Description of ^{150}Nd nucleus by a new alternative scheme *

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Abstract: A new scheme was recently proposed in which the usual $SU(3)$ quadrupole-quadrupole interaction was replaced by an $O(6)$ cubic interaction in the Interacting Boson Model, and also successfully applied to the description of ^{152}Sm for the $N=90$ rare earth isotones with $X(5)$ symmetry. By using this new scheme, in the present work, we further explore the properties of another candidate of ^{150}Nd for the $N=90$ with $X(5)$ symmetry. The low-lying energy levels and E2 transition rates are calculated and compared with the experimental data. The results show that the new scheme can also reasonably describe the experimental low-lying spectrum and the intraband and the interband E2 transitions for ^{150}Nd. However, for the low-lying spectrum, the $O(6)$ cubic interaction seems better in describing the energy levels, especially in higher excited states and γ band, yet the 0_2^+ level within the β band is lower than the corresponding experimental value and the $U(5)$-$SU(3)$ scheme seems better to describe the low-lying levels of β band; and for the B(E2) transition, for the intraband transitions within the ground band and some interband transitions between the β band and the ground band, the results from $O(6)$ cubic interaction are better than those from $SU(3)$ quadrupole-quadrupole interaction, yet of which seems better to describe the intraband E2 transitions within β band. The present work is very meaningful in helping us to understand in depth the new characteristics of symmetry by the higher order $O(6)$ cubic interaction.

Key words: interacting Boson model, $O(6)$ cubic interaction, $SU(3)$ quadrupole-quadrupole interaction

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1 Introduction

The nature of shape phase transitions in finite many-body systems is a fundamental issue and has been the subject of many investigations [1–4]. Transitional nuclei experienced renewed interest over the last decade and were extensively studied in the Interacting Boson Model (IBM) [5–9]. It is now widely accepted that the three limiting cases [10–12] of the theory correspond to three different geometric shapes of nuclei, referred to as spherical (vibrational limit with $U(5)$ symmetry), axially deformed (rotational limit with $SU(3)$ symmetry), and γ-soft (triaxial with $O(6)$ symmetry), respectively. The full range of the model can be parameterized in terms of the Casten triangle [13]. Interesting phenomena occur when a system falls between two limits of the theory, in which case a quantum phase transition occurs at a critical point [14–16] where the dominance of one of the symmetries yields to the dominance of the other. Indeed, the $U(5)$-$SU(3)$ transitional description for Nd, Sm, Gd, and Dy nuclei was first reported in Ref. [17]. Later on, the $X(5)$ symmetry at the critical point of this transition was studied in Ref. [18], in which Iachello proposed analytical solutions of a Bohr Hamiltonian in the situation appropriate for the description of nuclei near the critical point of the spherical to axially deformed shape-phase transition. Casten and Zamfir showed in Ref. [19] that ^{152}Sm and other $N=90$ isotones demonstrate these characteristics. Transitional patterns from the spherical, $U(5)$, to the axially deformed, $SU(3)$, limit of the IBM with a schematic Hamiltonian were studied in Refs. [20–22]; in particular, the transitional behavior of some physically relevant quantities across the entire span of the $U(5)$-$SU(3)$ transitional region were explored.

In fact another new idea was first suggested by van Isacker [23], in which the quantum phase transitional behavior for an alternative characterization of the spherical to axially deformed shape-phase transition was analyzed. In Ref. [23], the well-known $SU(3)$ quadrupole-quadrupole interaction in the schematic Hamiltonian ($U(5)$-$SU(3)$ scheme) is replaced by the $O(6)$ $[\hat{Q}(0)\times\hat{Q}(0)\times\hat{Q}(0)]^0$ cubic interaction, where $\hat{Q}_\mu(0)=s^\dagger\tilde{d}_\mu+d^\dagger_\mu s$ is generator of the $O(6)$ algebra. This idea was then further confirmed by Thiamova and Cejnar [24]. From

