbol{r}}\cdot\boldsymbol{b}) - \boldsymbol{a}\cdot\boldsymbol{b}$. Additionally, we also define the functions $Y(\Lambda, m, r)$, $Z(\Lambda, m, r)$ and $T(\Lambda, m, r)$ with the expressions

$$
Y(\Lambda,m_E,r) = \frac{1}{4\pi r}(e^{-m_E r} - e^{-\Lambda r}) - \frac{\Lambda^2 - m_E^2}{8\pi\Lambda}e^{-\Lambda r}, \tag{41}
$$

$$
Z(\Lambda,m_E,r) = \nabla^2 Y(\Lambda,m_E,r)
$$
$$
= \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}Y(\Lambda,m_E,r), \tag{42}
$$

$$
T(\Lambda,m_E,r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y(\Lambda,m_E,r). \tag{43}
$$

In Eqs. (32)–(33), $c = +1$ corresponds to the $Z_{B\bar{B}^*}^{(T)}$ and $Z_{B\bar{B}^*}^{(S)}$ states including these two charged Z_b states observed by Belle Collaboration while taking $c = -1$ corresponds to the $Z_{B\bar{B}^*}^{(T)\prime}$ and $Z_{B\bar{B}^*}^{(S)\prime}$ states which are partner states of X(3872).

As indicated in Eq. (27), we consider both S-wave and D-wave interactions between the B and $\bar{B}^*$ mesons. Finally the total effective potential can be obtained by making the replacement in the subpotentials

$$
\begin{array}{c}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)\\(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)\end{array}\Bigg\} \longmapsto \begin{pmatrix}1&0\\0&1\end{pmatrix}, \tag{44}
$$

$$
S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger) \longmapsto \begin{pmatrix}0&-\sqrt{2}\\-\sqrt{2}&1\end{pmatrix}, \tag{45}
$$

which results in the total effective potential of the $B\bar{B}^*$ system, i.e, a two by two matrix.

The effective potential of the $D\bar{D}^*$ system is similar to that of the $B\bar{B}^*$ system. The η, σ, ρ and ω meson exchange potentials of $D\bar{D}^*$ system can be easily obtained by replacing the parameters for the $B\bar{B}^*$ system with the ones for the $D\bar{D}^*$ system. Since the mass gap of m_D^* and m_D is larger than the mass of π, which is different from the case of the $B\bar{B}^*$ system, the π exchange potential of the $D\bar{D}^*$ system is [34, 35]

$$
V_\pi^{\text{Cross}} = -\frac{g^2}{f_\pi^2}\bigg[\frac{1}{3}(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_3^\dagger)Z_\pi^{DD^*}(\Lambda_4,m_4,r)
$$
$$
+\frac{1}{3}S(\hat{\boldsymbol{r}},\boldsymbol{\epsilon}_2,\boldsymbol{\epsilon}_3^\dagger)T_\pi^{DD^*}(\Lambda_4,m_4,r)\bigg], \tag{46}
$$

where

$$
Y_\pi^{DD^*}(\Lambda_4,m_4,r)
$$
$$
= \frac{1}{4\pi r}\bigg(-e^{-\Lambda_4 r} - \frac{r(\Lambda_4^2+m_4^2)}{2\Lambda_4}e^{-\Lambda_4 r} + \cos(m_4 r)\bigg), \tag{47}
$$

$$Z_\pi^{DD^*}(\Lambda_4, m_4, r) = \nabla^2 Y_\pi^{DD^*}(\Lambda_4, m_4, r) = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$$

$$\times Y_\pi^{DD^*}(\Lambda_4, m_4, r), \qquad (48)$$

$$T_\pi^{DD^*}(\Lambda_4, m_4, r) = r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r}Y_\pi^{DD^*}(\Lambda_4, m_4, r). \qquad (49)$$

