Optimization of magnet sorting in a storage ring using genetic algorithms*

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Abstract: In this paper, the genetic algorithms are applied to the optimization problem of magnet sorting in an electron storage ring, according to which the objectives are set so that the closed orbit distortion and beta beating can be minimized and the dynamic aperture maximized. The sorting of dipole, quadrupole and sextupole magnets is optimized while the optimization results show the power of the application of genetic algorithms in magnet sorting.

Key words: storage ring, magnet sorting, closed orbit distortion, beta beating, dynamic aperture, genetic algorithms

PACS: 29.20.-C **DOI:** 10.1088/1674-1137/37/12/127004

1 Introduction

There are three main kinds of magnet elements in storage rings: dipole, quadrupole and sextupole magnets.

The actual magnetic field in storage rings is somewhat different from the designed magnetic field. Most of the errors of the magnetic field can be partially corrected by trimming or shimming magnetic poles. However, the residual errors will induce some effects, such as closed orbit distortion (COD), lattice functions distortion, tune shift, and dynamic aperture shrinkage. These effects need to be minimized to improve the performance of storage rings. For example, the dynamic aperture shrinkage caused by the sextupole field errors cannot be ignored in advanced third-generation light sources, future-ring based colliders and ultimate storage rings [1–8], where the nonlinear performance is very sensitive to the errors. So the work of the optimization of magnet sorting is in high demand for installation of magnets during the construction of a storage ring.

There are several methods used for the optimization of magnet sorting, such as analytical methods, random search method [9] and artificial intelligence algorithms. In the analytical methods, the aim is set to solve linear equations or to find the pair wise dipoles whose phase difference is equal to integer multiples of pi [10]. The random search method is not suitable for the storage rings with numerous magnets. For a third-generation light source storage ring consisting of hundreds of magnets, the number of all possible permutations of magnets is $n!$ (n denotes the number of magnets). In the past decade, the artificial intelligence algorithms, which are used for the search of the global optimal solutions and high efficiency of problem optimization, have been successfully applied to many research fields. In this paper, the genetic algorithms (GAs) are applied to the solution of the magnet sorting problem in an electron storage ring. Sorting the dipole magnets gets a smaller COD than analytical methods, and sorting the quadrupole magnets gets a smaller beta beating than random search methods while sorting the sextupole magnets gets a bigger dynamic aperture than random search methods.

2 The magnet sorting method

The COD in storage rings mainly originates from the integral magnetic field errors; the lattice function distortions and tune shift mainly come from the gradient field errors when the sextupole field errors deteriorate the dynamic aperture and correction chromaticities. As the permutation of magnets changes, the above effects will change simultaneously. The purpose of magnet sorting is to find the permutation that can minimize the harmful effects. In principle, the best magnet permutation can be found by minimizing the objective functions of different sorting scheme.

In mathematics, the sorting problem is regarded as one of the NP-hard problems. To date, it is generally accepted that there is no effective analytical algorithm for NP-complete problem or NP-hard problem [11]. Such problems can only be solved by using approximate algorithms. For instance, the traveling salesman problem (TSP) is one of the typical NP-hard problems studied

Received 13 March 2013

* Supported by National Natural Science Foundation of China (10979045, 11175180, 11175182)

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in operations research and theoretical computer science. Given a list of cities and the distances between each other, the task is to find the shortest path along which one can visit each city only once and return to the original city. TSP is an exact problem, which is often used as a benchmark for many optimization methods. Till now, various algorithms based on artificial intelligence algorithms have been used to solve TSP, and yet, the optimal route can only be found when the city number is small.

Considering the similarity between TSP and the magnet sorting problem, the GAs are adopted as optimization algorithm to find the best sorting scheme. Compared with the traditional mathematical methods, GAs have stronger global optimization capability and have been used in accelerator physics research in recent years, such as linear lattice and nonlinear performance of storage ring optimization [12–15].

The basic processes of the GAs are:

(1) Input the errors, population size, maximum of generation number, crossover probability and mutation probability.

(2) Create a population consisting of a group of random permutations.

