Dynamical study of light scalar mesons below 1 GeV in a flux-tube model *

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Abstract: The light scalar mesons below 1 GeV configured as tetraquark systems are studied in the framework of the flux-tube model. Comparative studies indicate that a multi-body confinement, instead of the additive two-body confinement, should be used in a multiquark system. The σ and κ mesons could be well accommodated in the diquark-antidiquark tetraquark picture, and could be colour-confinement resonances. The $a_0(980)$ and $f_0(980)$ mesons are not described as $K\bar{K}$ molecular states and $ns\bar{n}\bar{s}$ diquark-antidiquark states. However, the mass of the first radial excited state of the diquark-antidiquark state, $nn\bar{n}\bar{n}$ is 1019 MeV, is close to the experimental data of the $f_0(980)$.

Key words: scalar meson, tetra-quark system, flux-tube model, multi-body interaction

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1 Introduction

The charged κ, a scalar meson, was recently observed by the BES collaboration [1]. The Breit-Wigner mass and the decay width were found to be $826\pm49^{+49}_{-34}$ MeV and $449\pm156^{+144}_{-81}$ MeV, and the pole position was determined to be $(764\pm63^{+71}_{-54})-i(306\pm149^{+143}_{-85})$ MeV/c^2. These are in good agreement with those of the neutral κ, whose mass and decay widths are $878\pm23^{+64}_{-55}$ MeV and $499\pm53^{+55}_{-87}$ MeV, respectively, as observed by the BES and other collaborations [2].

The understanding of scalar mesons, which have the same quantum numbers as the vacuum, is a crucial problem in low-energy quantum chromodynamics (QCD) since they could shed light on the chiral symmetry breaking mechanism, and presumably also on confinement in QCD. Although many of the properties of scalar mesons have been studied for decades, the understanding of the internal structure of scalar mesons is still a puzzle. Their masses do not fit into the quark model predictions [3, 4]. The flavor structures of these light scalar mesons below 1 GeV, $a_0(980)$, $f_0(980)$, σ and κ, are still an open question. In the $q\bar{q}$ configuration, the p-wave relative motion between q and $\bar{q}$ has to be invoked to account for the spin and parity of the scalar mesons. This leads to much higher masses for them. Another possible configuration for scalar mesons is a tetraquark state. In the tetraquark configuration, the light scalar mesons could be classified into an $SU(3)$ flavor nonet if the diquark picture is used [5–8]. Their quark contents can be expressed as

$$\sigma = [ud][\bar{u}\bar{d}],\ f_0^0=\frac{[su][\bar{s}\bar{u}]+[sd][\bar{s}\bar{d}]}{\sqrt{2}};$$

$$\kappa^+ = [ud][\bar{d}\bar{s}],\ \bar{\kappa^+}=[ds][\bar{u}\bar{d}],\ \kappa^0=[ud][\bar{u}\bar{s}],\ \bar{\kappa^0}=[us][\bar{u}\bar{d}];$$

$$a_0^+ = [su][\bar{s}\bar{d}],\ a_0^0=\frac{[su][\bar{s}\bar{d}]-[sd][\bar{s}\bar{u}]}{\sqrt{2}},\ a_0^-=[sd][\bar{s}\bar{u}].$$

Jaffe et al. interpreted light scalar mesons as tetraquark states with all the relative orbital angular momenta assumed to be zero [5–11]. Weinstein et al. described light scalar mesons as hadronic molecular states due to strong meson-meson interaction [12–19]. The properties of some of these light scalar mesons were also studied in the $q\bar{q}$ picture [20–22]. The spectrum of light scalar mesons below 1.0 GeV were studied in the $q\bar{q}$ picture by including instanton interaction [23]. Bhavyashri et al. studied the instanton-induced interaction in the light meson spectrum on the basis of the phenomenological harmonic models for quarks [24]. Vijande et al. studied the scalar mesons in terms of the mixing of a chiral nonet of tetraquarks with conventional $q\bar{q}$ states [25, 26].

