Side-effects of the space charge field introduced by a hollow electron beam in the electron cooler of CSRm*

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Abstract: An electron cooler is used to improve the quality of the ion beam in a synchrotron; however it also introduces a nonlinear electromagnetic field to the accelerator, which causes tune shift, tune spread and may drive resonances leading to ion beam loss. In this paper the tune shift and the tune spread caused by the nonlinear electromagnetic field of a hollow electron beam is investigated, and the resonance driving terms of the nonlinear electromagnetic field are analysed. The differences are presented compared with a solid electron beam. Calculations are performed for $^{238}U^{32+}$ ions of energy 1.272 MeV stored in the main Cooler Storage Ring (CSRm) at the Institute of Modern Physics, Lanzhou. It is found that in this situation the nonlinear field caused by the hollow electron beam does not lead to serious resonances.

Key words: electron cooler, tune shift, tune spread, resonance

PACS: 29.20.dk, 29.27.Bd, 41.85.Ew **DOI:** 10.1088/1674-1137/39/12/127003

1 Introduction

Two electron cooler devices, with the very important characteristic that the distribution of the electron beam is adjustable have been installed at the Cooler Storage Ring of Heavy Ion Research Facility in Lanzhou (HIRFL-CSR). Generally, a hollow electron beam is used to cool the ion beam stored in CSR, so analyzing the side-effects of the hollow electron beam on the ion beam is necessary for the HIRFL-CSR. Because the space charge field of the electron beam has larger effects on lower-energy ion beams, $^{238}U^{32+}$ ions of energy 1.272 MeV/u are chosen as a typical example in the calculation. The parameters used in the calculation are summarised in Table 1 and Table 2.

Table 1. Parameters used in the calculations.

particle	$^{238}U^{32+}$ 1.272 MeV
currents of the hollow and solid electron beam I_e	0.077 A
parameters for the electron beam distribution	$a_1=2.86\times10^{-4}$, $b_1=14.3\times10^{-2}$ $a_2=2.86\times10^{-4}$, $b_2=28.6\times10^{-2}$
cooling length Lcool	4 m
beta function in the cooler	β_x=10 m, β_y=17 m
tune of CSRm	Q_x=3.63, Q_y=2.61

The radial distribution of the electron beam in the cooler can be parameterized by the following equations [1]. Equation (1) is for a hollow electron beam and Eq. (2) is for a solid electron beam.

$$\rho(r)=\frac{\rho_h}{2}\left(\mathrm{erfc}((r-b_2)/a_2)-\mathrm{erfc}((r-b_1)/a_1)\right),$$
$$\text{with } \rho_h=\frac{I_e}{\pi v\int_0^\infty \mathrm{r}(\mathrm{erfc}((r-b_2)/a_2)-\mathrm{erfc}((r-b_1)/a_1))\mathrm{d}r}. \quad (1)$$

$$\rho(r)=\frac{\rho_s}{2}\left(\mathrm{erfc}((r-b_2)/a_2)\right),$$
$$\text{with } \rho_s=\frac{I_e}{\pi v\int_0^\infty r\mathrm{erfc}((r-b_2)/a_2)\mathrm{d}r}. \quad (2)$$

In the above equations, a_1, a_2, b_1, b_2 are constants determining the shape of the distribution (see Table 1), I_e is the current of the electron beam, and v is the velocity of the electron beam.

Figure 1 shows the normalized radial charge density distribution $\rho_n(R)$ ($\rho_n(R)=\rho(R)/\rho_{s,h}$) for solid electron beam and hollow electron beam.