(3) Calculate the fitness value for each permutation (the fitness value is determined by COD, beta beating or dynamic aperture).

(4) Based on the fitness value, select the subjects that will survive.

(5) Based on the given probability, crossover or mutate.

(6) Increase the counter of generation $t{=}t{+}1$, and repeat the steps (3)–(6). Stop the processes, if generation counter equals the maximum of generation number.

According to the typical GA procedure, we have developed a code to solve the magnet sorting problem, in which the inputted information is the magnet lattice datum and the magnetic field errors.

We conducted numerical simulation [16] to test the efficiency of the developed method. We choose the Shanghai Synchrotron Radiation Facility (SSRF) as the test model. The lattice of the SSRF is a four-fold symmetry structure, each super-period includes five DBA cells. The storage ring, whose circumference is about 432 meters, consists of 40 dipole magnets, 240 quadrupole magnets and 140 sextupole magnets [17]. The lattice functions of half super-period are shown in Fig. 1.

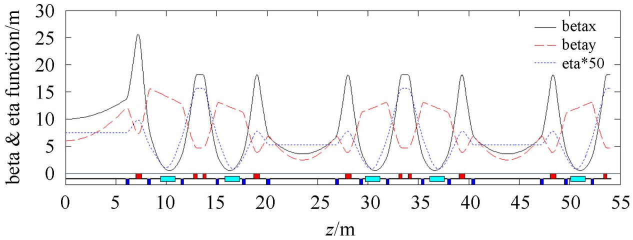

Fig. 1. (color online) The lattice functions of half super-period (SSRF).

3 The numerical simulation

3.1 Dipole sorting

The COD in the storage ring mainly comes from the integral magnetic field errors of dipole magnets. Thus, in dipole sorting, this paper will treat quadrupole magnets as ideal ones. The COD can be calculated using the following formula:

$$X_{\mathrm{c}}(z) = \frac{\beta_x(z)}{2\sin(\pi\nu_x)}, \sum_i \sqrt{\beta_{xi}}\cos(\Delta\varphi_i^z - \pi\nu_x)\Delta\theta_{xi}$$

$$\Delta\theta_{xi} = \frac{\Delta(BL)}{B\rho} = \frac{\Delta(BL)}{BL}\frac{L}{\rho};$$

$$\Delta\varphi_i^z = \int_i^z \frac{\mathrm{d}z}{\beta_x},$$

the index 'i' denotes the i-th dipole magnets, while 'z' marks the points of observation. For dipole magnets, whose length is usually long (in this paper the dipole length is 1.44 m), we insert 10 thin kicks with the same strength along each dipole magnet. In the simulation, we compared various numbers of kicks with 10 kicks and confirmed that the calculated result of COD was highly precise.

In dipole sorting, we expect to get the minimum COD. Therefore, we choose the maximum of COD and root mean square (RMS) of COD to be involved in optimization since the RMS of COD decreases proportionally with the maximum of COD. Thus, the objective function for dipole sorting is set as follows:

$$f = \max(|x_{ci}|).$$

In order to test the optimization capability of GAs approach, we implemented a numerical simulation and compared the result of GAs with Ref. [10].

In order to demonstrate the general optimization of various error distributions by the GAs methods, we selected three types of error distributions in the dipole, quadrupole and sextupole: normal distribution (error A & error B), uniform distribution (error C). Considering the actual error distributions are usually normal ones, we compared two groups of normal distributions, i.e., error A and error B. The distribution of error A: $\mu{=}0$, $\sigma{=}3$, the distribution of error B: $\mu{=}0$, $\sigma{=}6$. The letter μ indicates the average value of errors, σ denotes the standard deviation of errors.

In the dipole simulation, the differences of integral magnetic field errors between magnets are set to be about $\pm 5‰$ and generated randomly (as shown in Fig. 2).