A multi-quark state is quite different from ordinary hadrons ($q\bar{q}$ mesons and qqq baryons) because the multi-

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quark state has more color structures than those of ordinary hadrons. The color structures of a multi-quark state are no longer trivial and the properties of multi-quark states may be sensitive to the hidden color structure. A tetraquark state, if its existence is confirmed, may provide important information about the low-energy QCD interaction which is absent from the ordinary hadrons. Some authors have studied the tetraquark system with the three-body $qq\bar{q}$ and $q\bar{q}\bar{q}$ interaction [27, 28], and exotic hadrons have also been studied as multiquark states in the flux-tube model in our previous work [29, 30]. These studies suggest that the multi-body confinement, instead of the additive two-body confinement, might be more suitable in the quark model study of multiquark states. The newly updated experimental data might shed more light on the possibility of the existence of tetraquark states and QCD interaction for multi-quark states.

The aim of this paper is to study the properties of scalar mesons below 1 GeV in the flux-tube model with multi-body confinement potential. The powerful method for few-body systems with high precision, the Gaussian expansion method (GEM) [31], is used here. The paper is organized as follows: in Section 2, the flux-tube model with multi-body interaction is introduced. A brief introduction of GEM and the construction of the wave functions of tetra-quark states are given in Section 3. The numerical results and discussions are presented in Section 4, and a brief summary is given in the last section.

2 Quark model and multi-body confinement potential

Long-term studies on hadrons in the past several decades indicate that ordinary hadrons ($q\bar{q}$ mesons and qqq baryons) can be well described by QCD-inspired quark models. Low-energy QCD phenomena are dominated by two well known quark correlations: confinement and chiral symmetry breaking. The perturbative, effective one-gluon exchange properties of QCD should also be included. Hence, the main ingredients of the quark model are: constituent quarks with a few hundred MeV effective mass, phenomenological confinement potential, effective Goldstone bosons, and one-gluon exchange between these constituent quarks.

For ordinary hadrons, their color structures are unique and trivial. Naive models based on two-body color confinement interactions proportional to the color charges $\lambda_i \cdot \lambda_j$ can describe the properties of ordinary hadrons well. However, the structures of a multiquark state are abundant [29, 30, 32], which include important QCD information that is absent from ordinary hadrons. There is no theoretical reason to directly extend the two-body confinement in the naive quark model to a multi-quark system. Furthermore, the direct application of the two-body confinement to the multi-quark system induces many serious problems, such as anti-confinement [27] and color Van der Waals force. Much theoretical work has been done to try to amend these serious drawbacks. The string flip model for multi-quark systems was proposed by M. Oka to avoid the pathological Van der Waals force [33, 34]. Three-quark confinement is explored by introducing strings which connect quarks according to a certain configuration rule.

Recent lattice QCD studies [35–37] show that the confinement of multi-quark states is a multi-body interaction and is proportional to the minimum of the total length of strings which connect the quarks to form a multiquark state. Based on these studies, a naive flux-tube or string model [29, 30, 32] with multi-body confinement has been proposed for the multiquark systems. The harmonic interaction approximation, i.e., the total length of the strings is replaced by the sum of the square of the string lengths, is assumed to simplify the numerical calculation.

The diquark-antidiquark picture of tetraquark states has been discussed by several authors [3, 38–40]. In the present work, the scalar mesons below 1 GeV are studied as diquark-antidiquark systems in the flux-tube model. Two color structures for a tetraquark state are shown in Fig. 1, where the solid dot represents a quark, while the hollow dot represents an antiquark. $\boldsymbol{r}_i$ is the quark's position and $\boldsymbol{y}_i$ represents a junction where three strings (flux tubes) meet. A thin line connecting a quark and a junction (an antiquark) represents a fundamental representation, i.e. color triplet. A thick line connecting two junctions is for a color sextet or other representations, namely a compound string. The different types of string may have different stiffnesses [41–43]. In Fig. 1(b), color couplings satisfying the overall color singlet of the tetraquark are $[[qq]_{\bar{3}}[\bar{q}\bar{q}]_3]_1$ and $[[qq]_6[\bar{q}\bar{q}]_{\bar{6}}]_1$. The subscripts represent the dimensions of the color representations.