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these investigations one understands that similar results may be realized when the quadratic scalar formed with the $U(5)$-$SU(3)$ quadrupole operator is replaced by the cubic scalar formed with the quadrupole interaction of the $O(6)$ limit. Inspired by this new idea, in our very recent work [25], a systematic investigation of this alternative scheme was dynamically investigated, in which the transitional behaviors of the low-lying energy levels, eigenstates, isomer shifts, E2 transition rates, and expectation values of shape variables across the entire transitional region are all examined. A comparison with outcomes of the usual $U(5)$-$SU(3)$ transitional description shows that the spherical to axially deformed shape-phase transition can also be described within this alternative context, especially near the critical point. The results show that the transition with the $O(6)$ cubic interaction (UQ scheme) is considerably smoother than the $U(5)$-$SU(3)$ case, but with the nuclear shape less well defined, even in the axially deformed limit. It is also shown that the new scheme can also reasonably describe the low-lying spectrum and E2 transition rates of ^{152}Sm for $N=90$, and the new scheme seems better than the usual $U(5)$-$SU(3)$ scheme in describing the properties of ^{152}Sm with the $X(5)$ symmetry.

The purpose of the present study is to further investigate another candidate of N=90 rare earth isotones with $X(5)$ symmetry. The low-lying energy levels and the intraband and the interband E2 transitions relating ground, β, and γ bands for ^{150}Nd will be calculated within both the original $U(5)$-$SU(3)$ scheme and UQ scheme, and compared with the experimental data. We will show that the typical quantities in the new UQ scheme, including the low-lying spectrum and E2 transitions for ^{150}Nd, indeed display almost the same transitional patterns as found in the original $U(5)$-$SU(3)$ scheme. The analysis will further confirm that the $O(6)$ cubic interaction can play a role similar to that of the $SU(3)$ quadrupole-quadrupole interaction.

2 Hamiltonian

It is well known that the consistent-Q Hamiltonian [26] can be used to describe the most general situation in the IBM, which is given by

$$\hat{H}_{\rm Q}=c_1\left[x\hat{n}_{\rm d}+\frac{x-1}{f_1(N)}\hat{Q}(\chi)\cdot\hat{Q}(\chi)\right], \tag{1}$$

where N is the total number of bosons, $\widehat{n}_{\rm d}$ is the number operator for counting d-bosons, $\hat{Q}_\mu(\chi)=(d^\dagger s+s^\dagger\tilde{d})^{(2)}_\mu+\chi(d^\dagger\tilde{d})^{(2)}_\mu$, x and χ are control parameters, $f_1(N)$ is a linear function of N, and c_1 is the scaling factor. Therefore, three limited situations of the model are given by $x=1$ for the $U(5)$, $x=0$ and $\chi=0$ for the $O(6)$, and $x=0$ and $\chi=-\frac{\sqrt{7}}{2}$ for the $SU(3)$, respectively.

2.1 For $U(5)$-$SU(3)$ scheme

The original $U(5)$-$SU(3)$ scheme [20] is the special case of the consistent-Q formalism (1) with $\chi=-\frac{\sqrt{7}}{2}$, which is suitable to describe the spherical to axially deformed shape phase transition, thus the Hamiltonian is given by

$$\hat{H}_1=c_1\left[x\hat{n}_d+\frac{(x-1)}{f_1(N)}\hat{Q}(-\sqrt{7}/2)\cdot\hat{Q}(-\sqrt{7}/2)\right], \tag{2}$$

where $f_1(N)$ is a linear function of N, and as shown in Refs. [18, 20, 27], the critical point $x_{\rm c}$ will be different for different choices of the function $f_1(N)$, we adopt $f_1(N)=4N$ as used in Refs. [18, 20, 27]. Also, parameter $c_1>0$ is the scaling factor, x is the control parameter of the $U(5)$-$SU(3)$ scheme with $0\leqslant x\leqslant 1$.

The B(E2) transition is a very sensitive signature of the structure. The B(E2) reduced transition probability is defined by

$$B(E2;L_{\rm i}\rightarrow L_{\rm f})=\frac{|\langle L_{\rm f}\|T(E2)\|L_{\rm i}\rangle|^2}{2L_{\rm i}+1}. \tag{3}$$

Here the $T(E2)$ operator in the $U(5)$-$SU(3)$ scheme is chosen to be the same as that used in Ref. [20] with

$$T_\mu(E2)=q_1\hat{Q}_\mu(-\sqrt{7}/2), \tag{4}$$

where

$$\hat{Q}_\mu(-\sqrt{7}/2)=(s^\dagger\tilde{d}_\mu+d^\dagger_\mu s)-\sqrt{7}/2(d^\dagger\tilde{d})^{(2)}_\mu \tag{5}$$

is the $SU(3)$ generator, and q_1 is the effective charge.