In the present case, the parameters Λ_4 and m_4 are defined as

$$\Lambda_4 = \sqrt{\Lambda^2 - (m_{D^*} - m_D)^2}, \qquad (50)$$

$$m_4 = \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2}. \qquad (51)$$

2.3.2 The B*B̄* system

For the isoscalar and isovector B*B̄* systems, the general expressions of the total effective potentials are

$$\mathcal{V}^{Z^{(T)(\prime)}}_{B^*\bar{B}^*} = W_\sigma - \frac{1}{2}W_\rho + \frac{1}{2}W_\omega - \frac{1}{2}W_\pi + \frac{1}{6}W_\eta, \qquad (52)$$

$$\mathcal{V}^{Z^{(S)}{}^{(\prime)}}_{B^*\bar{B}^*} = W_\sigma + \frac{3}{2}W_\rho + \frac{1}{2}V_\omega + \frac{3}{2}W_\pi + \frac{1}{6}W_\eta, \qquad (53)$$

respectively, where the π, η, σ, ρ and ω meson exchanges can contribute to the effective potentials. The corresponding subpotentials are expressed as

$$W_\pi = -\frac{g^2}{f_\pi^2}\left[\frac{1}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\pi, r)\right.$$
$$\left.+\frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\pi, r)\right], \quad (54)$$

$$W_\eta = -\frac{g^2}{f_\pi^2}\left[\frac{1}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\eta, r)\right.$$
$$\left.+\frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\eta, r)\right], \quad (55)$$

$$W_\sigma = -g_s^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\sigma, r), \qquad (56)$$

$$W_\rho = -\frac{1}{4}\left\{2\beta^2 g_V^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\rho, r)\right.$$
$$-8\lambda^2 g_V^2\left[\frac{2}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\rho, r)\right.$$
$$\left.\left.-\frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\rho, r)\right]\right\}, \quad (57)$$

$$W_\omega = -\frac{1}{4}\left\{2\beta^2 g_V^2(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger)Y(\Lambda, m_\omega, r)\right.$$
$$-8\lambda^2 g_V^2\left[\frac{2}{3}(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)Z(\Lambda, m_\omega, r)\right.$$
$$\left.\left.-\frac{1}{3}S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger)T(\Lambda, m_\omega, r)\right]\right\}. \quad (58)$$

Here, the definitions of $Y(\Lambda, m, r)$, $Z(\Lambda, m, r)$, $T(\Lambda, m, r)$ and $S(\hat{\boldsymbol{r}}, \boldsymbol{a}, \boldsymbol{b})$ are given in Sec. 2.3.1.

In this work, we consider both S-wave and D-wave interactions between the B* and B̄* mesons, which are illustrated in Eq. (27). Thus, the total effective potential of the B*B̄* with $J = 0$, 1, 2 is 2×2, 3×3, 4×4 matrices, which can be obtained by replacing the corresponding terms in the subpotentials, i.e.,

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad (59)$$

$$(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (60)$$

$$S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \qquad (61)$$

for the B*B̄* states with $J = 0$,

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (62)$$

$$(\boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger)\cdot(\boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad (63)$$

$$S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1\times\boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2\times\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 0 & -\sqrt{2} & 0 \\ -\sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (64)$$

for the B*B̄* states with $J = 1$, and

$$(\boldsymbol{\epsilon}_1\cdot\boldsymbol{\epsilon}_3^\dagger)(\boldsymbol{\epsilon}_2\cdot\boldsymbol{\epsilon}_4^\dagger) \rightarrowtail \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (65)$$

$$(\boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger) \cdot (\boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger) \longmapsto \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad (66)$$

$$S(\hat{\boldsymbol{r}}, \boldsymbol{\epsilon}_1 \times \boldsymbol{\epsilon}_3^\dagger, \boldsymbol{\epsilon}_2 \times \boldsymbol{\epsilon}_4^\dagger)$$

$$\longmapsto \begin{pmatrix} 0 & \sqrt{\dfrac{2}{5}} & 0 & -\sqrt{\dfrac{14}{5}} \\ \sqrt{\dfrac{2}{5}} & 0 & 0 & -\dfrac{2}{\sqrt{7}} \\ 0 & 0 & -1 & 0 \\ -\sqrt{\dfrac{14}{5}} & -\dfrac{2}{\sqrt{7}} & 0 & -\dfrac{3}{7} \end{pmatrix} \qquad (67)$$

for the $B^*\bar{B}^*$ states with $J = 2$.

