# Systematic analysis of $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740), \mathrm{D}_{\mathrm{J}}^{*}(2760)$ ， $\mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ in the D meson family ${ }^{*}$ 

YU Guo－Liang（于国梁 $)^{1 ; 1)}$ WANG Zhi－Gang（王志刚 $)^{1 ; 2)}$ LI Zhen－Yu（李振宇）$)^{2}$ MENG Gao－Qing（蒙高庆）${ }^{1}$<br>${ }^{1}$ Department of Mathematics and Physics，North China Electric Power University，Baoding 071003，China<br>${ }^{2}$ School of Physics and Electronic Science，Guizhou Normal College，Guiyang 550018，China


#### Abstract

In this work，we tentatively assign the charmed mesons $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740), \mathrm{D}_{\mathrm{J}}^{*}(2760)$ ， $\mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ observed by the LHCb collaboration according to their spin，parity and masses，then sys－ tematically study their strong decays to ground state charmed mesons plus pseudoscalar mesons with the ${ }^{3} P_{0}$ decay model．Based on these studies，we assign the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ as the $1 D \frac{5}{2} 3^{-}$state，the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ as the $1 F \frac{5}{2} 2^{+}$or $1 F \frac{7}{2} 4^{+}$ state，the $\mathrm{D}_{\mathrm{J}}(3000)$ as the $1 F \frac{7}{2} 3^{+}$or $2 P \frac{1}{2} 1^{+}$state in the D meson family．As a byproduct，we also study the strong decays of the states $2 P \frac{1}{2} 0^{+}, 2 P \frac{3}{2} 2^{+}, 3 S \frac{1}{2} 1^{-}, 3 S \frac{1}{2} 0^{-}$etc，which will be valuable in searching for the partners of these D mesons．


Key words：${ }^{3} P_{0}$ model，D mesons，decay width
PACS：13．25．Ft，14．40．Lb DOI：10．1088／1674－1137／39／6／063101

## 1 Introduction

In 2013，the LHCb collaboration announced several $\mathrm{D}_{\mathrm{J}}$ resonances by studying the $\mathrm{D}^{+} \pi^{-}, \mathrm{D}^{0} \pi^{+}$and $\mathrm{D}^{*+} \pi^{-}$ invariant mass spectra，which were obtained from pp col－ lisions at a center－of－mass energy of 7 TeV ［1］．The LHCb collaboration observed two natural parity resonances， $\mathrm{D}_{\mathrm{J}}^{*}(2650)^{0}$ and $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}$ ，and two unnatural parity res－ onances， $\mathrm{D}_{\mathrm{J}}(2580)^{0}$ and $\mathrm{D}_{\mathrm{J}}(2740)^{0}$ ，in the $\mathrm{D}^{*+} \pi^{-}$mass spectrum，and tentatively identified $\mathrm{D}_{\mathrm{J}}(2580)$ as the $2 S 0^{-}$state， $\mathrm{D}_{\mathrm{J}}^{*}(2650)$ as the $2 S 1^{-}$state， $\mathrm{D}_{\mathrm{J}}(2740)$ as the $1 D 2^{-}$state，$D_{J}^{*}(2760)$ as the $1 D 1^{-}$state．

The $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}$ resonance observed in the $\mathrm{D}^{*+} \pi^{-}$and $\mathrm{D}^{+} \pi^{-}$decay modes has consistent parameters，and its charged partner $D_{J}^{*}(2760)^{+}$was observed in the $\mathrm{D}^{0} \pi^{+}$fi－ nal state［1］．Furthermore，the LHCb collaboration also observed one unnatural parity resonance $\mathrm{D}_{\mathrm{J}}(3000)^{0}$ in the $\mathrm{D}^{*+} \pi^{-}$final state，and two resonances $\mathrm{D}_{\mathrm{J}}^{*}(3000)^{0}$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)^{+}$in the $\mathrm{D}^{+} \pi^{-}$and $\mathrm{D}^{0} \pi^{+}$mass spectra， respectively［1］．The relevant parameters are presented in Table 1.

In 2010，the BaBar collaboration observed four ex－ cited charmed mesons $\mathrm{D}(2550), \mathrm{D}(2600), \mathrm{D}(2750)$ and $\mathrm{D}(2760)$ in the decays $\mathrm{D}^{0}(2550) \rightarrow \mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{0}(2600) \rightarrow$