Because the longitudinal length of the electron beam is far larger than its transverse width, and the distribution of the electron beam is axisymmetric, then using

Received 24 April 2015
* Supported by National Natural Science Foundation of China (11375245)
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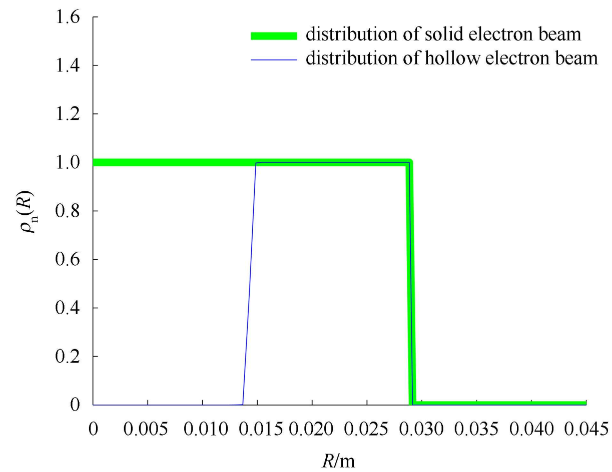

Fig. 1. (color online) Normalized radial distribution of the electron beam.

Ampere's law and Gauss's law, the magnetic field and the electric field produced by the electron beam can be obtained:

$$\vec{E}(R)=\frac{\int_0^R \rho(r)r\mathrm{d}r}{R\varepsilon_0}\hat{r}. \tag{3}$$

$$\vec{B}(R)=\frac{\mu_0\beta c\int_0^R \rho(r)r\mathrm{d}r}{R}\hat{\phi}. \tag{4}$$

ϵ_0 is the permittivity, μ_0 is the permeability, β is the relativistic factor, R is the transverse location, and $\rho(r)$ is the radial distribution of the electron beam in Eq. (1) and (2). A particle with charge q at location R will encounter the force:

$$F(R)=qE(R)+qv\times B(R). \tag{5}$$

Substituting Eq. (3) and (4) into Eq. (5), the force becomes:

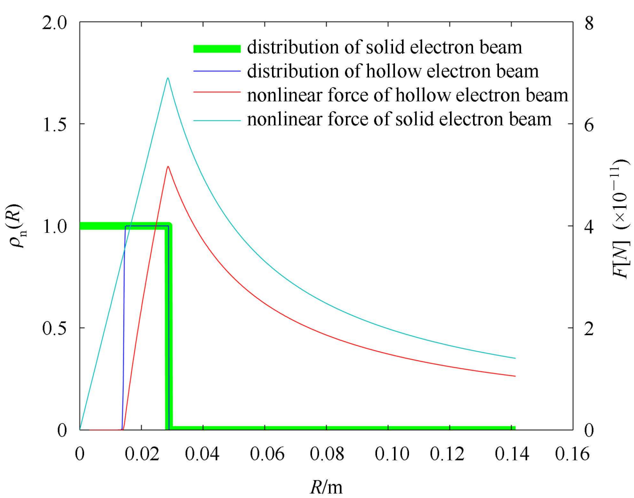

Fig. 2. (color online) Nonlinear force exerted on the particles at different radii.

$$F(R)=\frac{q}{R\varepsilon_0\gamma^2}\int_0^R \rho(r)r\mathrm{d}r, \tag{6}$$

where q is the charge of the particle, and γ is the Lorentz factor. As Fig. 2 and Eq. (6) show, the force that the particle encounters in the electron beam is nonlinear. So because of nonlinear tune shift, tune spread will be caused and resonances may be driven.

In Sections 2, 3 and 4 the calculations of tune shift, tune footprint and the tune spread for hollow electron beam and solid electron beam are presented respectively, in Section 5 the resonance driving terms are analyzed, and in Section 6 the conclusions are summarised.

2 Tune shift

With the thin lens approximation, the nonlinear transverse kick can be obtained [2, 3]:

$$\Delta r'=\frac{qq'N'}{2\pi\varepsilon_0\beta^2m_0c^2\gamma^3R}\frac{\int_0^R r\rho(r)\mathrm{d}r}{\int_0^\infty r\rho(r)\mathrm{d}r}. \tag{7}$$

$$N'=\left|\frac{L_{\mathrm{cool}}I_{\mathrm{e}}}{q'\beta c}\right|, \tag{8}$$

where q' is the charge of the electron, L_{cool} is the length of the cooler section, m_0 is the rest mass of the particle, and N' is the number of electrons in the cooler section [4].