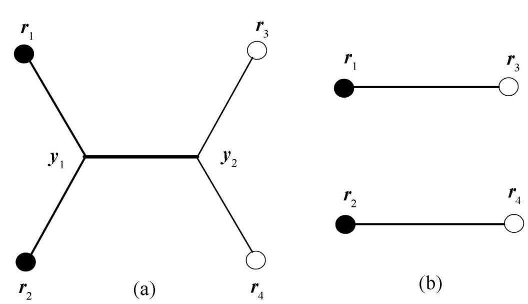

Fig. 1. Two-color structures for a tetraquark state.

In the flux-tube model with quadratic confinement potential, which is believed to be flavor independent, the tetraquark state with a diquark-antidiquark structure has the following form [29],

$$V^{\rm CH} = k[(\boldsymbol{r}_1-\boldsymbol{y}_1)^2+(\boldsymbol{r}_2-\boldsymbol{y}_1)^2+(\boldsymbol{r}_3-\boldsymbol{y}_2)^2 + (\boldsymbol{r}_4-\boldsymbol{y}_2)^2+\kappa_{\rm d}(\boldsymbol{y}_1-\boldsymbol{y}_2)^2], \tag{1}$$

where k is the stiffness of the string with the fundamental representation 3, which is determined by the meson spectrum, and $k\kappa_{\rm d}$ is the compound string stiffness. The compound string stiffness parameter $\kappa_{\rm d}$ [43] depends on the color representation, $\boldsymbol{d}$, of the string,

$$\kappa_{\rm d}=\frac{C_{\rm d}}{C_3}, \tag{2}$$

where $C_{\rm d}$ is the eigenvalue of the Casimir operator associated with the $SU(3)$ color representation $\boldsymbol{d}$ of the string. $C_3=\frac{4}{3}$, $C_6=\frac{10}{3}$ and $C_8=3$.

For given quark positions $\boldsymbol{r}_i$ $(i=1,\cdots,4)$, we can fix the positions of the junctions $\boldsymbol{y}_i$ $(i=1,2)$ by minimizing the energy of the system. After fixing $\boldsymbol{y}_i$, a set of canonical coordinates, $\boldsymbol{R}_i$ $(i=1,\cdots,4)$, is introduced to simplify the expressions of the potential, which are read as,

$$\begin{aligned} \boldsymbol{R}_1 &= \sqrt{\frac{1}{2}}(\boldsymbol{r}_1-\boldsymbol{r}_2), \quad \boldsymbol{R}_2=\sqrt{\frac{1}{2}}(\boldsymbol{r}_3-\boldsymbol{r}_4), \\ \boldsymbol{R}_3 &= \sqrt{\frac{1}{4}}(\boldsymbol{r}_1+\boldsymbol{r}_2-\boldsymbol{r}_3-\boldsymbol{r}_4), \\ \boldsymbol{R}_4 &= \sqrt{\frac{1}{4}}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4). \end{aligned} \tag{3}$$

Therefore, the minimum of the confinement interaction has the following form,

$$V^{\rm CH}_{\rm min} = k\left(\boldsymbol{R}_1^2+\boldsymbol{R}_2^2+\frac{\kappa_{\rm d}}{1+\kappa_{\rm d}}\boldsymbol{R}_3^2\right). \tag{4}$$

Taking into account the potential energy shift, the confinement potential $V^{\rm C}_{\rm min}$ used in the present calculation has the following form

$$V^{\rm CH}_{\rm min}=k\left[(\boldsymbol{R}_1^2-\Delta)+(\boldsymbol{R}_2^2-\Delta)+\frac{\kappa_{\rm d}}{1+\kappa_{\rm d}}(\boldsymbol{R}_3^2-\Delta)\right]. \tag{5}$$

Carlson and Pandharipande also considered a similar flux-tube energy shift, which they assumed to be proportional to the number of quarks N [44]. Obviously, the confinement potential $V^{\rm C}$ is a multi-body interaction rather than a two-body interaction. It should be emphasized here that our approach is different from that in Iwasaki's work [45], where the four-body problem is simplified to a two-body one by treating the diquark as an antiquark and antidiquark as a quark.