The potentials of the $D^*\bar{D}^*$ system and $B^*\bar{B}^*$ system have the same form. We only need to replace the parameters for the $B^*\bar{B}^*$ system with the ones for the $D^*\bar{D}^*$ system.

3   Numerical results

With the obtained effective potentials, we can find the bound state solution by solving the coupled-channel Schrödinger equation. Corresponding to the systems in Eqs. (32)–(33), the kinetic terms for the $Z_{B\bar{B}^*}^{(\alpha)'}$ and $Z_{B^*\bar{B}^*}^{(\alpha)}$ $[J]$ (J = 0, 1, 2) systems are

$$K_{Z_{B\bar{B}^*}^{(\alpha)'}} = \mathrm{diag}\left(-\frac{\Delta}{2\tilde{m}_1}, -\frac{\Delta_2}{2\tilde{m}_1} \right), \qquad (68)$$

$$K_{Z_{B^*\bar{B}^*}^{(\alpha)}[0]} = \mathrm{diag}\left(-\frac{\Delta}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2} \right), \qquad (69)$$

$$K_{Z_{B^*\bar{B}^*}^{(\alpha)}[1]} = \mathrm{diag}\left(-\frac{\Delta}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2} \right), \qquad (70)$$

$$K_{Z_{B^*\bar{B}^*}^{(\alpha)}[2]}$$
$$= \mathrm{diag}\left(-\frac{\Delta}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2}, -\frac{\Delta_2}{2\tilde{m}_2} \right), \qquad (71)$$

respectively. Here, $\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}$, $\Delta_2 = \Delta - \frac{6}{r^2}$. $\tilde{m}_1 = m_B m_{B^*}/(m_B + m_{B^*})$ and $\tilde{m}_2 = m_{B^*}/2$ are the reduced masses of the $Z_{b1}^{(i)}$ and $Z_{b2}^{(i)}$ systems, where m_B and m_{B^*} denote the masses of the pseudoscalar and vector bottom mesons [54], respectively. Of course,

the kinematic terms for the $D\bar{D}^*$ and $D^*\bar{D}^*$ systems are of the same forms as those for the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems, where we replace the mass of $D^{(*)}$ with that of $B^{(*)}$.

In this work, the FESSDE program [55, 56] is adopted to produce the numerical values for the binding energy and the relevant root-mean-square r with the variation of the cutoff in the region of $0.8 \leqslant \Lambda \leqslant 5$ GeV. Moreover, we also use MATSCE [57], a MATLAB package for solving coupled-channel Schrödinger equation, to perform an independent cross-check.

3.1   The $B\bar{B}^*$ and $D\bar{D}^*$ systems

In the following, we first present the numerical results for the $Z_{B\bar{B}^*}^{(T)}$ and $Z_{B\bar{B}^*}^{(S)}$ states where $Z_{B\bar{B}^*}^{(T)}$ corresponds to $Z_b(10610)$ observed by Belle [36]. As shown in Table 1, there exist two systems with $c = -1$ and $C = +1$ in the flavor wave functions, marked as $Z_{B\bar{B}^*}^{(T)'}$ and $Z_{B\bar{B}^*}^{(S)'}$.