Imitating the calculating method in Ref. [2], the differential equation for the transverse motion with the kicker of the electron beam can be written as follows [2]:

$$\frac{\mathrm{d}^2z}{\mathrm{d}s^2}+K(s)z=\Delta z'\delta(s). \tag{9}$$

$$\Delta z'=\frac{qq'N'}{2\pi\varepsilon_0\beta^2m_0c^2\gamma^3z}\frac{\int_0^z r\rho(r)\mathrm{d}r}{\int_0^\infty r\rho(r)\mathrm{d}r}. \tag{10}$$

Using the Courant–Snyder transformation

$$\eta=z/\sqrt{\beta_z},\theta=\int\mathrm{d}s/(Q\beta_z), \tag{11}$$

Eq. (9) will become:

$$\frac{\mathrm{d}\eta^2}{\mathrm{d}\theta^2}+Q^2\eta=Q\sqrt{\beta_z}\Delta z'(\eta,\theta)\delta(\theta), \tag{12}$$

where Q is the tune of the ring, β_z is the beta function of the ring. Transforming the equation to ε, ϕ:

$$\eta=\sqrt{\varepsilon}\cos(\phi),\ \eta'=\sqrt{\varepsilon}\sin(\phi). \tag{13}$$

ε is the parameter equal to $2J$, and J is the action variable of the particle. ϕ is the phase variable of the particle.

Replacing the periodic Dirac function in Eq. (12) by its Fourier expansion:

$$\delta(\theta)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\exp(-\mathrm{i}m\theta). \tag{14}$$

Then the equation of transverse motion can be transformed to two first-order differential equations:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\theta}=\frac{\sqrt{\varepsilon}\sin(\phi)\sqrt{\beta}_z\Delta z'}{\pi}\sum_{m=-\infty}^{\infty}\exp(-\mathrm{i}m\theta). \tag{15}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}\theta}=Q+\left(\frac{\sqrt{\beta}_z\cos(\phi)\Delta z'}{2\pi\sqrt{\varepsilon}}\right)\sum_{m=-\infty}^{\infty}\exp(-\mathrm{i}m\theta). \tag{16}$$

Now the nonlinear tune shift can be calculated by averaging the right hand side of Eq. (16) over θ and ϕ:

$$\Delta\nu=\frac{qq'N}{8\pi^3\varepsilon\beta^2c^2m_0\varepsilon_0\gamma^3}\int_0^{2\pi}\frac{\int_0^{\sqrt{\beta_z\varepsilon}\cos(\phi)}r\rho(r)\mathrm{d}r}{\int_0^{\infty}r\rho(r)\mathrm{d}r}\mathrm{d}\phi. \tag{17}$$

Integrating the right side of Eq. (17) with different ε, the variations of tune shift with the transverse oscillating amplitude of the particle ($A=\mathrm{sqrt}(\beta_z\epsilon)$) can be obtained.

Using the parameters in Table 1, the tune shift can be worked out. The maximum tune shift for hollow electron beam is $\Delta\nu_x=0.015$, $\Delta\nu_y=0.025$ and for solid electron beam is $\Delta\nu_x=0.024$, $\Delta\nu_y=0.031$. The variations of the tune shift with transverse oscillating amplitude of the particle is illustrated in Fig. 3.

From Fig. 3 the following phenomena can be observed:

1) The tune shift caused by the hollow electron beam is smaller than the tune shift caused by the solid electron beam.