2.2 For UQ scheme

Alternatively, in the UQ scheme, the suitable Hamiltonian to describe the same shape phase transition in this region is given by

$$\hat{H}_2=c_2\left[y\hat{n}_{\rm d}+\frac{(1-y)}{f_2(N)}[Q(0)\times Q(0)\times Q(0)]^0\right], \tag{6}$$

where parameter $c_2>0$ is the scaling factor, y is the control parameter of the UQ scheme with $0\leqslant y\leqslant 1$, $f_2(N)$ is a quadratic function of N. The critical point $y_{\rm c}$ will be different for different choices of the function $f_2(N)$, and we adopt $f_2(N)=0.8N^2$ as used in Ref. [25] which puts the critical point $y_{\rm c}$ of the UQ scheme close to $x_{\rm c}$ of the $U(5)$-$SU(3)$ scheme.

The T(E2) operator in the UQ scheme is chosen to be

$$T_\mu({\rm E2})=q_2\hat{Q}_\mu(0), \tag{7}$$

where

$$\hat{Q}_\mu(0)=s^\dagger\tilde{d}_\mu+d^\dagger_\mu s \tag{8}$$

is the $O(6)$ quadrupole operator and q_2 is the effective charge.

2.3 How to solve the equation

In order to diagonalize Hamiltonians (2) and (6), we expand the corresponding eigenstates in terms of the $U(6)\supset SU(3)\supset SO(3)$ basis vectors $|N(\lambda\mu)KL\rangle$ as

$$|NL_\xi\rangle=\sum_{(\lambda\mu)K} C^{L_\xi}_{(\lambda\mu)K}|N(\lambda\mu)KL\rangle, \tag{9}$$

where ξ is used to denote the ξ-th level with angular momentum quantum number L, and the $C^{L_\xi}_{(\lambda\mu)K}$ are expansion coefficients. Since the total number of bosons N is fixed for a given nucleus, the eigenstates given in Eq. (9) are also denoted $|L_\xi;z\rangle$ with $z=x$ for the $U(5)$-$SU(3)$ scheme or $z=y$ for the UQ scheme in the following, where the value of the control parameter z is explicitly shown. In our calculations, the orthonormalization process [28, 29] with respect to the additional quantum number K needed to label the basis vectors of $SU(3)\supset SO(3)$ and the phase convention for the $U(6)\supset SU(3)$ basis vectors proposed in Ref. [30] are adopted. By using analytic expressions for $U(6)\supset SU(3)$ reduced matrix elements of the d-boson creation or annihilation operator [30] and an algorithm [28, 29] for generating the $SU(3)\supset SO(3)$ Wigner coefficients, the eigenequation that simultaneously determines the eigenenergy and the corresponding set of the expansion coefficients $C^{L_\xi}_{(\lambda\mu)K}$ can be established, with results that can then be used to calculate physical quantities in both schemes.

3 Results and discussion

In order to demonstrate that the UQ scheme indeed serves as an alternative description of the spherical to axially deformed phase transition, recently we have systematically investigated the possible $X(5)$ candidate of ^{152}Sm for $N=90$, and shown that the new scheme can indeed reasonably describe the low-lying spectrum of ^{152}Sm and E2 transition rates with the $X(5)$ critical point symmetry [25] .

Now in this subsection, another possible $X(5)$ symmetry candidate, ^{150}Nd nucleus, will be further investigated by the new UQ scheme, of which the results are compared with the experimental data [31–34] and those obtained from the $U(5)$-$SU(3)$ scheme. In the following, we present the calculated results in both $U(5)$-$SU(3)$ and UQ schemes, including some low-lying energy levels, the intraband and the interband E2 transitions relating ground, β, and γ bands. Since the proton number is 60 and the neutron number is 90, so the total number of bosons for ^{150}Nd is $N=9$. In the calculation, the taken value of x for the $U(5)$-$SU(3)$ scheme and y for the UQ scheme will be decided by the experimental low-lying levels of ^{150}Nd.

