Comprehensive analysis of $e^+e^- \to \gamma\eta_c(2S)$ *

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Abstract: We discuss the production of $\eta_c(2S)$ through the process $e^+e^- \to \gamma\eta_c(2S)$, where the leading contribution originates from 1-loop electroweak corrections. Adopting some reasonable light-cone distribution amplitudes, we analyze the cross section of this process. As the electron-positron center of mass energy $\sqrt{s}$=3770 MeV, the typical production cross section of this process is about 1 fb.

Key words: OZI-forbidden, hadronization, $\eta_c(2S)$

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1 Introduction

From an experimental and theoretical point of view, the charmonium draws great attention because it provides an ideal laboratory in which to test predictions from the perturbative- and nonperturbative-chromodynamics. Since $\eta_c(2S)$ is the first radially excited S-wave spin singlet state in the charmonium system, it is very interesting to study its properties in detail and many works have been done recently.

For example, the authors of Ref. [1] estimated the decay rates of $\eta_c(2S) \to \gamma\gamma$, by taking into account both relativistic and QCD radiative corrections. Similarly, in Ref. [2], the author presented a relativistic calculation of two-photon decays for heavy and light mesons in the framework of the Salpeter equation for quark-antiquark states. In addition, observation of $\eta_c(2S)$ production in $\gamma\gamma$ fusion at CLEO was discribed in Ref. [3]. The authors of Ref. [4] presented the complexion of pseudoscalar mesons, and gave the mass and decay constant of $\eta_c(2S)$, The authors of Ref. [5] also presented the decay constants and the radiative decay widths of $\eta_b(nS)$ and $\eta_c(nS)$ which are computed within a semirelativistic quark model, using a potential found through the AdS/QCD correspondence. In Ref. [6], the authors neglected the mass of the light quark mass of the light meson and obtained an improved analytical expression for the rates of $J/\psi \to \eta\gamma, \eta'\gamma$.

However, it is very difficult to detect $\eta_c(2S)$ through the process $\psi(2S) \to \gamma\eta_c(2S)$, and the CLEO Collaboration set an upper bound on the branch ratio $B(\psi(2S) \to \gamma\eta_c(2S)) < 0.2\%$ at 90% confidence level (C.L.) as the mass of $\eta_c(2S) \sim 3594$ MeV. Since the relevant signal is overwhelmed by that from the background processes $\psi(2S) \to \gamma X$, they did not obtain evidence for the decay process $\psi(2S) \to \gamma\eta_c(2S), \eta_c(2S) \to \pi^+\pi^-\eta_c(1S)$ [7]. So in this work, we discuss the production of $\eta_c(2S)$ through the process $e^+e^- \to \gamma\eta_c(2S)$ which may be a new channel to product $\eta_c(2S)$.

The Okubo-Zweig-Iizuka (OZI) rule implies that the cross section or the branch ratio of the relevant processes is suppressed when there is no quark line connecting the initial and final hadron states. The branch ratio for the OZI-forbidden radiative decays in perturbative QCD has been investigated by Körner et al. [8], where the 4-point and 5-point loop functions are approximated in a weak-binding approach for both heavy and light mesons, that is the quark q and anti-quark $\bar{q}$ in mesons are all assumed to have the same momentum and satisfy the on-mass-shell conditions. Here, we abandon the weak-binding approach in this approximation to get the non-zero cross section, namely, we consider the relative momentum between q and $\bar{q}$ and not let them be on the mass shell.

Since three momenta of the final meson are larger than Λ_{QCD}, we adopt some typical light-cone wavefunctions [9–11] to evaluate the hadronic matrix elements which contain the effects from non-perturbative QCD. In principle, there is contribution to the cross section of $e^+e^- \to \gamma\eta_c(2S)$ from the tree level Feynman diagrams drawn in Fig. 1, which were calculated based on non-relativistic QCD a few years ago [12–14]. We also discuss the contribution to the cross section of the process $e^+e^- \to \gamma\eta_c(2S)$ from those one loop Feynman diagrams drawn in Fig. 2.

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In concrete calculation, the amplitudes corresponding to the one loop diagrams drawn in Fig. 2 are some simplified linear combinations of the Lorentz-covariant operators where those corresponding coefficients are reduced to standard scalar Passarino-Veltman integrals.