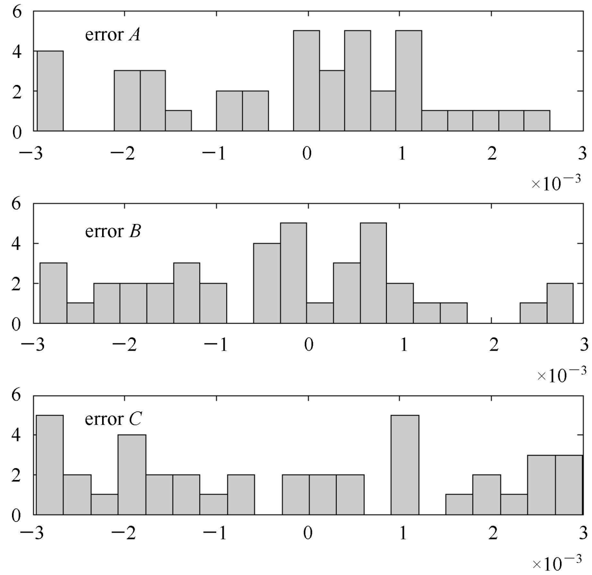

Fig. 2. Error distributions.

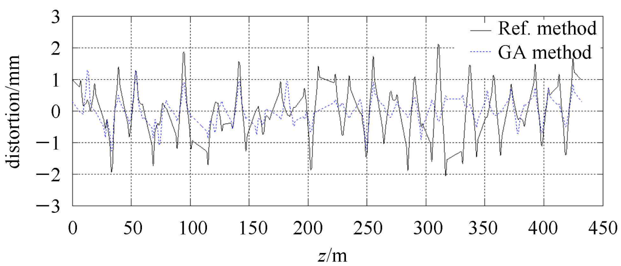

Fig. 3. (color online) The COD of two sorting approaches in the case of error A.

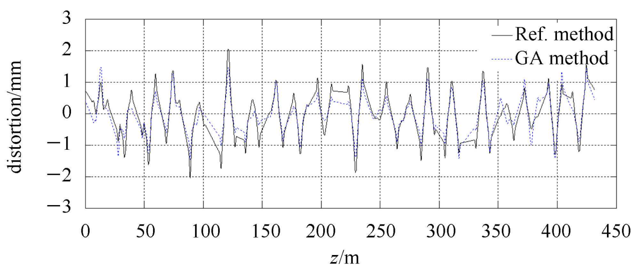

Fig. 4. (color online) The COD of two sorting approaches in the case of error B.

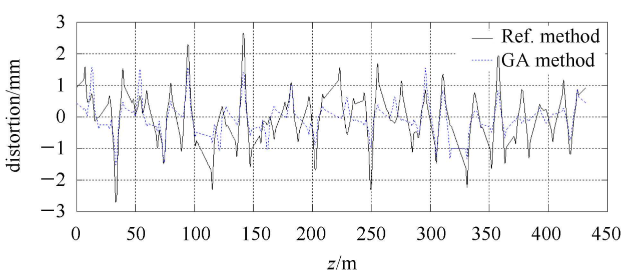

Fig. 5. (color online) The COD of two sorting approaches in the case of error C.

The results of GAs approach sorting and Ref. [10] are shown in Figs. 3–5.

As shown in Figs. 3–5 and Table 1, whatever type the error distribution is, the maximum value and RMS of COD of our results are almost half of the results of Ref. [10].

Table 1. The data of two sorting approaches.

	Max of COD/mm	RMS of COD/mm
Ref. [10] error A	2.108	0.814
GAs error A	1.284	0.421
Ref. [10] error B	2.043	0.728
GAs error B	1.469	0.584
Ref. [10] error C	2.712	0.849
GAs error C	1.556	0.524

The sorting process in Refs. [10, 18] is shown as below:

1) Place the dipole magnet whose absolute value of field error is the largest to the location whose objective function is the minimum one among all the locations that it could be placed.

2) Place the dipole magnet of the second absolute value in field error where the object function is the minimum one among all the locations that remain for it to be placed.

3) Follow the above step to place the dipole magnets by the ranking of the absolute value of the field errors from large to small one by one.

The parameters of the GAs code were set as: population size=200, max generation=50, mutation probability=0.2, crossover probability=0.65. The running time of this program is about 1 hour, on personal computer (CPU: Intel i5-2300 2.8 GHz).