With regard to the mesons $\rm f_0(980)$ and $\rm a_0(980)$, these are also interpreted as $\rm K\bar{K}$ molecular states with $I=0$ and $I=1$, respectively [12–19]. In the flux-tube model, the confinement potential of the $\rm K\bar{K}$ molecular states can be written as

$$V^{\rm CM}_{\rm min}=k[((\boldsymbol{r}_1-\boldsymbol{r}_3)^2-\varDelta)+((\boldsymbol{r}_2-\boldsymbol{r}_4)^2-\varDelta)], \tag{6}$$

where $\rm q_1$ ($\rm q_2$) and $\rm \bar{q}_3$ ($\rm \bar{q}_4$) compose a K ($\rm \bar{K}$) meson, see Fig. 1(b). In fact, the mesons $\rm f_0(980)$ and $\rm a_0(980)$, if they are really tetraquark systems, should be the superposition of the diquark-antidiquark state and $\rm K\bar{K}$ molecular state. When two mesons, K and $\rm \bar{K}$, are largely separated, the dominant component of the system should be two isolated color singlet mesons, because other hidden color flux-tube structures are suppressed due to a confinement. With the separation reduction, a loose $\rm K\bar{K}$ molecular state may be formed if the attractive force between $\rm K\bar{K}$ is strong enough. In particular, when they are close enough to be within the range of a confinement (about 1 fm), the diquark-antidiquark state and the $\rm K\bar{K}$ molecular state may appear due to the excitation and rearrangements of flux tubes and junctions. In this case, the confinement potential of a tetraquark system, $\rm ns\bar{n}\bar{s}$, should be taken to be the minimum of two flux-tube structures. It reads

$$V^{\rm C}_{\rm min}=\min\left[V^{\rm CM}_{\rm min},V^{\rm CH}_{\rm min}\right]. \tag{7}$$

The other parts of the Hamiltonian are the rest masses, kinetic energies, one-gluon-exchange potential and Goldstone-boson-exchange potentials [29],

$$H = \sum_{i=1}^{4}\left(m_i+\frac{\boldsymbol{p}_i^2}{2m_i}\right)-T_{\rm CM}+V^{\rm C}+\sum_{i>j}^{4}(V^{\rm G}_{ij}+V^{\rm B}_{ij}), \tag{8}$$

$$V^{\rm B}_{ij} = v^{\pi}_{ij}\sum_{a=1}^{3}F^a_iF^a_j+v^{\rm K}_{ij}\sum_{a=4}^{7}F^a_iF^a_j+v^{\eta}_{ij}(F^8_i\cdot F^8_j\cos\theta_{\rm P}-F^0_i\cdot F^0_j\sin\theta_{\rm P}), \tag{9}$$

$$v^{\chi}_{ij} = \frac{g^2_{\rm ch}}{4\pi}\frac{m^3_{\chi}}{12m_im_j}\frac{\varLambda^2_{\chi}}{\varLambda^2_{\chi}-m^2_{\chi}}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\left[Y(m_{\chi}r_{ij})-\frac{\varLambda^3_{\chi}}{m^3_{\chi}}Y(\varLambda_{\chi}r_{ij})\right], \quad \chi=\pi,{\rm K},\eta. \tag{10}$$

$$V^{\rm G}_{ij} = \frac{\alpha_{\rm s}}{4}\lambda^c_i\cdot\lambda^c_j\left[\frac{1}{r_{ij}}-\frac{\pi}{2}\delta(\boldsymbol{r}_{ij})\left(\frac{1}{m^2_i}+\frac{1}{m^2_j}+\frac{4}{3m_im_j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right)\right], \tag{11}$$

where $T_{\rm CM}$ is the center-of-mass kinetic energy, and $\boldsymbol{F}_i,\lambda_i$ are the flavor, color SU_3 Gell-Mann matrices. $Y(x)$ is the standard Yukawa function, and all other symbols have their usual meanings. The δ-function should be regularized [46, 47]

$$\delta(\boldsymbol{r}_{ij})=\frac{1}{4\pi}\frac{\mathrm{e}^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}, \tag{12}$$

where μ is the reduced mass of q_i and q_j, and $r_0(\mu)=\hat{r}_0/\mu$. The effective scale-dependent strong coupling constant is given by [47]

$$\alpha_{\rm s}(\mu)=\frac{\alpha_0}{\ln\left[\dfrac{\mu^2+\mu_0^2}{\Lambda_0^2}\right]}. \tag{13}$$