Firstly, the low-lying levels will be directly obtained for ^{150}Nd by solving the Hamiltonian (2) for $U(5)$-$SU(3)$ scheme or Hamiltonian (6) for UQ scheme.

We must choose the parameters. In order to achieve global quality of fits to low-lying spectrum, the parameters c_1 and x for the $U(5)$-$SU(3)$ scheme, or parameters c_2 and y for the UQ scheme, are chosen when the mean-square deviation for excitation energies

$$\sigma(E)=\sqrt{\sum_i^{\mathcal{N}_1}|E^i_{\rm exp}-E^i_{\rm th}|^2/\mathcal{N}_1}, \tag{10}$$

reaches the corresponding minimum, where $E^i_{\rm th}$, and $E^i_{\rm exp}$ are energy of the i-th level calculated, and that of the corresponding experimental value, respectively, and $\mathcal{N}_1$ is the total number of levels fitted.

In Table 1, we show the calculated results from both the $U(5)$-$SU(3)$ scheme and the UQ scheme of low-lying excitation energies $E(L_i^+)$ (in keV) normalized to the 2_1^+ state, and the corresponding mean-square deviation $\sigma(E)$ for excitation energies. It is shown that $\sigma(E)$ deviation is 80 keV and 72 keV for $U(5)$-$SU(3)$ scheme and the UQ scheme, respectively, indicating that the overall fitting of low-lying excitation energies from UQ scheme is better than those of $U(5)$-$SU(3)$ scheme. This can also be clearly seen in Fig. 1, in which the low-lying levels from the ground band, β and γ bands are drawn, the experimental data and the calculated results of $U(5)$-$SU(3)$ scheme and UQ scheme are represented, respectively. We can see that both the $U(5)$-$SU(3)$ scheme and the UQ scheme can reasonably describe the low-lying levels of ^{150}Nd. However, the UQ scheme seems better in describing the low-lying spectrum, especially in higher excited levels and γ band, yet the 0_2^+ level is lower than the corresponding experimental value, indicating that the $U(5)$-$SU(3)$ scheme seems better to describe the levels of β band.

Table 1. The energy levels (in keV) for ^{150}Nd.

L_i^+	$E(L_i^+)$		
	Exp. [33, 34]	$U(5)$-$SU(3)$	UQ
2_1^+	130.2	130.2	130.2
4_1^+	381.1	382.0	381.7
6_1^+	720.4	741.1	736.4
8_1^+	1129.6	1198.0	1171.2
10_1^+	1598.5	1746.4	1659.4
0_2^+	675.9	610.7	460.1
2_2^+	850.8	879.9	790.5
4_2^+	1137.8	1245.0	1165.2
2_3^+	1062.1	1090.6	1058.7
4_3^+	1350.5	1510.2	1379.1
$\sigma(E)$		80.0	72.0

Secondly, B(E2) transition, as another quantity which acts as a sensitive signature of the structure, will

be further studied. In fact in Ref. [34] the experimental reduced transition probabilities in ^{150}Nd were obtained and compared to the predictions of the critical point symmetry $X(5)$ of the phase shape transition that occurs for the N=90 rare earth isotones, and very good agreement was observed, revealing this as the case for the realization of the $X(5)$ symmetry.

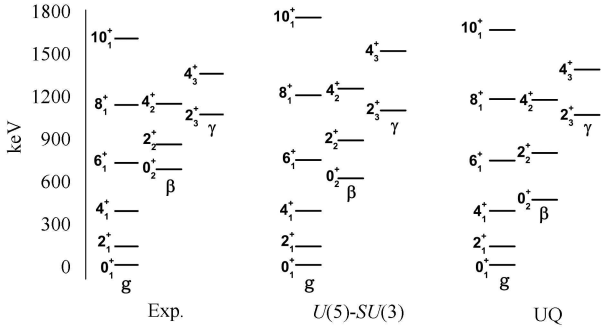

Fig. 1. The energy levels for ^{150}Nd.