3.2 Quadrupole sorting

The gradient magnetic field errors would produce the tune shift and lattice functions distortion, which has a serious harmful effect on beam quality. Recently, to improve the beam quality, quadrupole magnets are usually independently powered, which facilitate optical calibration and correction. However, the series-mode power supplies are still adopted for quadrupoles in several machines, such as damping ring and booster, etc. To demonstrate the effectiveness of our sorting code, we implemented the quadrupole magnet sorting.

The purpose of quadrupole magnet sorting is to minimize beta function distortion and tune shift, which can be described by one function, the beta beating:

$$\Delta\beta_{ui}=(\beta_{ui}-\beta_{0i})/\beta_{0i},$$

where 'u' represents the horizontal or vertical direction, the index 'i' marks the point of observation, '0' denotes the undisturbed beta function. We optimized the maximum and RMS of beta beating in both horizontal and vertical directions. Since the RMS reduces as the maximum of the beta beating decreases, the objective function is set as follows:

$$f=a\times\max|\beta_x|+b\times\max|\beta_y|,$$

where a and b are the weights of their corresponding function, respectively. After a few simulation tests, we found that the equal values of their weights leads to the minimum of beta beating and the specific values of the weights are not important (but they should be positive). Thus, we let $a=b=1$.

Considering that the tune shift is not strictly related to beta beating, we gave a constraint condition: if the tune shift is over ±0.03, let the value of the objective function be infinite.

In numerical simulation, the discrepancy between quadrupoles are assumed as random numbers, which are smaller than $\pm5‰$ (as shown in Fig. 6).

We implemented the GAs approach and random search approach to find the best permutations of quadrupole magnets. In the random search approach, 1×10^5 cases were studied while in the GAs approach the number of cases is 2×10^4. The two results are presented in Figs. 7–9.

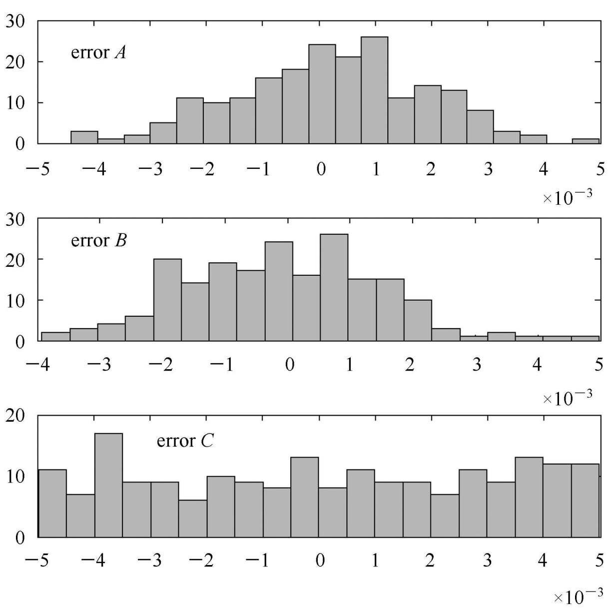

Fig. 6. Error distributions.

As shown in Table 2, better beta beating is achieved in shorter time by the GA approach, which indicates that the GAs approach is more efficient than the random search approach in the magnet sorting.

The parameters of the GA code were set as: population size=200, max generation=100, mutation probability=0.2 and crossover probability=0.65. The running time of the GA program is about 2 h.

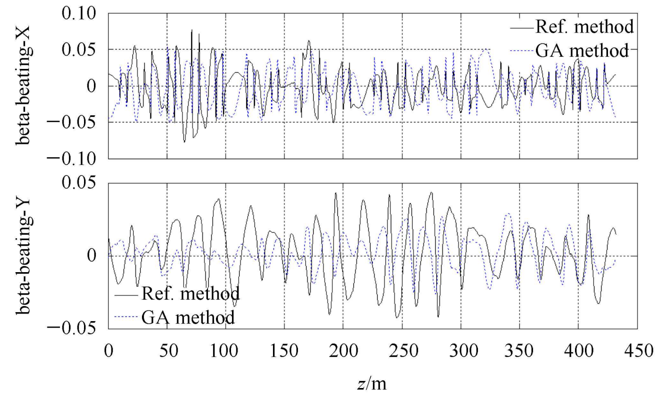

Fig. 7. (color online) Beta beating in the case of error A.