3 Wave functions and the Gaussian expansion method

The total wave function of a diquark-antidiquark state can be written as a sum of the following direct products of color, isospin and spatial-spin terms,

$$\begin{aligned}\Phi_{IJ_{\rm T}M_{\rm T}}^{[\rm qq][\bar{q}\bar{q}]} = & \sum_{l,s,c,I}\xi_{l,s,c,I}\Big[\Big[\big[\phi_{l_1m_1}^{\rm G}(\boldsymbol{r})\eta_{s_1m_{s_1}}\big]_{J_1M_1} \\ & \times\big[\psi_{l_2m_2}^{\rm G}(\boldsymbol{R})\eta_{s_2m_{s_2}}\big]_{J_2M_2}\Big]_{J_{12}M_{12}} \\ & \times\chi_{LM}^{\rm G}(\boldsymbol{X})\Big]_{J_{\rm T}M_{\rm T}}\Big[\eta_{i_1m_{i_1}}\eta_{i_2m_{i_2}}\Big]_I \\ & \times[\chi_{c_1w_1}\chi_{c_2w_2}]_1.\end{aligned} \tag{14}$$

Here, I and $J_{\rm T}$ are the total isospin and total angular momentum, respectively. $\eta_{s_1m_{s_1}}$ $(\eta_{s_2m_{s_2}})$, $\eta_{i_1m_{i_1}}$ $(\eta_{i_2m_{i_2}})$ and $\chi_{c_1w_1}$ $(\chi_{c_2w_2})$ are the spin, flavor and color wave functions of the diquark (antidiquark), respectively. [ ]'s denote the Clebsh-Gordan coefficients coupling. The coefficient $\xi_{l,s,i,c,L}^{\rm IJ_T}$ is determined by diagonalizing the Hamiltonian, and subscripts l, s, i, c, and L represent all the possible intermediate quantum numbers, therefore our calculations are multi-channel coupling calculations. The Jacobi coordinates of the tetraquark are defined as

$$\begin{aligned}\boldsymbol{r} &= \boldsymbol{r}_1-\boldsymbol{r}_2,\quad \boldsymbol{R}=\boldsymbol{r}_3-\boldsymbol{r}_4, \\ \boldsymbol{X} &= \frac{m_1\boldsymbol{r}_1+m_2\boldsymbol{r}_2}{m_1+m_2}-\frac{m_3\boldsymbol{r}_3+m_4\boldsymbol{r}_4}{m_3+m_4}, \\ \boldsymbol{R}_{\bf CM} &= \frac{m_1\boldsymbol{r}_1+m_2\boldsymbol{r}_2+m_3\boldsymbol{r}_3+m_4\boldsymbol{r}_4}{m_1+m_2+m_3+m_4},\end{aligned} \tag{15}$$

where particles 1 and 2 are two quarks and particles 3 and 4 are two antiquarks. L, l_1 and l_2 are the orbital angular momenta associated with the coordinates of $\boldsymbol{X}$, $\boldsymbol{r}$ and $\boldsymbol{R}$, respectively. The calculation is done in the center-of-mass coordinate system ($\boldsymbol{R}_{\bf CM}=0$). The tetraquark state is an overall color singlet with well defined parity $P=(-1)^{l_1+l_2+L}$, isospin I and total angular momentum $J_{\rm T}$. For scalar mesons, we set the angular momentum L, l_1 and l_2 to be zero.

For the color part, the color singlet is constructed in the following two ways: $\chi_c^1=\bar{3}_{12}\otimes 3_{34}$, $\chi_c^2=6_{12}\otimes\bar{6}_{34}$; both "good" diquark and "bad" diquark are included. With respect to the flavor part, the flavor wave function reads as $\eta_I=\eta_{12}\otimes\eta_{34}$. Taking into account all degrees of freedom, the Pauli principle must be satisfied for each subsystem of the identical quarks or antiquarks. To obtain a reliable solution to the few-body problem, a high-precision method is indispensable. In this work, the GEM [31], which has been proven to be rather powerful in solving the few-body problem, is used to perform the calculations. In GEM, three relative motion wave functions are written as,