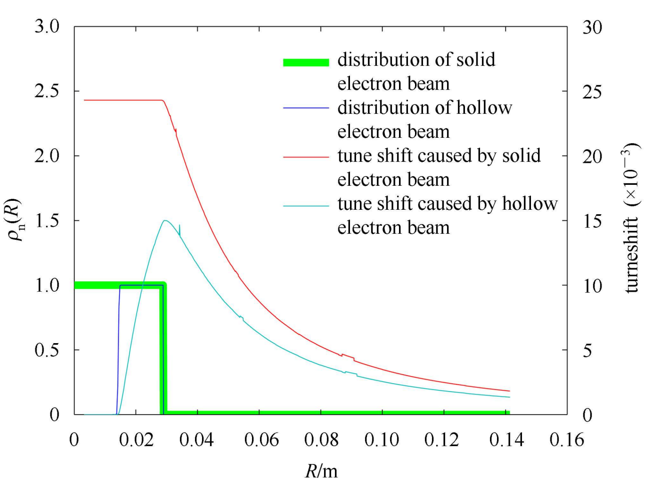

Fig. 3. (color online) Variation of tune shift caused by electron beam (horizontal direction only).

2) For hollow electron beam the tune shift is zero when the transverse oscillating amplitude of the particle is smaller than the inner radius of the hollow electron beam, while for the solid electron beam the tune shift is a constant when the transverse oscillating amplitude of the particle is smaller than the range of the electron beam.

3) Once the oscillating amplitude of the particle is larger than the range of electron beam, both tune shifts decrease with the increase of the transverse oscillating amplitude of the particle.

3 Tune footprint

From Equation (5) the two-dimensional equations of transverse motion can be derived:

$$\frac{\mathrm{d}\eta_x{}^2}{\mathrm{d}\theta^2}+Q_x{}^2\eta_x=Q_x\sqrt{\beta_x}\Delta x'\delta(\theta). \tag{18}$$

$$\frac{\mathrm{d}\eta_y{}^2}{\mathrm{d}\theta^2}+Q_y{}^2\eta_y=Q_y\sqrt{\beta_y}\Delta y'\delta(\theta). \tag{19}$$

$$\Delta x'=\frac{qq'N'x}{2\pi\varepsilon_0\beta^2m_0c^2\gamma^3R^2}\frac{\int_0^R r\rho(r)\mathrm{d}r}{\int_0^{\infty}r\rho(r)\mathrm{d}r}. \tag{20}$$

$$\Delta y'=\frac{qq'N'y}{2\pi\varepsilon_0\beta^2m_0c^2\gamma^3R^2}\frac{\int_0^R r\rho(r)\mathrm{d}r}{\int_0^{\infty}r\rho(r)\mathrm{d}r}. \tag{21}$$

$$x=\sqrt{\beta_x\varepsilon_x}\cos(\phi_x). \tag{22}$$

$$y=\sqrt{\beta_y\varepsilon_y}\cos(\phi_y). \tag{23}$$

$$R=\sqrt{\beta_x\varepsilon_x\cos^2(\phi_x)+\beta_y\varepsilon_y\cos^2(\phi_y)}. \tag{24}$$

Using the method in Ref. [2], the periodic Dirac function in Eqs. (18) and (19) is replaced by its Fourier expansion (Eq. (14)). Using the same method to get Eq. (17), the following equations are derived:

$$\Delta\nu_x(\varepsilon_x,\varepsilon_y)=\frac{1}{4\pi^2}\int_0^{2\pi}\mathrm{d}\phi_y\int_0^{2\pi}\frac{\Delta x'\beta_x\cos^2(\phi_x)}{2\pi\sqrt{\varepsilon_x\beta_x\cos^2(\phi_x)+\varepsilon_y\beta_y\cos^2(\phi_y)}}\mathrm{d}\phi_x. \tag{25}$$

$$\Delta\nu_y(\varepsilon_x,\varepsilon_y)=\frac{1}{4\pi^2}\int_0^{2\pi}\mathrm{d}\phi_y\int_0^{2\pi}\frac{\Delta y'\beta_y\cos^2(\phi_y)}{2\pi\sqrt{\varepsilon_x\beta_x\cos^2(\phi_x)+\varepsilon_y\beta_y\cos^2(\phi_y)}}\mathrm{d}\phi_x. \tag{26}$$