Here in the present work, we will use both the $U(5)$-$SU(3)$ scheme and the UQ scheme to study the B(E2) transition. The values of the intraband and the interband E2 transitions relating ground, β, and γ bands will be calculated for ^{150}Nd using the above constructed E2 transition operators from Eq. (4) and Eq. (7) for $U(5)$-$SU(3)$ scheme and the UQ scheme, respectively. In Table 2, we list the E2 transitions of experimental data [31–34] and calculated results from both the $U(5)$-$SU(3)$ scheme and the UQ scheme for ^{150}Nd, in which the B(E2) values (in W.u.) are normalized to the $2_1^+ \to 0_1^+$ transition. The corresponding mean-square deviation

$$\sigma(\mathrm{BE2})=\sqrt{\sum_i^{\mathcal{N}_2}|B(\mathrm{E2})^i_{\mathrm{exp}}-B(\mathrm{E2})^i_{\mathrm{th}}|^2/\mathcal{N}_2}, \qquad (11)$$

where $\mathcal{N}_2$ is the total number of transitions calculated in the present work, and the deviation is 20.1 W.u. for $U(5)$-$SU(3)$ scheme and 25.6 W.u. for the UQ scheme. It seems that both schemes can reasonably describe the transition rates of ^{150}Nd, however the UQ scheme is better in describing the intraband transitions within the ground band, and some interband transitions between the β band and the ground band. Yet the $U(5)$-$SU(3)$ scheme seems better in describing the intraband E2 transitions within the β band.

In Fig. 2, we shows some typical B(E2) values for ^{150}Nd, the experimental data and calculated results from both the $U(5)$-$SU(3)$ scheme and the UQ scheme are presented, where (a), (b), (c), and (d) represent $B(\mathrm{E2};4_1^+ \to 2_1^+)$, $B(\mathrm{E2};10_1^+ \to 8_1^+)$, $B(\mathrm{E2};2_2^+ \to 0_1^+)$, and $B(\mathrm{E2};4_2^+ \to 2_1^+)$, respectively. It is clearly shown in Fig. 2 that the intraband transitions within the ground band, $B(\mathrm{E2};4_1^+ \to 2_1^+)$ and $B(\mathrm{E2};10_1^+ \to 8_1^+)$, the results from UQ scheme are a little better than those from $U(5)$-$SU(3)$ scheme. While the interband transitions between the β band and the ground band, $B(\mathrm{E2};2_2^+ \to 0_1^+)$ and $B(\mathrm{E2};4_2^+ \to 2_1^+)$, especially the $B(\mathrm{E2};2_2^+ \to 0_1^+)$ transition calculated from the two schemes are different. From Table 2, the experimental data for $B(\mathrm{E2};2_2^+ \to 0_1^+)$ transition is 0.7 W.u., however the calculated result from $U(5)$-$SU(3)$ scheme is 0.03 W.u., one order of magnitude smaller than the experimental data. It is improved quite a lot with the result of 0.16 W.u. in the UQ scheme.

Table 2. The B(E2) values (in W.u.) normalized to the $2_1^+ \to 0_1^+$ transition for ^{150}Nd.

$L_i^+ \to L_f^+$	B(E2)		
	Exp.	$U(5)$-$SU(3)$	UQ
$2_1^+ \to 0_1^+$	116.0	116.0	116.0
$4_1^+ \to 2_1^+$	180.7	172.8	173.7
$6_1^+ \to 4_1^+$	206.0	190.8	192.0
$8_1^+ \to 6_1^+$	216.0	192.6	195.0
$10_1^+ \to 8_1^+$	201.0	182.8	188.2
$2_2^+ \to 0_2^+$	113.9	71.0	56.6
$4_2^+ \to 2_2^+$	170.2	108.5	89.7
$0_2^+ \to 2_1^+$	39.1	44.3	32.1
$2_2^+ \to 0_1^+$	0.7	0.03	0.16
$2_2^+ \to 2_1^+$	10.0	8.6	7.82
$2_2^+ \to 4_1^+$	19.0	15.0	8.06
$4_2^+ \to 2_1^+$	0.015	0.067	0.044
$4_2^+ \to 4_1^+$	7.015	6.7	10.61
$4_2^+ \to 6_1^+$	9.2	9.86	2.51
$2_3^+ \to 0_1^+$	3.0	2.23	12.15
$2_3^+ \to 2_1^+$	>2.9	1.77	14.48
$2_3^+ \to 4_1^+$	1.7	4.18	4.20
σ(BE2)		20.1	25.5