$$\begin{aligned}\phi_{l_1m_1}^{\rm G}(\boldsymbol{r}) &= \sum_{n_1=1}^{n_{1\max}}c_{n_1}N_{n_1l_1}r^{l_1}\mathrm{e}^{-\nu_{n_1}r^2}Y_{l_1m_1}(\hat{\boldsymbol{r}}), \\ \psi_{l_2m_2}^{\rm G}(\boldsymbol{R}) &= \sum_{n_2=1}^{n_{2\max}}c_{n_2}N_{n_2l_2}R^{l_2}\mathrm{e}^{-\nu_{n_2}R^2}Y_{l_2m_2}(\hat{\boldsymbol{R}}), \\ \chi_{LM}^{\rm G}(\boldsymbol{X}) &= \sum_{n_3=1}^{n_{3\max}}c_{n_3}N_{LM}X^{L}\mathrm{e}^{-\nu_{n_3}X^2}Y_{LM}(\hat{\boldsymbol{X}}).\end{aligned}$$

The Gaussian size parameters are taken as the following geometric progression numbers

$$\nu_n=\frac{1}{r_n^2},\ r_n=r_1a^{n-1},\ a=\left(\frac{r_{n_{\max}}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{16}$$

Within the framework of the flux-tube model, the wavefunctions of a $\rm K\bar{K}$ molecular state can be expressed as

$$\begin{aligned}\Phi_{IJ_{\rm T}M_{\rm T}}^{\rm K\bar{K}} = & \sum_{M,S,I}\xi_{M,S,I}\big[\big[\phi_{\rm K}^{\rm G}(\boldsymbol{r})\psi_{\rm \bar{K}}^{\rm G}(\boldsymbol{R})\chi_{LM}^{\rm G}(\boldsymbol{X})\big] \\ & \times\eta_{\rm S}\big]_{J_{\rm T}M_{\rm T}}\eta_{\rm I}\chi_{\rm c}.\end{aligned} \tag{17}$$

The details of the wavefunctions are omitted and are similar to those of a diquark-antiquark state.

4 Numerical results and discussions

Now we turn to the calculation of tetraquark states with diquark-antiquark structures. The model parameters are fixed by reproducing the ordinary meson spectrum and are listed in Table 1. the meson spectrum can be reproduced very well. Because the flux-tube model is reduced to the ordinary quark model for a $\rm q\bar{q}$ system, the obtained meson spectra (from light to heavy) are similar to those found in other work, e.g. Ref. [47]. Parts of the calculated meson spectra are shown in Table 2. The experimental values are taken from the PDG compilation [48].

Table 1. The model parameters (Set I). The masses of π,K,η take the experimental values.

m/MeV	m_s/MeV	k/(MeV·fm^{-2})	$\hat{r}_0$/(MeV·fm)	Λ_0/fm^{-1}	μ_0/fm^{-1}	Δ/fm^2	α_0	Λ_π/fm^{-1}	$\Lambda_K=\Lambda_\eta$/fm^{-1}	θ_P/(°)
313	520	213.3	30.85	0.187	0.113	0.5	4.25	4.2	5.2	15

Table 2. The meson spectra (unit: MeV).

meson	π	K	ρ	K*	ω	φ
Cal.	139	502	761	897	735	1023
Exp.	139	496	770	898	780	1020
$\sqrt{\langle r^2\rangle}$/fm	0.57	0.60	1.05	0.96	1.02	0.85

The energies of scalar meson states can be obtained by solving the four-body Schrödinger equation

$$(H-E)\Phi_{I J_T M_T}=0 \tag{18}$$

with Rayleigh-Ritz variational principle. In GEM the calculated results are converged with $n_{1\max}=6$, $n_{2\max}=6$ and $n_{3\max}=6$. The minimum and maximum ranges of the bases are 0.1 fm and 2.0 fm for coordinates $\boldsymbol{r}$, $\boldsymbol{R}$ and $\boldsymbol{X}$, respectively.