Table 1．Experimental results from the LHCb collaboration，where the N and U denote the natural and unnatural parity，respectively．

|  | mass $/ \mathrm{MeV}$ | width $/ \mathrm{MeV}$ | decay channel | significance |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{D}_{\mathrm{J}}^{*}(2650)^{0}(\mathrm{~N})$ | $2649.2 \pm 3.5 \pm 3.5$ | $140.2 \pm 17.1 \pm 18.6$ | $\mathrm{D}^{*+} \pi^{-}$ | $24.5 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}(\mathrm{~N})$ | $2761.1 \pm 5.1 \pm 6.5$ | $74.4 \pm 3.4 \pm 37.0$ | $\mathrm{D}^{*+} \pi^{-}$ | $10.2 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}(2580)^{0}(\mathrm{U})$ | $2579.5 \pm 3.4 \pm 5.5$ | $177.5 \pm 17.8 \pm 46.0$ | $\mathrm{D}^{*+} \pi^{-}$ | $18.8 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}(2740)^{0}(\mathrm{U})$ | $2737.0 \pm 3.5 \pm 11.2$ | $73.2 \pm 13.4 \pm 25.0$ | $\mathrm{D}^{*+} \pi^{-}$ | $7.2 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}(3000)^{0}(\mathrm{U})$ | $2971.8 \pm 8.7$ | $188.1 \pm 44.8$ | $\mathrm{D}^{*+} \pi^{-}$ | $9.0 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}(\mathrm{~N})$ | $2760.1 \pm 1.1 \pm 3.7$ | $74.4 \pm 3.4 \pm 19.1$ | $\mathrm{D}^{+} \pi^{-}$ | $17.3 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}^{*}(3000)^{0}$ | $3008.1 \pm 4.0$ | $110.5 \pm 11.5$ | $\mathrm{D}^{+} \pi^{-}$ | $21.2 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{+}(\mathrm{N})$ | $2771.7 \pm 1.7 \pm 3.8$ | $66.7 \pm 6.6 \pm 10.5$ | $\mathrm{D}^{0} \pi^{+}$ | $18.8 \sigma$ |
| $\mathrm{D}_{\mathrm{J}}^{*}(3000)^{+}(\mathrm{N})$ | 3008.1 （fixed $)$ | $110.5($ fixed $)$ | $\mathrm{D}^{0} \pi^{+}$ | $6.6 \sigma$ |

[^0]Table 2. Experimental results from the BaBar collaboration. The particles in brackets are the possible corresponding ones observed by the LHCb collaboration.

|  | mass $/ \mathrm{MeV}$ | width/MeV | decay channel |
| :---: | :---: | :---: | :---: |
| $\mathrm{D}^{0}(2550)\left[\mathrm{D}_{\mathrm{J}}(2580)^{0}\right]$ | $2539.4 \pm 4.5 \pm 6.8$ | $130 \pm 12 \pm 13$ | $\mathrm{D}^{*+} \pi^{-}$ |
| $\mathrm{D}^{0}(2600)\left[\mathrm{D}_{\mathrm{J}}^{*}(2650)^{0}\right]$ | $2608.7 \pm 2.4 \pm 2.5$ | $93 \pm 6 \pm 13$ | $\mathrm{D}^{+} \pi^{-}, \mathrm{D}^{*+} \pi^{-}$ |
| $\mathrm{D}^{0}(2750)\left[\mathrm{D}_{\mathrm{J}}(2740)^{0}\right]$ | $2752.4 \pm 1.7 \pm 2.7$ | $71 \pm 6 \pm 11$ | $\mathrm{D}^{*+} \pi^{-}$ |
| $\mathrm{D}^{0}(2760)\left[\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}\right]$ | $2763.3 \pm 2.3 \pm 2.3$ | $\mathrm{D}^{+} \pi^{-}$ |  |
| $\mathrm{D}^{+}(2600)$ | $2621.3 \pm 3.7 \pm 4.2$ | $\mathrm{D}^{0} \pi^{+}$ |  |
| $\mathrm{D}^{+}(2760)\left[\mathrm{D}_{\mathrm{J}}^{*}(2760)^{+}\right]$ | $2769.7 \pm 3.8 \pm 1.5$ | $93.1 \pm 3.6$ | $\mathrm{D}^{0} \pi^{+}$ |

$\mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{+} \pi^{-}, \mathrm{D}^{0}(2750) \rightarrow \mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{0}(2760) \rightarrow \mathrm{D}^{+} \pi^{-}$, $\mathrm{D}^{+}(2600) \rightarrow \mathrm{D}^{0} \pi^{+}$and $\mathrm{D}^{+}(2760) \rightarrow \mathrm{D}^{0} \pi^{+}$respectively in the inclusive $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow c \bar{c}$ interactions [2]. The BaBar collaboration also analyzed the helicity distributions to determine the spin-parity, and tentatively identified the $(\mathrm{D}(2550), \mathrm{D}(2600))$ as the $2 S$ doublet $\left(0^{-}, 1^{-}\right)$and the $\mathrm{D}(2750)$ and $\mathrm{D}(2760)$ as the $D$-wave states. The relevant parameters are presented in Table 2, where we also present the possible correspondences among the particles observed by the LHCb and BaBar collaborations. Physicists have also studied the decay behaviors of these charmed mesons using the heavy meson effective theory [3], constituent quark model [4] and the Eichten-Hill-Quigg formula [5].