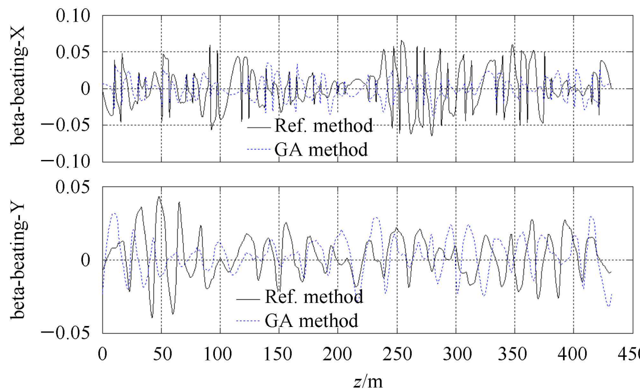

Fig. 8. (color online) Beta beating in the case of error B.

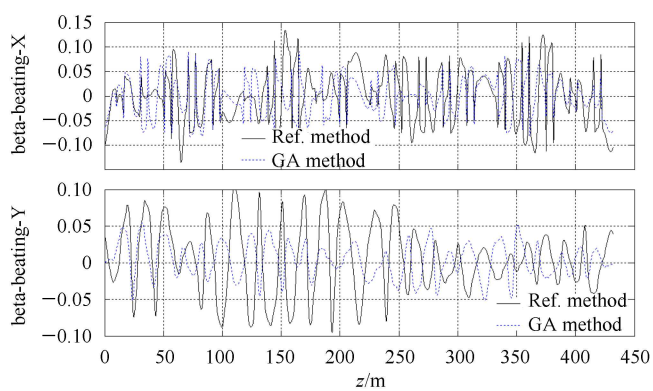

Fig. 9. (color online) Beta beating in the case of error C.

Table 2. The beta beating of the two approaches.

	horizontal direction		vertical direction	
	max	RMS	max	RMS
random error A	0.077	0.024	0.044	0.019
GAs error A	0.053	0.024	0.029	0.012
random error B	0.065	0.026	0.043	0.015
GAs error B	0.039	0.014	0.032	0.012
random error C	0.136	0.053	0.105	0.045
GAs error C	0.091	0.040	0.054	0.023

3.3 Sextupole sorting

In the storage ring with strong nonlinearity, magnetic field errors will produce dynamic aperture reduction, and distort correction chromaticities. These effects may trigger difficulty of beam injection and degeneration of beam lifetime. The reliable method to measure the harmful effects of sextupole field errors is numerical calculation. We have developed a particle tracking code, in which we adopted the second-order symplectic integrator method, and the integrated kick was placed in the middle of the sextupole [19, 20]. When the particles pass the kick, their direction will change immediately. The formula for the model reads as:

$$\begin{aligned} x^f &= x^i, \\ p_x^f &= p_x^i - \frac{1}{2}\lambda(z_0)\Delta z[(x^i)^2-(y^i)^2], \\ y^f &= y^i, \\ p_y^f &= p_y^i + \lambda(z_0)\Delta z(x^i y^i), \end{aligned}$$

the index i and f denote the status of particles before and after the kick, respectively. The particle tracking starts from 0 point as shown in Fig. 1. Dynamic aperture is obtained by multi-particle tracking.

We compared the dynamic aperture by our code with ELEGANT. See Figs. 10–12.