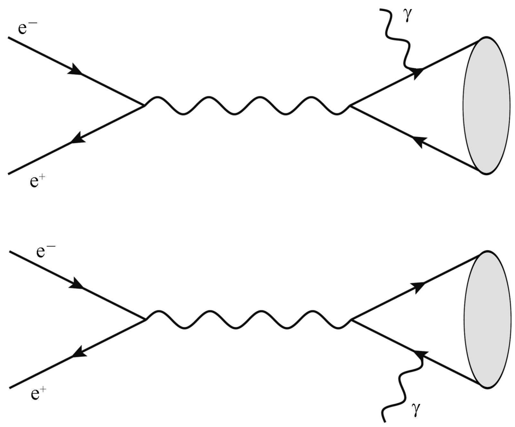

Fig. 1. The possible tree level Feynman diagrams contributing to the process $e^+e^- \to \gamma\eta_c(2S)$, where the gray bulb denotes the meson.

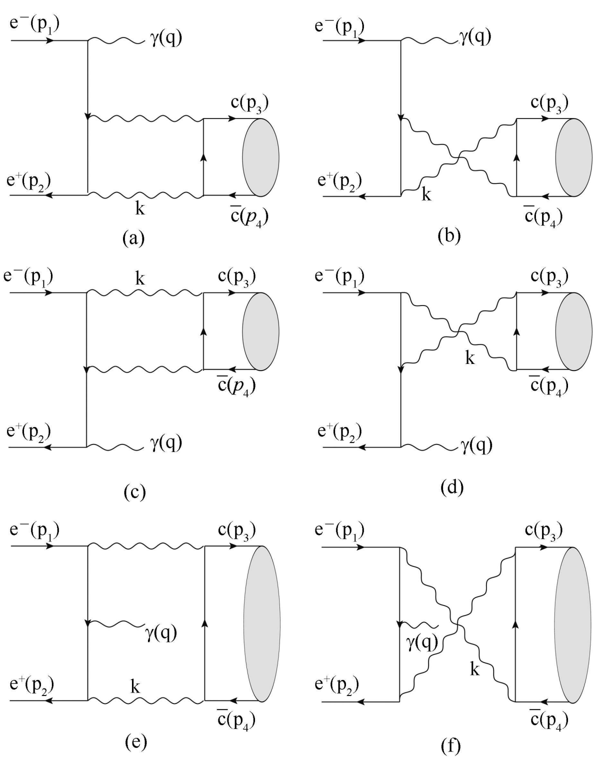

Fig. 2. The one loop Feynman diagrams contribute to the process $e^+e^- \to \gamma\eta_c(2S)$, where the gray bulb denotes the meson.

This paper is composed of the following sections. In section 2, we present the amplitudes of the diagrams drawn in Fig. 2 as the linear combinations of the Lorentz-covariant operators, and verify the contributions from those one loop diagrams which are infrared-safe when we consider the end-point behaviors of the wave functions. Section 3 is devoted to the numerical analysis and discussion. In section 4, we give our conclusion.

2 The leading contributions from one loop diagrams

As mentioned above, the leading contributions to the cross section of $e^+e^-\to\gamma\eta_c(2S)$ originate from those one loop diagrams drawn in Fig. 2 where the charm and anti-charm in the final state compose the meson $\eta_c(2S)$. In order to obtain the corrections from these diagrams properly, we employ some model-dependent wavefunctions to evaluate the relevant hadronic matrix elements. For the bound state $\eta_c(2S)$, the matrix element of nonlocal operators sandwiched between the vacuum and the meson could be written as:

$$\langle\eta|\bar{q}^a_\alpha(0)q^b_\beta(x)|0\rangle=\frac{\delta_{ab}}{4N_c}\Big\{\langle\eta|\bar{q}(0)q(x)|0\rangle +\gamma_5\langle\eta|\bar{q}(0)\gamma_5 q(x)|0\rangle+\gamma^\mu\langle\eta|\bar{q}(0)\gamma_\mu q(x)|0\rangle -\gamma^\mu\gamma_5\langle\eta|\bar{q}(0)\gamma_\mu\gamma_5 q(x)|0\rangle +\frac{1}{2}\sigma^{\mu\nu}\gamma_5\langle\eta|\bar{q}(0)\sigma_{\mu\nu}\gamma_5 q(x)|0\rangle\Big\}_{\beta\alpha}. \quad (1)$$