1) In Table 2, we present the numerical results of the obtained bound state solutions in both OME and OPE cases. We find the bound state solutions for the two isoscalar $Z_{B\bar{B}^*}^{(S)}$ and $Z_{B\bar{B}^*}^{(S)'}$ with reasonable Λ values ($\Lambda \sim 1$ GeV), which indicates the existence of the $Z_{B\bar{B}^*}^{(S)}$ and $Z_{B\bar{B}^*}^{(S)'}$ molecular states.

2) For the $Z_{B\bar{B}^*}^{(T)}$ state, we also find the bound state solution with Λ around 2.2 GeV. Our result shows that $Z_{B\bar{B}^*}^{(T)}$ could be a molecular state with a very shallow binding energy. In addition, its binding energy is not strongly dependent on Λ. Thus, it is quite natural to interpret $Z_b(10610)$ as a $B\bar{B}^*$ molecular state with isospin $I = 1$.

3) For the $Z_{B\bar{B}^*}^{(T)'}$ system, the bound state solution can be found in the region $\Lambda > 4.7$ GeV. To some extent, the value of Λ for the $Z_{B\bar{B}^*}^{(T)'}$ seems a little large compared with 1 GeV.

4) We also discuss the case when we only consider the OPE potential. For the $\{Z_{B\bar{B}^*}^{(T)}, Z_{B\bar{B}^*}^{(T)'}, Z_{B\bar{B}^*}^{(S)'}\}$ or $Z_{B\bar{B}^*}^{(S)}$, we need to decrease or increase the Λ value to obtain the same binding energy as that from OME. The one pion meson exchange potential indeed plays the crucial role in the formation of the BB^* bound states.

We extend the formalism in Sec. 2 to study the $D\bar{D}^*$ systems. As shown in Table 3, we can exclude the existence of the $Z_{D\bar{D}^*}^{(T)}$ and $Z_{D\bar{D}^*}^{(T)'}$ since we do not find any bound state solution for the $Z_{D\bar{D}^*}^{(T)}$ and $Z_{D\bar{D}^*}^{(T)'}$ states. For the two isoscalar $Z_{D\bar{D}^*}^{(S)}$ and $Z_{D\bar{D}^*}^{(S)'}$, there exist loosely bound states with reasonable Λ values. If only considering the OPE exchange potential, we notice: (1) the bound state solution of the $Z_{D\bar{D}^*}^{(T)}$ appears when $\Lambda \sim 4.6$ GeV, which largely deviates from

1 GeV; (2) there is still no bound state solution for $Z_{D\bar{D}^*}^{(T)'}$; (3) for $Z_{D\bar{D}^*}^{(S)}$ and $Z_{D\bar{D}^*}^{(S)'}$, Λ becomes larger in order to find the bound state solution. The comparison between the OME and OPE results also reflects the importance of one pion exchange in the $D\bar{D}^*$ systems. We need to specify that $Z_{D\bar{D}^*}^{(S)'}$ with $0^+(1^{++})$ directly corresponds to the observed X(3872) [58].

The BaBar Collaboration measured the radiative decay of X(3872) and found a ratio of B(X(3872)→ ψ(2S)γ)/B(X(3872)→ J/ψγ) = 3.4 ± 1.4 [59], which contradicts the prediction with a purely $D\bar{D}^*$ molecular assignment to X(3872) [9]. However, very re-

cently, Belle reported a new measurement of the radiative decay of X(3872), where only the decay mode X(3872)→ J/ψγ was observed and the upper limit B(X(3872)→ ψ(2S)γ)/B(X(3872)→ J/ψγ) < 2.1 was given [60]. The inconsistence between the Belle and BaBar results indicate that the study of X(3872) is still an important research topic. Our numerical results suggest that the mass of the loosely bound molecular state $Z_{D\bar{D}^*}^{(S)'}$ is consistent with that of X(3872). The assignment of X(3872) as a molecular candidate is still very attractive.

Table 2.   The obtained bound state solutions (binding energy E and root-mean-square radius r_{RMS}) for the $B\bar{B}^*$ systems. Here, we discuss two situations, i.e., including all one meson exchange (OME) contribution and only considering one pion exchange (OPE) potential.