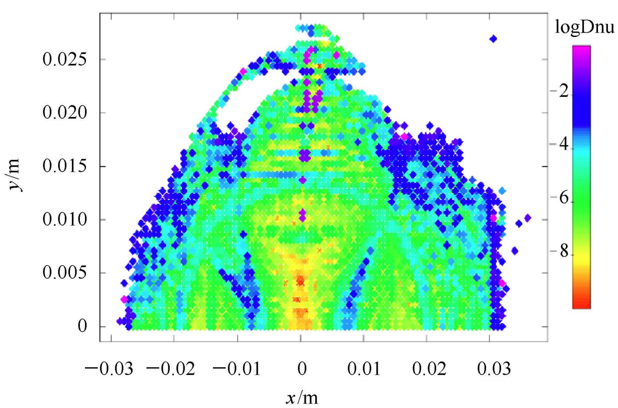

Fig. 10. (color online) Dynamic aperture by ELEGANT (1000 turns).

As shown in Figs. 10–12, the dynamic aperture by 10 kicks model is highly similar to the ELEGANT ones while the dynamic aperture by 1 kick model is acceptable. We adopt 1 kick model to optimize the sextupole magnets sorting, as the 1 kick model is 10 times faster than the 10 kick model.

To find the best sextupole magnet permutation based on the dynamic aperture tracking method, the dynamic aperture tracking results should be ranked according to some rules. The rules would reveal the area size and symmetry information of dynamic aperture. In order to construct the objective function more conveniently, the particles are set to distribute in concentric ellipses.

Since the chromaticities are quite important to the nonlinear effects of the storage ring, a constraint condition is adopted: if the chromaticities are over the range of 0–1, we let the value of the objective function be infinite.

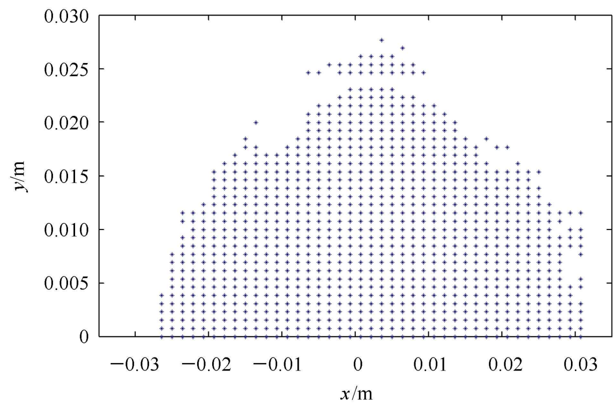

Fig. 11. (color online) Dynamic aperture by 10 kicks model (1000 turns).

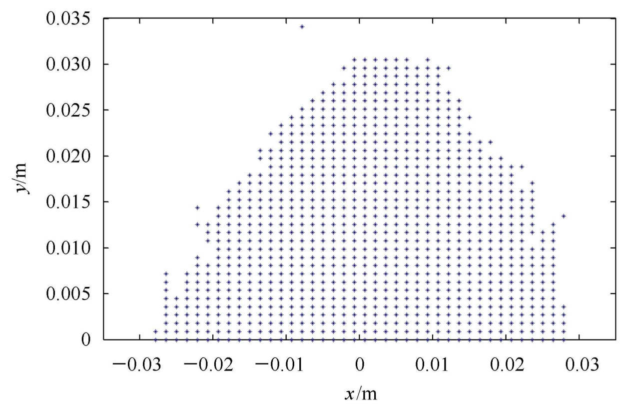

Fig. 12. (color online) Dynamic aperture by 1 kick model (1000 turns).

In the sextupole magnet sorting simulation, we compared the results of the GA approach and random search approach. In the random search approach, 3×10^5 cases were studied while in the GA approach the number of cases is 3×10^4. We set the deviations to a range of ±3% in order to show significant difference between the two approaches (as shown in Fig. 13), since the dynamic aperture of the storage ring is not very sensitive to the sextupole field errors. Figs. 14–16 give the dynamic aperture of the two approaches.

The circles mark the dynamic aperture of GAs optimization while the stars represent the best aperture of random search.