The quark contents and corresponding masses in the three different quark models for the light scalar mesons as tetra-quark states are shown in Table 3, where n stands for a non-strange quark (u or d), s stands for a strange quark, I and N denote the total isospin and principal quantum number of the total radial excitation, and S, L and J have their usual meanings. "Naive" stands for the naive quark model, where only one-gluon-exchange potential is taken into account in addition to the additive two-body confinement [49]. "Chiral" stands for the chiral quark model, where one-gluon-exchange and one-Goldstone-boson-exchange are included besides the additive two-body confinement [47]. The masses in the naive and chiral model are much higher (several hundred MeV) than those in the flux-tube model. The origin of this discrepancy mainly comes from the different type of confinement interaction, a two-body confinement potential is applied in the naive and chiral model, whereas a multi-body interaction confinement is used in the flux-tube mode. Zou et al. studied scalar mesons in the quark model by introducing three-body confinement interaction. Their study also indicates that the multi-body confinement potential, instead of two-body interaction, should be applied in the study of multi-quark states [50]. The naive quark model gives the highest masses, due to the absence of Goldstone boson exchange, which induces additional attraction for the tetraquark system.

In the framework of the flux-tube model, it can be seen from Table 3 that the lowest masses of the $nn\bar{n}\bar{n}$ and $nn\bar{n}\bar{s}$ systems are 587 MeV and 948 MeV, which are close to the masses of the σ and κ mesons. If the existence of the σ and κ mesons is further confirmed, then the tetraquark state is a possible interpretation. This interpretation is in agreement with many other studies [5–11]. Prelovsek et al. recently studied the light scalar mesons σ and κ by lattice QCD simulation, and they also found that σ and κ have sizable tetra-quark components, $nn\bar{n}\bar{n}$ and $nn\bar{n}\bar{s}$, respectively [51]. In order to check the dependence of the numerical results on the model parameters, we make the same calculations of scalar mesons with another set of parameters, which are listed in Table 4 (the unchanged parameters are not listed). A meson spectrum that is almost the same is obtained. The results for the tetraquark states are shown in Table 5. Comparing tables 4 and 5, our results are quite stable against the variation in model parameters.

Table 3. The numerical results for three models (unit: MeV).

flavour	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{n}$
IJ^P	00^+	00^+	10^+
$N^{2S+1}L_J$	0^1S_0	1^1S_0	0^1S_0
naive	938	1431	1431
chiral	666	1237	1406
flux-tube	587	1019	1210
candidate	σ	$f_0(980)$?	—
mass	541±39 [?]	980±10 [48]	
flavour	$nn\bar{n}\bar{s}$	$ns\bar{n}\bar{s}$	$ns\bar{n}\bar{s}$
IJ^P	$\frac{1}{2}0^+$	00^+	10^+
$N^{2S+1}L_J$	0^1S_0	0^1S_0	0^1S_0
naive	1216	1456	1456
chiral	1122	1454	1454
flux-tube	948	1314	1318
candidate	κ	—	—
mass	$826\pm49^{+49}_{-34}$ [1]		

Table 4. The model parameters (Set II).

k/(MeV·fm^{-2})	$\hat{r}_0$/(MeV·fm)	Δ/fm^2	α_0
267	30.0	0.6	4.09

Table 5. The numerical results in the flux-tube model (unit: MeV).

flavour	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{n}$	$nn\bar{n}\bar{s}$	$ns\bar{n}\bar{s}$	$ns\bar{n}\bar{s}$
IJ^P	00^+	00^+	10^+	$\frac{1}{2}0^+$	00^+	10^+
$N^{2S+1}L_J$	0^1S_0	1^1S_0	0^1S_0	0^1S_0	0^1S_0	0^1S_0
flux-tube	531	969	1180	908	1270	1275

The σ and κ mesons, if they have diquark-antidiquark structures, cannot decay into two colorful hadrons directly due to color confinement. They must transform into color singlet mesons by means of breaking and rejoining flux tubes before decaying into color singlet mesons. This decay mechanism is similar to the compound nucleus formation and therefore should induce a resonance, which is named a "color confined, multi-quark resonance" state [30, 52]. The large decay width of the σ and κ mesons may be qualitatively explained if the arrangement and rupture of the flux tubes are fast enough. The systematic investigation of the decay is left for future work.