So the leading-twist distribution amplitude could be written as

$$\langle\eta(p')|\bar{q}(0)\gamma_\mu\gamma_5 q(x)|0\rangle=-\mathrm{i}f_\eta p'\int_0^1 \mathrm{d}u\,\mathrm{e}^{\mathrm{i}up'\cdot x}\phi(u), \quad (2)$$

in the momentum representation which is [9, 15]

$$\langle\eta(p')|\bar{q}^a_\alpha q^b_\beta|0\rangle=\mathrm{i}\frac{\delta_{ab}f_\eta}{4N_c}\int_0^1 \mathrm{d}u\{\not{p}_0\gamma_5\phi(u)\}_{\beta\alpha}, \quad (3)$$

where $\bar{q}$, q are the spinors of the valence anti-quark and quark in the meson, a, b=1, 2, $\cdots$, N_c are the color indices, p_0 is the momentum of the meson, f_η denotes the decay constant of $\eta_c(2S)$, respectively. In addition, $\phi(u)$ is the light-cone wavefunction of the meson, and denotes the leading-twist distribution of the momenta of valence quarks in the bound state. The momenta of valence quarks are $p(\bar{q})=up_0$, $p(q)=(1-u)p_0$ or $p(\bar{q})=(1-u)p_0$, $p(q)=up_0$. Certainly, the light-cone wavefunction of the meson satisfies the normalization condition

$$\int_0^1 \mathrm{d}u\phi(u)=1. \quad (4)$$

Here we adopt three typical light-cone wavefunctions of the meson [9, 15–19] to evaluate the hadronic matrix elements:

$$\begin{aligned}&\phi_1(u)=6u(1-u),\\&\phi_2(u)=30u^2(1-u)^2,\\&\phi_3(u)=\frac{15}{2}(1-2u)^2[1-(1-2u)^2].\end{aligned}\tag{5}$$

For example, the amplitude of Fig. 2(a) at quark level can be written as

$$\begin{aligned}\mathcal{M}_a =& \frac{e^5Q_\mathrm{q}^2}{(p_1-q)^2-m_\mathrm{e}^2}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\\&\times\overline{u}_\mathrm{q}(p_3)\gamma^\rho(\not{k}-\not{p}_4+m_\mathrm{q})\gamma^\mu\upsilon_\mathrm{q}(p_4)\\&\times\frac{\overline{\upsilon}_\mathrm{e}(p_2)\gamma_\mu(\not{k}-\not{p}_2+m_\mathrm{e})\gamma_\rho(\not{p}_1-\not{q}+m_\mathrm{e})\not{\varepsilon}^*u_\mathrm{e}(p_1)}{k^2(k-p_3-p_4)^2((k-p_2)^2-m_\mathrm{e}^2)((k-p_4)^2-m_\mathrm{c}^2)},\end{aligned}\tag{6}$$

with ε^ν denoting the photon polarization vector, Q_q is the quark electric charge, and m_e and m_c are the masses of the electron and the quark, respectively.

Applying Eq. (1), the hadronic matrix elements from this diagram is given as

$$\begin{aligned}&\langle\eta\gamma|\mathcal{M}_a|e^+e^-\rangle\\=&\frac{\mathrm{i}e^5Q_\mathrm{q}^2f_\eta\delta_{ab}}{4N_\mathrm{c}((p_1-q)^2-m_\mathrm{e}^2)}\\&\times\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int_0^1\mathrm{d}u\mathrm{Tr}[\not{p}_0\gamma_5\phi(u)\gamma^\rho(\not{k}-\not{p}_4+m_\mathrm{c})\gamma^\lambda]\\&\times\frac{\overline{\upsilon}_\mathrm{e}(p_2)\gamma_\mu(\not{k}-\not{p}_2+m_\mathrm{e})\gamma_\rho(\not{p}_1-\not{q}+m_\mathrm{e})\not{\varepsilon}^*u_\mathrm{e}(p_1)}{k^2(k-p_3-p_4)^2((k-p_2)^2-m_\mathrm{e}^2)((k-p_4)^2-m_\mathrm{c}^2)}.\end{aligned}\tag{7}$$