In the following calculations the transverse coupled oscillating amplitude of the particle is defined by:

$$A=\sqrt{\beta_x\varepsilon_x+\beta_y\varepsilon_y}. \quad (27)$$

In the above equation, ε_x equals $2J_x$, ε_y equals $2J_y$, and J_x J_y are the action variables of a particle in the horizontal direction and vertical direction respectively. β_x and β_y are the beta function at the cooler section.

To calculate the tune footprint of the particle caused by nonlinear electromagnetic field of the electron beam in CSRm, we assume that the ion beam obeys a Gaussian distribution in transverse phase space, and the emittance of the ion beam is equal to the acceptance of the ring. Generating 5×10^4 Gaussian particles with the parameters in Table 2 and integrating Eqs. (25) and (26) numerically, we get the tune shift for different particles. Plotting the $\Delta\nu_x$, $\Delta\nu_y$ in the ($\Delta\nu_x$, $\Delta\nu_y$) plane with the transverse oscillating amplitude of the particle (defined by Eq. (27)) as a parameter, the figures of the tune footprint can be obtained (Fig. 4, Fig. 5)[3].

Table 2. Parameters used to generate particles.

particle	$^{238}U^{32+}$ 1.272 MeV
horizontal emittance	200 πmm·mrad
vertical emittance	30 πmm·mrad
twiss parameters at	β_x=10 m, β_y=17 m
generating point	γ_x=1/10m^{-1}, γ_x=1/17m^{-1}, α=0
dispersion function	0 m

Figure 4 shows that when the transverse oscillating amplitude of the particle is smaller than the inner radius of the hollow electron beam ($A<R_{\text{inner-radius}}$), $\Delta\nu_x$, $\Delta\nu_y$ stay close to 0. When the transverse oscillating amplitude of the particle is in the range of the hollow electron beam ($R_{\text{inner-radius}}<A<R_{\text{outer-radius}}$), the $\Delta\nu_x$, $\Delta\nu_y$ increase with the increase of transverse oscillating amplitude of the particle. When the transverse oscillating amplitude of the particle is larger than the range of the electron beam ($A>R_{\text{outer-radius}}$), the $\Delta\nu_x$, $\Delta\nu_y$ decrease with the increase of the transverse oscillating amplitude of the particle.

Figure 5 shows that the particle has the largest tune shift when the transverse oscillating amplitude of the particle is smaller than the range of the solid electron beam, as the bluest point at the top right corner shows. When the transverse oscillating amplitude of the particle increases, the tune shift $\Delta\nu_x$, $\Delta\nu_y$ decreases.

Comparing the tune footprint in Fig. 4 and Fig. 5, the following conclusions can be drawn: The area of tune footprint caused by the hollow electron beam is larger than that caused by the solid electron beam, while the maximum of the tune shift caused by the hollow electron beam is smaller than that caused by the solid electron beam.

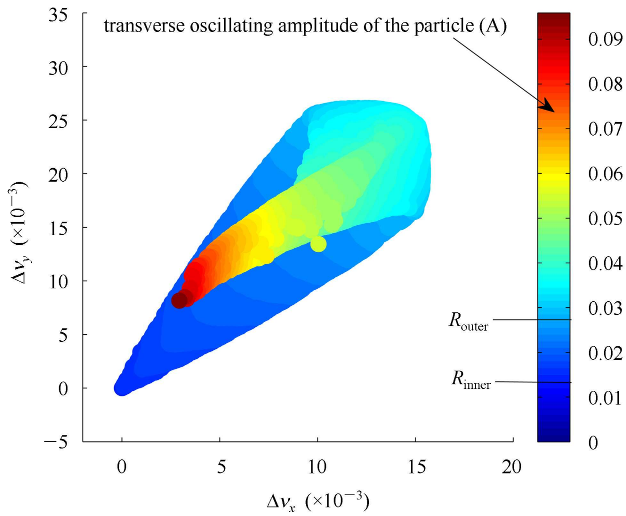

Fig. 4. (color online) Tune footprint caused by the hollow electron beam. R_{inner}, R_{outer} represent the ranges of the hollow electron beam.