$I^G(J^{PC})$	state	OME			OPE		
		Λ	E/MeV	r_{RMS}/fm	Λ	E/MeV	r_{RMS}/fm
$1^+(1^+)$	$Z_{B\bar{B}^*}^{(T)}$	2.1	−0.22	3.05	2.2	−8.69	0.62
		2.3	−1.64	1.31	2.4	−20.29	0.47
		2.5	−4.74	0.84	2.6	−38.54	0.36
$1^-(1^+)$	$Z_{B\bar{B}^*}^{(T)'}$	4.9	−0.14	3.64	4.5	−17.79	0.56
		5.0	−0.41	2.45	4.6	−22.65	0.52
		5.1	−0.85	1.80	4.7	−28.29	0.48
$0^-(1^{+-})$	$Z_{B\bar{B}^*}^{(S)}$	1.0	−0.28	3.35	1.8	−10.09	0.96
		1.05	−1.81	1.71	1.9	−15.11	0.84
		1.1	−5.36	1.18	2.0	−21.53	0.76
$0^+(1^{++})$	$Z_{B\bar{B}^*}^{(S)'}$	0.8	−0.95	1.84	1.0	−7.68	0.82
		0.9	−6.81	0.91	1.1	−15.30	0.65
		1.0	−19.92	0.65	1.2	−26.53	0.53

Table 3.   The obtained bound state solutions (binding energy E and root-mean-square radius r_{RMS}) for the $D\bar{D}^*$ systems.

$I^G(J^{PC})$	state	OME			OPE		
		Λ	E/MeV	r_{RMS}/fm	Λ	E/MeV	r_{RMS}/fm
$1^+(1^{+-})$	$Z_{D\bar{D}^*}^{(T)}$	—	—	—	4.6	−0.85	1.46
					4.7	−3.42	1.17
					4.8	−7.18	0.93
					4.9	−12.40	0.75
$1^-(1^{++})$	$Z_{D\bar{D}^*}^{(T)'}$	—	—	—	—	—	—
$0^-(1^{+-})$	$Z_{D\bar{D}^*}^{(S)}$	1.3	—	—	3.4	−0.11	1.74
		1.4	−1.56	1.61	3.5	−2.03	1.50
		1.5	−12.95	0.98	3.6	−4.79	1.26
		1.6	−35.73	0.69	3.7	−9.62	1.06
$0^+(1^{++})$	$Z_{D\bar{D}^*}^{(S)'}$	1.1	−0.61		1.7	−3.01	1.37
		1.2	−4.42	1.38	1.8	−7.41	1.06
		1.3	−11.78	1.05	1.9	−14.15	0.84
		1.4	−21.88	0.86	2	−23.82	0.68

3.2 The $B^*\bar{B}^*$ and $D^*\bar{D}^*$ systems

The numerical results of the $B^*\bar{B}^*$ systems are presented in Table 4, which include the obtained binding energy and the corresponding root-mean-square radius. We find the bound state solution for all the $B^*\bar{B}^*$ states with reasonable Λ values:

1)A loosely bound state exists for $Z^{(T)}_{B^*\bar{B}^*}$ [1] corresponding to the observed $Z_b(10650)$ with Λ slightly above 2 GeV. With only considering the OPE potential, the obtained binding energy becomes deeper with the same Λ value.

2) In addition, the $B^*\bar{B}^*$ can form loosely bound molecular states $Z^{(S)}_{B^*\bar{B}^*}$ [0], $Z^{(T)}_{B^*\bar{B}^*}$ [0], $Z^{(S)}_{B^*\bar{B}^*}$ [1] and $Z^{(S)}_{B^*\bar{B}^*}$ [2] with very reasonable Λ values. Comparing the results between OME and OPE cases, one notices again that the one pion exchange indeed is very important to form the $B^*\bar{B}^*$ bound state.