This hadronic matrix elements can be expressed as the linear combinations of some Lorentz-covariant tensors constructed by the metric tensor $g_{\mu\nu}$ and a linearly independent set of external momenta [20]:

$$\begin{aligned}&\langle\eta\gamma|\mathcal{M}_a|e^+e^-\rangle\\=&-\frac{e^5\pi^2f_\eta Q_\mathrm{q}^2}{N_\mathrm{c}((p_1-q)^2-m_\mathrm{e}^2)}\times\int_0^1\mathrm{d}u\phi(u)\overline{\upsilon}_\mathrm{e}(p_2)(A_0\not{\varepsilon}(q)\gamma_5\\&+A_1p_1\cdot\varepsilon(q)\gamma_5+A_2p_2\cdot\varepsilon(q)\gamma_5+A_3(p_1\cdot\varepsilon(q)\not{p}_2\gamma_5\\&+p_2\cdot\varepsilon(q)(\not{q}-\not{p}_1)\gamma_5+\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu(p_1^\nu p_2^\rho+p_2^\nu q^\rho)\varepsilon^\sigma(q))\\&+A_4\epsilon_{\mu\nu\rho\sigma}\sigma^{\mu\nu}p_2^\rho\varepsilon^\sigma(q)\\&+A_5\epsilon_{\mu\nu\rho\sigma}\sigma^{\mu\nu}(p_1-q)^\rho\varepsilon^\sigma(q))u_\mathrm{e}(p_1),\end{aligned}\tag{8}$$

with $\epsilon_{\mu\nu\rho\sigma}$ denoting the totally antisymmetric tensor, and the form factors A_i (i=0, 1, $\cdots$, 5) are defined as

$$\begin{aligned}A_0 =& 6D_{00}(m_\mathrm{e}^2+p_1\cdot p_2-2p_1\cdot q-p_2\cdot q)\\&+2C_2\Big(m_\mathrm{e}^4-2p_1\cdot qm_\mathrm{e}^2-(p_1\cdot p_2-p_2\cdot q)^2\Big),\\A_1 =& -2m_\mathrm{e}\Big(C_2m_\mathrm{e}^2+3D_{00}-C_1p_1\cdot p_2+C_1p_2\cdot q\Big),\\A_2 =& 2m_\mathrm{e}\Big(C_1m_\mathrm{e}^2+3D_{00}-C_2p_1\cdot p_2-2C_1p_1\cdot q\\&+C_2p_2\cdot q\Big),\\A_3 =& 2\Big(C_1m_\mathrm{e}^2+3D_{00}-C_2p_1\cdot p_2+2C_3p_1\cdot q\\&+C_2p_2\cdot q\Big),\\A_4 =& m_\mathrm{e}\Big(C_1m_\mathrm{e}^2+3D_{00}-C_2p_1\cdot p_2-2C_1p_1\cdot q\\&+C_2p_2\cdot q\Big),\\A_5 =& m_\mathrm{e}\Big(C_2m_\mathrm{e}^2+3D_{00}-C_1p_1\cdot p_2+C_1p_2\cdot q\Big),\end{aligned}\tag{9}$$

where

$$\begin{aligned}C_1 =& D_0+D_1+D_{13}+D_2+D_{23}+2D_3+D_{33}\\&-(u-1)(D_1+D_{11}+D_{12}+D_{13}),\\C_2 =& D_{12}+D_2+D_{22}+D_{23}\\&+u(D_1+D_{11}+D_{12}+D_{13}),\\C_3 =& -D_{13}-D_{23}-D_3-D_{33}\\&+(u-1)(D_1+D_{11}+D_{12}+D_{13}),\end{aligned}\tag{10}$$

with

$$\begin{aligned}D_{\{i,\ ij\}} =& D_{\{i,\ ij\}}((p_2-p_4)^2,m_c^2,m_\eta^2,(p_0-p_2)^2,\\&m_\mathrm{e}^2,m_\mathrm{c}^2,m_\mathrm{e}^2,m_\mathrm{c}^2,0,0),\end{aligned}\tag{11}$$

being the 4-point standard scalar Passarino-Veltman integrals [20], and they could be calculated by using 'LoopTools'. The hadronic matrix elements in Eq. (6) are infrared safe since we adopt the light-cone distribution amplitudes $\phi_i(u)$ (i=1, 2, 3). In other words, we do not need to worry about the infrared problem here [15]. Similarly, we can derive the correction to matrix elements from other diagrams, and the hadronic matrix elements of the 5-point figure can also be expressed as the linear combinations of some Lorentz-covariant tensors [21].