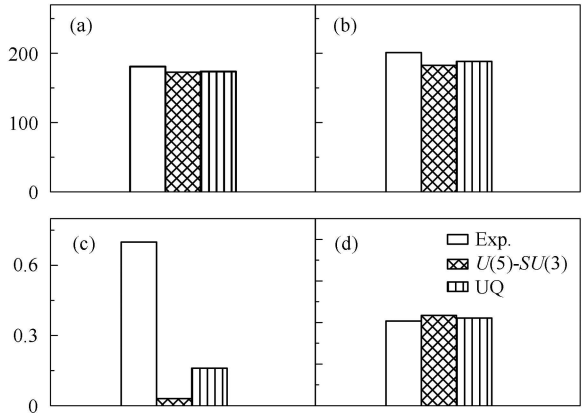

Fig. 2. B(E2) values calculated from the $U(5)$-$SU(3)$ scheme and the UQ scheme and the corresponding experimental data of ^{150}Nd, and where (a) $B(\mathrm{E2};4_1^+ \to 2_1^+)$, (b) $B(\mathrm{E2};\ 10_1^+ \to 8_1^+)$, (c) $B(\mathrm{E2};\ 2_2^+ \to 0_1^+)$, (d) $B(\mathrm{E2};\ 4_2^+ \to 2_1^+)$.

Because the value of $B(\mathrm{E2};2_2^+ \to 0_1^+)$ from the $U(5)$-$SU(3)$ scheme is too small in comparison with the experimental data, it seems that the result from the UQ scheme is much better than those from the $U(5)$-$SU(3)$ scheme in describing this intraband transition.

4 Conclusion

In the present work, the quantum phase transitional behavior of an alternative characterization of the spherical to axially deformed shape-phase transition in IBM is further explored, in which the usual $SU(3)$ quadrupole-quadrupole interaction is replaced by an $O(6)$ cubic interaction. We apply this alternative scheme to further investigate another candidate ^{150}Nd of N=90 with $X(5)$ symmetry. The low-lying energy levels and E2 transitions are calculated and compared with the experimental data, and the results show that the new scheme can also reasonably describe the experimental low-lying spectrum and E2 transitions for ^{150}Nd. Thus the UQ scheme can display almost the same transitional patterns as found in the original $U(5)$-$SU(3)$ scheme. Our analysis confirms that the $O(6)$ cubic interaction can indeed play a role similar to that of the $SU(3)$ quadrupole-quadrupole interaction.

However, for the low-lying spectrum, the UQ scheme seems better in describing the energy levels, especially in higher excited levels and γ band, yet the $U(5)$-$SU(3)$ scheme seems better to describe the β band; and for the $B(\mathrm{E2})$ transition, the results of the intraband transitions within the ground band from the UQ scheme seem better than those from the $U(5)$-$SU(3)$ scheme, the UQ scheme can also improve some interband transitions between the β band and the ground band. Yet the $U(5)$-$SU(3)$ scheme seems better to describe the intraband E2 transitions within β band.

Indeed, as shown in the application of the $U(5)$-$SU(3)$ and UQ schemes to the critical point symmetry candidate, ^{150}Nd, the overall fitting quality of the UQ scheme is almost similar to that of the $U(5)$-$SU(3)$ scheme. As has been stated in Ref. [23], whether the $O(6)$ cubic interaction in place of the usual $SU(3)$ quadrupole-quadrupole interaction is required in the deformed limit is not clear at the moment since comprehensive phenomenological studies of this question are still lacking, hence it is our aim to provide more theoretical studies by the $O(6)$ cubic interaction in comparison with the $SU(3)$ quadrupole-quadrupole interaction and experimental data. It is important that more candidates with $X(5)$ symmetry should be further explored along this idea to help us understand deeply the new characteristics of symmetry by the higher order $O(6)$ cubic interaction.

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