3) For the $Z^{(T)}_{B^*\bar{B}^*}$ [2] state, the existence of the loosely bound state requires the value of Λ around 4.4 GeV.

In the following, we also present the numerical results for the $D^*\bar{D}^*$ systems in Table 5. Our calculation indicates:

1) We find the bound state solutions for the three

isoscalar states $Z^{(S)}_{D^*\bar{D}^*}$ [0], $Z^{(S)}_{D^*\bar{D}^*}$[1] and $Z^{(S)}_{D^*\bar{D}^*}$ [2], where the corresponding Λ is around 1 GeV. If only considering the OPE contribution for the $Z^{(S)}_{D^*\bar{D}^*}$ [0], $Z^{(S)}_{D^*\bar{D}^*}$ [1] states, we need to largely increase the Λ value in order to obtain a loosely bound state. Here, either $Z^{(S)}_{D^*\bar{D}^*}$ [0] or $Z^{(S)}_{D^*\bar{D}^*}$ [2] could correspond to the observed Y(3930) by Belle [61] and BABAR [62], which is consistent with the conclusion in Ref. [31].

2) There does not exist the bound state $Z^{(T)}_{D^*\bar{D}^*}$ [2]. The value of Λ is about 3.6 GeV in order to form a bound state $Z^{(T)}_{D^*\bar{D}^*}$ [0]. In the range $0.8 < \Lambda < 5$ GeV, we cannot find the bound state solution for $Z^{(T)}_{D^*\bar{D}^*}$ [1] in the OME case. Thus, we exclude the existence of the $Z^{(T)}_{D^*\bar{D}^*}$ [1] molecular state.

4 Conclusion

In this work, by the OBE model we systematically carry out the dynamical study of the $B\bar{B}^*$ and $B^*\bar{B}^*$ systems, where both the S-wave and D-wave interactions between the $B^{(*)}$ and $\bar{B}^*$ mesons are considered. Besides $Z_b(10610)$ and $Z_b(10650)$ explained as $B^{(*)}$ and $\bar{B}^*$ molecular states, respectively, we also predict the existences of six other $B\bar{B}^*$ and $B^*\bar{B}^*$ bound

Table 4. The obtained bound state solutions (binding energy E and root-mean-square radius r_{RMS}) for the $B^*\bar{B}^*$ systems.

$I^G(J^{PC})$	state	OME			OPE		
		Λ	E/MeV	r_{RMS}/fm	Λ	E/MeV	r_{RMS}/fm
$1^+(0^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [0]	1.2	—	—	1	—	—
		1.4	−1.44	1.24	1.2	−0.32	1.53
		1.6	−6.16	0.77	1.4	−5.69	0.78
		1.8	−15.15	0.54	1.6	−18.82	0.50
$0^-(0^{+-})$	$Z^{(S)}_{B^*\bar{B}^*}$ [0]	0.9	—	—	1	—	—
		1	−0.81	2.11	1.2	−0.52	2.76
		1.1	−9.98	1.02	1.4	−5.74	1.12
		1.2	−35.16	0.70	1.6	−20.92	0.77
$1^+(1^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [1]	2.2	−0.81	1.38	2	−2.17	1.15
		2.4	−3.31	0.95	2.2	−8.01	0.68
		2.6	−7.80	0.68	2.4	−19.00	0.48
		2.8	−14.94	0.52	2.6	−36.36	0.38
$0^-(1^{+-})$	$Z^{(S)}_{B^*\bar{B}^*}$ [1]	1	−0.01	2.07	1.4	−0.51	1.90
		1.1	−5.50	1.17	1.6	3.65	−1.32
		1.2	−21.76	−0.75	1.8	−10.26	0.96
		1.3	−53.68	0.55	2.0	−21.81	0.75
$1^+(2^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [2]	4.4	−0.44	1.59	3.6	−2.82	1.12
		4.6	−1.59	1.28	3.8	−6.21	0.85
		4.8	−3.42	1.01	4.0	−11.41	0.68
		5	−6.16	0.81	4.2	−18.77	0.57
$0^-(2^{+-})$	$Z^{(S)}_{B^*\bar{B}^*}$ [2]	0.8	−2.33	1.32	0.8	−1.81	1.48
		0.9	−10.45	0.84	0.9	−5.64	1.01
		1.0	−27.14	0.63	1.0	−12.28	0.76