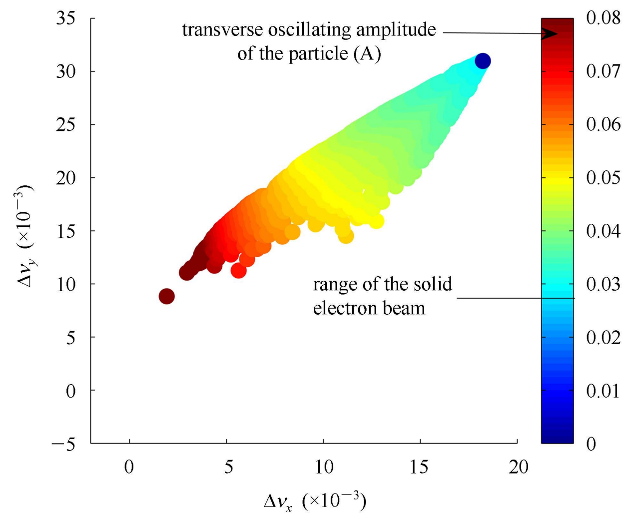

Fig. 5. (color online) Tune footprint caused by the solid electron beam.

4 Tune spread

Generating 5×10^4 particles with the parameters in Table 2, we use the Eqs. (25) and (26) again to calculate the tune shift for the generated particles. To obtain the tune spread, the numbers of particles at different tune shift intervals are counted. The results are shown in Figs. 6 and 7.

The tune spread caused by the solid electron beam is shown in Fig. 6. The horizontal tune spread has a range from near 0 to 0.018, and the vertical tune spread has a range from near 0 to 0.031. A peak appears at the maximum tune shift for both horizontal and vertical direction. About 65% of particles are near the peak for both horizontal and vertical direction.

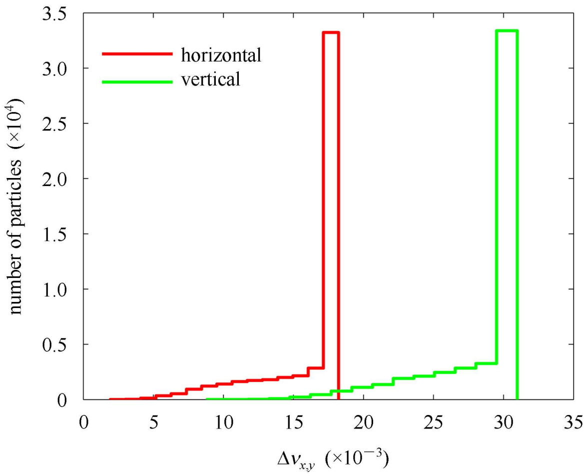

Fig. 6. (color online) Tune spread caused by the solid electron beam.

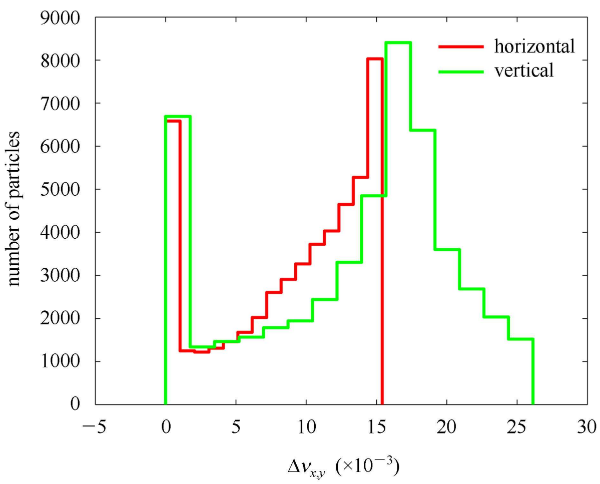

Fig. 7. (color online) Tune spread caused by the hollow electron beam.