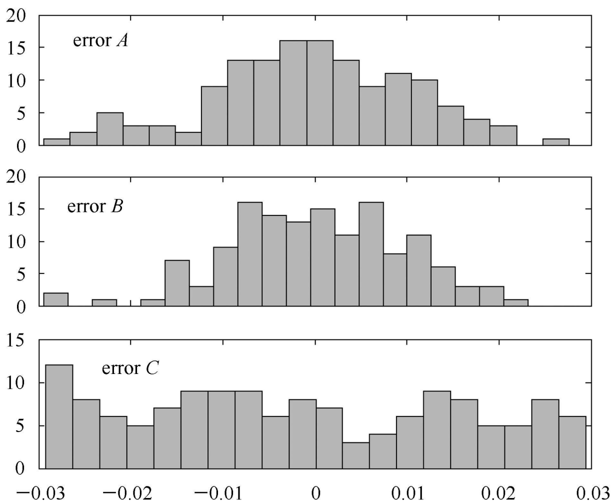

Fig. 13. Error distributions.

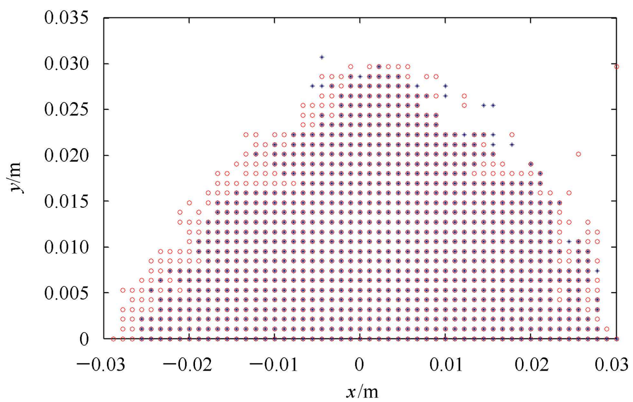

Fig. 14. (color online) The dynamic aperture in the case of error A (1000 turns).

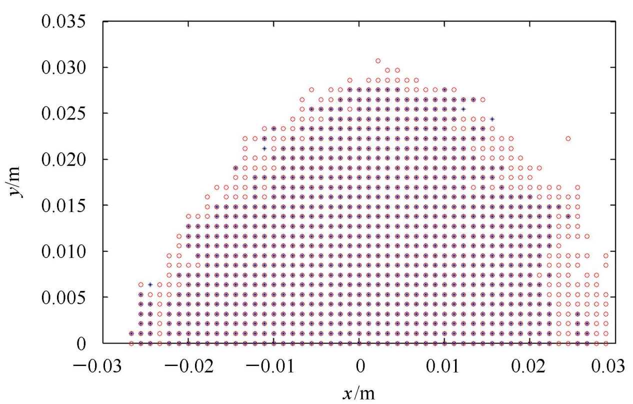

Fig. 15. (color online) The dynamic aperture in the case of error B (1000 turns).

The figures show that all the optimized dynamic apertures by the GAs approach have more particles surviving, and the distributions of particles are more symmetric. The sorting results of the two approaches indicate that the GAs approach has more powerful global search capability: less search space and better results.

The parameters of GAs code were set as: population size=300, max generation=100, mutation probability=0.2, crossover probability=0.7. The running time of this program is about 9 hours.

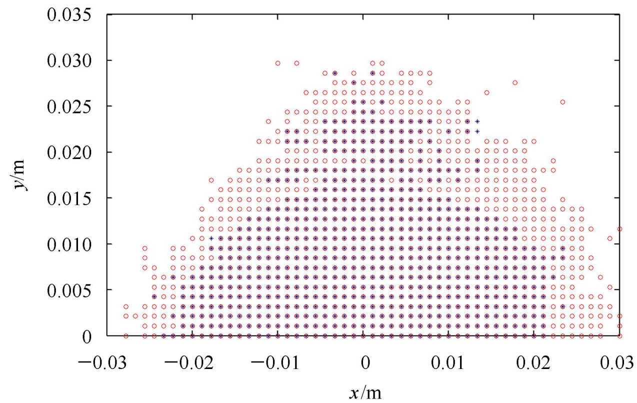

Fig. 16. (color online) The dynamic aperture in the case of error C (1000 turns).