Table 5.   The obtained bound state solutions (binding energy E and root-mean-square radius $r_{\rm RMS}$) for the $D^*\bar{D}^*$ systems.

$I^G(J^{PC})$	state	OME			OPE		
		Λ	E/MeV	$r_{\rm RMS}$/fm	Λ	E/MeV	$r_{\rm RMS}$/fm
$1^+(0^+)$	$Z^{(T)}_{D^*D^*}$ [0]	3.6	-0.94	1.74	2.8	-2.03	1.47
		3.8	-6.16	1.00	2.9	-6.10	1.00
		4	-16.44	0.66	3	-12.51	0.74
		4.2	-33.23	0.49	3.1	-21.56	0.59
$0^-(0^{+-})$	$Z^{(S)}_{D^*D^*}$ [0]	1.4	-1.72	1.62	3	-5.70	1.24
		1.5	-17.98	0.88	3.1	-12.15	0.96
		1.6	-54.60	0.47	3.2	-21.83	0.78
$1^+(1^+)$	$Z^{(T)}_{D^*D^*}$ [1]	—	—	—	4.7	-6.96	0.94
					4.8	-12.29	0.73
					4.9	-19.36	0.60
					5	-28.31	0.51
$0^-(1^{+-})$	$Z^{(S)}_{D^*D^*}$ [1]	1.3	—		3.6	-9.91	1.01
		1.4	-3.44	1.44	3.7	-15.25	0.87
		1.5	-16.57	0.90	3.8	-22.07	0.76
		1.6	-41.25	0.66	3.9	-30.53	0.68
$1^+(2^+)$	$Z^{(T)}_{D^*D^*}$ [2]	—	—	—	—	—	—
$0^-(2^{+-})$	$Z^{(S)}_{D^*D^*}$ [2]	1.1	-0.61	1.72	1.6	-3.89	1.28
		1.2	-7.50	1.19	1.7	-9.64	0.98
		1.3	-19.22	0.89	1.8	-18.38	0.77
		1.4	-35.93	0.73	1.9	-30.71	0.64

Table 6.   A summary of the $B\bar{B}^*$, $B^*\bar{B}^*$, $D\bar{D}^*$, $D^*\bar{D}^*$ systems. Here, we use ✓ and × to mark the corresponding systems with and without the bound states solution when taking a reasonable Λ value, respectively. The criteria of the choice of the reasonable Λ may be strongly biased.

$I^G(J^P)$	system	remark	experiment [36]	system	remark	experiment
$1^+(1^+)$	$Z^{(T)}_{B\bar{B}^*}$	✓	$Z_b(10610)$	$Z^{(T)}_{D\bar{D}^*}$	×	
$0^-(1^{+-})$	$Z^{(S)}_{B\bar{B}^*}$	✓		$Z^{(S)}_{D\bar{D}^*}$	✓	
$1^-(1^+)$	$Z^{(T)}_{B\bar{B}^*}{}'$	×		$Z^{(T)}_{D\bar{D}^*}{}'$	×	
$0^+(1^{++})$	$Z^{(S)}_{B\bar{B}^*}{}'$	✓		$Z^{(S)}_{D\bar{D}^*}{}'$	✓	$X(3872)$ [58]
$1^-(0^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [0]	✓		$Z^{(T)}_{D^*\bar{D}^*}$ [0]	×	
$0^+(0^{++})$	$Z^{(S)}_{B^*\bar{B}^*}$ [0]	✓		$Z^{(S)}_{D^*\bar{D}^*}$ [0]	✓	$Y(3930)$ [63–65]
$1^+(1^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [1]	✓	$Z_b(10650)$	$Z^{(T)}_{D^*\bar{D}^*}$ [1]	×	
$0^-(1^{+-})$	$Z^{(S)}_{B^*\bar{B}^*}$ [1]	✓		$Z^{(S)}_{D^*\bar{D}^*}$ [1]	✓	
$1^-(2^+)$	$Z^{(T)}_{B^*\bar{B}^*}$ [2]	×		$Z^{(T)}_{D^*\bar{D}^*}$ [2]	×	
$0^+(2^{++})$	$Z^{(S)}_{B^*\bar{B}^*}$ [2]	✓		$Z^{(S)}_{D^*\bar{D}^*}$ [2]	✓	$Y(3940)$ [63–65]