The tune spread caused by the hollow electron beam is shown in Fig. 7. The horizontal tune spread has a range from 0 to about 0.015, and the vertical tune spread has a range from 0 to about 0.025. The distributions of the particles have two peaks for both horizontal and vertical direction. For the horizontal direction the first peak appears at the tune shift close to 0, and the second peak appears at the maximum tune shift. For the vertical direction the first peak also appears at the tune shift close to 0, while the tune shift at the second peak is smaller than the maximum tune shift. About 13% of particles have the tune shift close to 0 for both horizontal and vertical directions.

5 Resonances

Beside causing tune shift and tune spread, the nonlinear force of the electron beam also results in resonances. To calculate the resonance driving terms, we again follow the calculation of Ref. [2]. Replacing the part in the bracket of Eq. (16) by its Fourier transform:

$$\frac{\sqrt{\beta}\cos(\phi)\Delta r'}{2\pi\sqrt{\varepsilon}}=\sum_{0}^{\infty}A_n\cos(n\phi). \tag{28}$$

After some algebra, the following expression can be obtained:

$$\frac{\mathrm{d}\phi}{\mathrm{d}\theta}=Q+\sum_{n=0}^{\infty}A_n\sum_{m=-\infty}^{\infty}\cos(n\phi-m\theta). \tag{29}$$

A_n is the Fourier coefficient

$$A_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{eqN'\cos(n\phi)}{4\pi^2\varepsilon_0\beta^2c^2m_0\varepsilon_0\gamma^3}\left[\frac{\int_0^{\sqrt{\beta\varepsilon_z}\cos(\phi)}r\rho(r)\mathrm{d}r}{\int_0^{\infty}r\rho(r)\mathrm{d}r}\right]\mathrm{d}\phi. \tag{30}$$

A_n is the resonance width defined in Refs. [2, 3], and is a function of ε, related to the transverse oscillating amplitude of the particle.

Integrating the right side of Eq. (30) numerically with the parameters in Table 1, the A_n can be worked out. Fig. 8 shows the variation of $\log_{10}(A_n)$ with the transverse oscillating amplitude of the particle.

The phenomena observed from Fig. 8 are summarized as follows:

1) Even order resonances can be driven by both the hollow beam and the solid beam.

2) When the transverse oscillating amplitude of the particle is smaller than the inner radius of the hollow

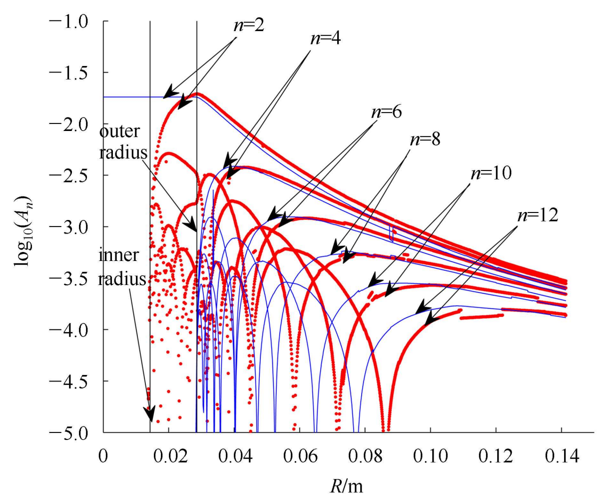

Fig. 8. (color online) Blue lines show the resonance width for the solid electron beam. Red dotted lines show the resonance width for the hollow electron beam. R is the transverse oscillating amplitude of the particle. The black vertical lines are the ranges of the electron beam.

electron beam, the resonance width is zero. In other words, the resonance never occurs when the particle is moving in the hollow part of the electron beam.