Using the hadronic matrix elements derived above, we write the cross section of $e^+e^-\to\gamma\eta_c(2S)$ as

$$\sigma=\frac{1}{16\pi s(s-4m_e^2)}\int|\mathcal{M}|^2\mathrm{d}t, \quad (12)$$

with $s=(p_1+p_2)^2$, $t=(p_1-q)^2=(p_2-p_0)^2$ are the Mandelstam variables.

In addition, we also consider the twist-3 distribution amplitudes defined in the matrix elements [11, 22]

$$\langle\eta(p')|\overline{q}(0)\mathrm{i}\gamma_5 q(x)|0\rangle = \frac{f_\eta m_\eta^2}{2m_q}\int_0^1 \mathrm{d}u e^{\mathrm{i}up'\cdot x}\phi_p^\eta(u),$$

$$\langle\eta(p')|\overline{q}(0)\sigma_{\mu\nu}\gamma_5 q(x)|0\rangle = -\frac{\mathrm{i}}{6}\frac{f_\eta m_\eta^2}{2m_q}\left[1-\left(\frac{2m_q}{m_\eta}\right)^2\right] \times(p'_\mu x_\nu-p'_\nu x_\mu)2m_q\int_0^1\mathrm{d}u e^{\mathrm{i}up'\cdot x} \times\phi_\sigma^\eta(u). \quad (13)$$

They can be expanded in terms of Gegenbauer polynomials:

$$\phi_p(u)=1+aC_2^{1/2}(u)+bC_4^{1/2}(u)+\cdots$$
$$\phi_\sigma(u)=6u(1-u)(1+dC_2^{3/2}(u)+\cdots), \quad (14)$$

where the coefficients a,b,d can be found in Refs. [11, 22].

3 Numerical analysis

Using the above preparation, we present the finally numerical results here. In our numerical analysis, we adopt the decay constant of $\eta_c(2S)$ $f_\eta=266$ MeV [5], the mass of $\eta_c(2S)$ $M_\eta=3637$ MeV, the charm quark mass $M_c=1270$ MeV, and the electron mass $M_e=0.511$ MeV, respectively [23]. The three possible distribution amplitudes $\phi_1(u)$, $\phi_2(u)$ and $\phi_3(u)$ are already given in Eq. (14).

We give the theoretical predictions on the corresponding cross section of $e^+e^-\to\gamma\eta_c(2S)$ at the electron-positron center of mass energy $\sqrt{s}=3770$ MeV for three distribution amplitudes in Table 1. We can see that the cross section from the distribution amplitude ϕ_3 is about five times that from the distribution amplitude ϕ_1, and ten times that from the distribution amplitude ϕ_2 quantitatively, which are similar to the case discussed in [15, 18]. In addition, we also consider the higher twist and get the cross section at twist-3, which is about 0.923 fb. So we could see that the effects are small for the twist-3 case.

Table 1. The cross section $e^+e^-\to\gamma\eta_c(2S)$ with three different distribution amplitudes $\phi(u)$ at $\sqrt{s}=3770$ MeV.

$\sigma(\phi_1)$/b	$\sigma(\phi_2)$/b	$\sigma(\phi_3)$/b
6.15E−16	2.53E−16	3.66E−15

The cross section of this process is typically about 10^{-15}b, which maybe implies that we can study the properties of $\eta_c(2S)$ through this window.

4 Conclusion

In this work, we investigate the production of $\eta_c(2S)$ through the process $e^+e^-\to\gamma\eta_c(2S)$. Using the popular light-cone wave function method to evaluate the hadronic matrix elements, we find the theoretical prediction on the corresponding cross section is typically 10^{-15}b. This result may imply that we can study the properties of $\eta_c(2S)$ through this window. In other words, the cross section with this order is large enough to be detected in future, and can be used to product the bound state $\eta_c(2S)$ at electron-positron colliders.

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