In the case of the $f_0(980)$ and $a_0(980)$ mesons, many theoretical studies assumed them to be tetra-quark states with quark content $ns\bar{n}\bar{s}$ and isospin $I=0$ and $I=1$, respectively. The masses for the tetraquark states $ns\bar{n}\bar{s}$ are much higher, about 300 MeV, than the experimental values, even in the flux-tube model; see Table 3 and Table 5. Therefore, their main components do not seem to be the tetraquark state $ns\bar{n}\bar{s}$ in the quark models. Instead, the mass of the first radial excited state of the $nn\bar{n}\bar{n}$ state is 1019 MeV, which is close to the mass of the $f_0(980)$ meson. Taking the $f_0(980)$ meson as the $nn\bar{n}\bar{n}$ state is consistent with Vijande's work on the nature of scalar mesons [25]. The observed $f_0(980)\rightarrow K\bar{K}$ process can be explained by the mixing of the $nn\bar{n}\bar{n}$, $ns\bar{n}\bar{s}$ and $s\bar{s}$ et al. strange quark components [25]. This work is being done in our group. Peláez also suggested that three scalar mesons, σ, κ and $f_0(980)$, have dominant tetraquark components, whereas the $a_0(980)$ meson might be a more complicated system [53], which is also consistent with our results.

The other well known interpretations of the $f_0(980)$ and $a_0(980)$ mesons are the $K\bar{K}$ bound states with isospin $I=0$ and $I=1$, respectively, because the experimental values are very close to the threshold of two mesons, K and $\bar{K}$. Within the quark models, the interactions between two quarks related to the color and spin factors, $\lambda_i\cdot\lambda_j$ and $\sigma_i\cdot\sigma_j$, are zero between the K and $\bar{K}$ mesons, therefore it is hard for the $K\bar{K}$ bound state to be formed. The coupling calculations on the diquark-antidiquark state and the $K\bar{K}$ state indicate that the $K\bar{K}$ bound state still cannot be formed. The arguments for this are: (i) the interactions between K and $\bar{K}$ are equal to zero, and the coupling interaction between the diquark-antidiquark state and the $K\bar{K}$ state is weak; and (ii) the relative kinetic energy between two mesons, K and $\bar{K}$, is not small due to the small mass of the K ($\bar{K}$) meson. These two factors are not beneficial to forming a bound state.

5 Summary

The comparative studies of the three quark models on light scalar mesons indicate that a multibody confinement potential, instead of a two-body confinement potential proportional to a color factor, plays an important role in a multiquark state, which can reduce the energy of a multiquark state because it avoids the appearance of anti-confinement in color symmetrical quark (antiquark) pairs.

In the flux-tube model, the σ and κ mesons can be assigned as diquark-antidiquark states $nn\bar{n}\bar{n}$ and $nn\bar{n}\bar{s}$ with $J^P=0^+$, respectively, which can be named as "color confined, multi-quark resonance" states. The interpretation of the $f_0(980)$ and $a_0(980)$ mesons as tetraquark states $ns\bar{n}\bar{s}$ with $I=0$ and $I=1$, respectively, would give a much higher mass (about 300 MeV) than the experimental data. The studies on the mixing of the diquark-antidiquark state $ns\bar{n}\bar{s}$ and the $K\bar{K}$ state indicate that the $K\bar{K}$ molecular state does not exist in the quark models due to weak coupling and a large relative kinetic energy between the K and $\bar{K}$ mesons. However, in our calculation the mass of the first radial excitation of the $nn\bar{n}\bar{n}$ diquark-antidiquark state is close to the mass of the $f_0(980)$ meson. The problem with this assignment, the small decay width of the $f_0(980)\rightarrow K\bar{K}$ meson, can be accounted for by the mixing of the $ns\bar{n}\bar{s}$ and $s\bar{s}$ et al. strange quark components with the $nn\bar{n}\bar{n}$ state.

At present, the nature of scalar mesons is still an open question, and the interpretation of scalar mesons as tetraquark states is a possibility. In fact, scalar mesons should be the superpositions of $q\bar{q}$, $qq\bar{q}\bar{q}$ and other components in a Fock space expansion approach, and the dominant one determined by quark dynamics. The mixing between two-body and four-body configurations would require knowledge of the quark-antiquark pair creation-annihilation interaction, which is being calculated in our group by tentatively using a $^3P_0$ model.

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