states (see Table 6) within the same framework. We want to stress that the long-range interaction between the heavy meson pair arises from the one-pion-exchange force, which is clearly known. This OPE force alone is strong enough to form the above loosely bound molecular states, which makes the present results quite model-independent and robust.

The observation of these $Z_b(10610)$ and $Z_b(10650)$ states shows that the hidden-bottom decays are very

important decay channels. This is characteristic and helpful to the search of the molecular bottomonium. After taking into account the phase space [54, 63–65] and the conservation of quantum number, the $Z^{(S)}_{B\bar{B}^*}$, $Z^{(S)'}_{B\bar{B}^*}$, $Z^{(T)}_{B^*\bar{B}^*}[0]$, $Z^{(S)}_{B^*\bar{B}^*}[0]$, $Z^{(S)}_{B^*\bar{B}^*}$ [1] and $Z^{(S)}_{B^*\bar{B}^*}$ [2] molecular states can decay into

$$\{\Upsilon(1S)\eta,\ \Upsilon(2S)\eta,\ h_b(1P)\eta,\ \eta_b(1S)\omega\},$$
$$\{\Upsilon(1S)\omega,\ \chi_{b0}(1P)\eta,\ \chi_{b1}(1P)\eta,\ \chi_{b2}(1P)\eta\},$$

$$\{\chi_{b1}(1P)\pi, \chi_{b1}(2P)\pi, \Upsilon(1S)\rho, \eta_b(1S)\pi\},$$

$$\{\Upsilon(1S)\omega, \chi_{b1}(1P)\eta, \eta_b(1S)\eta\},$$

$$\{\chi_{b0}(1P)\omega, \Upsilon(1S)\eta, \Upsilon(2S)\eta, \eta_b(1S)\omega, h_b(1P)\eta\},$$

$$\{\Upsilon(1S)\omega, \chi_{b1}(1P)\eta, \chi_{b2}(1P)\eta, \eta_b(1S)\eta\},$$

respectively. The above modes can be used in the future experimental search of the partner states of $Z_b(10610)$ and $Z_b(10650)$.

We also extend our formalism to study the molecular charmonia. The observed possible molecular charmonia are listed in Table 6. The possible hidden-

charm decay channels of the molecular states $Z_{D\bar{D}^*}^{(S)}$, $Z_{D^*\bar{D}^*}^{(S)}$ [0], $Z_{D^*\bar{D}^*}^{(S)}$ [1] and $Z_{D^*\bar{D}^*}^{(S)}$ [2] are

$$\{\eta_c(1S)\omega, J/\psi(1S)\eta\},$$

$$\{J/\psi\omega, \eta_c(1S)\eta\},$$

$$\{\eta_c(1S)\omega, J/\psi(1S)\eta\},$$

$$\{J/\psi(1S)\omega, \eta_c(1S)\eta\},$$

respectively. Due to the limit of phase space, the hidden-charm decays for the other one $Z_{D\bar{D}^*}^{(S)'}$ molecular state are $J/\psi(1S)$ or $\eta_c(1S)$ plus multi-pions.

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