3) For the solid electron beam, when the transverse oscillating amplitude of the particle is smaller than the range of the electron beam, the resonance width for n=4,6,8,10,12 is equal to zero, only the resonance width for n=2 is a constant.

4) With the increase of the resonance order, the resonance width for both electron beams decreases.

5) When the transverse oscillating amplitude of the particle is large, the resonance width of the hollow electron beam and the solid electron beam are nearly the same.

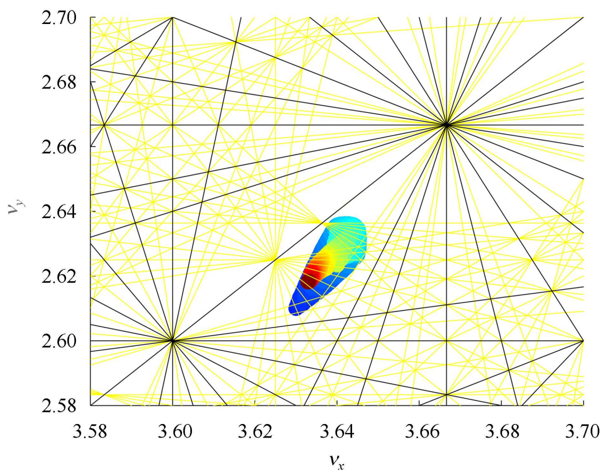

Fig. 9. (color online) The yellow lines represent resonance-lines of order equal to 8, 9, 10, 11, 12; the black lines represent resonance-lines of order smaller than 8.

To know whether the tune footprint caused by the hollow electron beam calculated for $^{238}U^{32+}$ is large enough to cross resonance-lines, we plot the resonance-lines on the tune footprint figure (Fig. 9).

As Fig. 9 shows, the tune footprint only crosses the resonance-lines of order larger than 7. In other words, under the conditions defined in Table 1, the nonlinear field caused by the hollow electron beam does not lead to resonances of order smaller than 7.

6 Conclusions

The tune shift, tune footprint and tune spread caused by a hollow electron beam have been calculated, and a comparison of the tune shift, tune footprint and tune spread caused by a solid electron beam has also been calculated. The resonance driving terms for space charge field of the hollow electron beam and the solid electron beam have been obtained. As an example, the calculation was performed for a beam of $^{238}U^{32+}$ ions of energy 1.272 MeV/u. The main conclusions are summarised: 1) The tune shift caused by the solid electron beam is larger than that caused by the hollow electron beam. 2) The area of the tune footprint caused by the hollow electron beam is larger than that caused by the solid electron beam. 3) The even order resonances will be driven both by the hollow electron bean and the solid electron beam when the tune hits the resonance lines. 4) Under the conditions defined in Table 1, resonances of order smaller than 7 do not happen for $^{238}U^{32+}$ ions of energy 1.272 MeV/u in CSRm.

Now the tune shift, tune spread and the resonance driving terms caused by the electron beam are clear. To further research the effects of the space charge field of the electron beam, the increase of the ion beam size and the ion beam emittance due to the resonances should be studied, and the tune shift and tune spread caused by the space charge field of the ion beam should also be analysed.

References

1 Ziemann V. Resonances Driven by the Electric Field of the Electron Cooler. TSL Note 98–43

2 Keil E. Beam-Beam Dynamics, in CERN 95-06, 539

3 Sorge S, Boine-Frankenheim O, Franchetti G. Analysis of Resonances Induced by the SIS-18 Electron Cooler. Proceedings of COOL 2007, Bad Kreuznach, Germany. 121–124

4 MAO L J. The Simulation and Test of Electron Cooling in HIRFL-CSRm. PH.D 2009. Institute of Modern Physics, Chinese Academy of Sciences, 2009