The fast calculation of dynamic aperture can be attributed to two reasons: 1) The 1 kick model consists of only addition and multiplication, making the operation simpler. 2) In the tracking process, nodes are set in the 5th, 10th, 20th, 50th, 100th, 200th, 500th, 700th and 1000th turns. If the dynamic aperture is worse than the standard ones, the operation will be aborted. Consequently, the operation speed is significantly improved.

4 Conclusion

The numerical simulations indicate that whatever type the error distribution is, the GAs-based magnet sorting code is more efficient than the traditional or random search approaches. We also applied the GAs codes to the magnet sorting of the HLS II storage ring upgrade project and get the desired results, which will be employed in the HLS II storage ring magnets installation.

Magnet sorting is very common in machine installation, especially for dipole and sextupole magnets, which are powered in series-mode. On the basis of GAs associated with the creation of approximate objective function, we developed a code to find the desired magnet sorting permutation. Numerical simulations demonstrate the superiority of the code in the search of the quasi-optimal solution as compared with other methods.

References

1 BEI M, Borland M, CAI Yun-Hai et al. Nuclear Instruments and Methods in Physics Research Section A-Accelerators Spectrometers Detectors and Associated Equipment, 2010, **622**(3): 518–535

2 CAI Yun-Hai, Bane K, Hettel R et al. Physical Review Special Topics-Accelerators and Beams, 2012, **15**(5): 054002–054018

3 LI Wei-Min, WANG Lin, FENG Guang-Yao et al. Chinese Physics C, 2009, **33**: 128–130

4 Shaftan T, Rose J, Pinayev I et al. Particle Accelerator Conf., New York, 2005. 3473–3475

5 ZHANG Hao, LI Wei-Min, FENG Guang-Yao et al. Chinese Physics C (HEP & NP), 2012, **36**(1): 91–95

6 ZHANG Hao, WANG Lin, HE Duo-Hui. High Energy Physics and Nuclear Physics, 2006, **30**: 138–140 (in Chinese)

7 TIAN Shun-Qiang, ZHANG Wen-Zhi, LI Hao-Hu et al. Chinese Physics C (HEP & NP), 2009, **33**: 83–85

8 FAN Wei, BAI Zheng-He, GAO Wei-Wei et al. Sci. China-Phys. Mech. Astron, 2012, **55**(5): 802–807

9 Priano C, Bazzano G, Bianculli D et al. 1st International Particle Accelerator Conf. Kyoto, Japan, 2010, pp.MOPEB004

10 Raparia D, Fedotov A, Lee Y Y et al. www.bnl.gov/isd/ documents/26476.pdf, 2004: Medium: ED; Size: 4 pages

11 LIN Feng-Tse, KAO Cheng-Yan, HSU Ching-Chi. IEEE Transactions on Systems, Man and Cybernetics, 1993, **23**(6): 1752-17671767

12 GAO Wei-Wei, WANG Lin, LI Wei-Min. Physical Review Special Topics-Accelerators and Beams, 2011, **14**(9): 094001–094011

13 GAO Wei-Wei, WANG Lin, LI Wei-Min et al. Chinese Physics C, 2011, **35**(9): 859–864

14 YANG Ling-Yun, LI Yong-Jun, GUO Wei-Ming et al. Physical Review Special Topics-Accelerators and Beams, 2011, **14**(5): 054001–054010

15 YANG Ling-Yun, Robin D, Sannibale F et al. Nuclear Instruments Methods in Physics Research, Section A-Accelerators, Spectrometers, Detectors and Associated Equipment, 2009, **609**(1): 50–57

16 LEE S Y. Singapore: World Scientific, 2004

17 LIU Gui-Min, DAI Zhi-Min, LI Hao-Hu et al. Chinese Physics C (HEP & NP), 2006, **30**(Supp. Ⅰ): 144–146

18 HOU Jie, LIU Gui-Min, LI Hao-Hu et al. Chinese Physics C, 2008, **32**(4): 285–289

19 Ripken G, Schmidt F. cds.cern.ch/record/281283/files/sl-95-012.pdf, 1995

20 CAI Yun-Hai. www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-7793.pdf,1998