Heavy meson effective theory is a powerful tool in studying the properties of hadrons with a single heavy quark. With this method, P. Colangelo et al. proposed a classification of many observed $c \bar{q}$ and $b \bar{q}$ mesons in doublets [6]. In Ref. [7], we study the strong decays of the charmed mesons $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740)$, $\mathrm{D}_{\mathrm{J}}^{*}(2760), \mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ with the heavy meson effective theory in the leading order approximation. The ratios of decay widths of different channels were calculated, but the exact values of the decay widths were not given, which constitutes the first motivation of this study. The quark pair creation (QPC) model, which is also known as the ${ }^{3} P_{0}$ decay model, is another effective method to study the strong decays of mesons. It was originally introduced by L. Micu [8] and further developed by A. Le Yaouanc et al. [9]. This model has been widely used to evaluate the strong decays of hadrons [1021], since it gives a good description of many observed decay amplitudes and partial widths of the hadrons. Y. Sun et al. [22] studied the strong decays of the $\mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ with the ${ }^{3} P_{0}$ decay model, and identified $\mathrm{D}_{\mathrm{J}}(3000)$ as the $2 P\left(1^{+}\right)$state and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ as the $2^{3} P_{0}$ state. However, $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740)$ and $\mathrm{D}_{\mathrm{J}}^{*}(2760)$, which were also observed by the LHCb Collaboration, were not analyzed in their studies. This is the second motivation of our work. Besides, they chose the simple harmonic oscillator ( $\mathrm{SHO} \mathrm{)} \mathrm{wave} \mathrm{functions} \mathrm{with}$ the effective oscillator parameter $R$ as the meson's radial wave functions. From Ref. [14], we can see that there are two types of SHO wave function: those with a
common oscillator parameter $R$ and those with an effective oscillator parameter $R$. According to a series of least squares fits of the model predictions to the decay widths of 28 of the best known meson decays, it seems that the SHO wave functions with a common $R$ can lead to better results [14]. Thus, in order to identify the $\mathrm{D}_{\mathrm{J}}(2580)$, $D_{J}^{*}(2650), D_{J}(2740), D_{J}^{*}(2760), D_{J}(3000)$ and $D_{J}^{*}(3000)$, it is necessary and interesting to systematically study the strong decays of these charmed mesons by the ${ }^{3} P_{0}$ decay model with the common oscillator parameter $R$.

In the heavy quark limit, the heavy-light mesons $Q \bar{q}$ can be classified in doublets according to the total angular momentum of the light antiquark $\vec{s}_{1}, \vec{s}_{1}=\vec{s}_{\bar{q}}+\vec{L}$, where the $\vec{s}_{\mathrm{q}}$ and $\vec{L}$ are the spin and orbital angular momentum of the light antiquark, respectively [23]. In the case of the radial quantum number $n=1$, the doublet ( $\mathrm{P}, \mathrm{P}^{*}$ ) has spin-parity $J_{\mathrm{s}_{1}}^{P}=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}}$ for $L=0$; the two doublets $\left(\mathrm{P}_{0}^{*}, \mathrm{P}_{1}\right)$ and $\left(\mathrm{P}_{1}, \mathrm{P}_{2}^{*}\right)$ have spin-parity $J_{\mathrm{s}_{1}}^{P}=\left(0^{+}, 1^{+}\right)_{\frac{1}{2}}$ and $\left(1^{+}, 2^{+}\right)_{\frac{3}{2}}$ respectively for $L=1$; the two doublets $\left(\mathrm{P}_{1}^{*}, \mathrm{P}_{2}\right)$ and $\left(\mathrm{P}_{2}, \mathrm{P}_{3}^{*}\right)$ have spin-parity $J_{\mathrm{s}_{1}}^{P}=\left(1^{-}, 2^{-}\right)_{\frac{3}{2}}$ and $\left(2^{-}, 3^{-}\right)_{\frac{5}{2}}$ respectively for $L=2$ and the two doublets $\left(\mathrm{P}_{2}^{*}, \mathrm{P}_{3}\right)$ and $\left(\mathrm{P}_{3}, \mathrm{P}_{4}^{*}\right)$ have spin-parity $J_{\mathrm{s}_{1}}^{P}=\left(2^{+}, 3^{+}\right)_{\frac{5}{2}}$ and $\left(3^{+}, 4^{+}\right)_{\frac{7}{2}}$ respectively for $L=3$, where the superscript $P$ denotes the parity. The $n=2,3,4, \cdots$ states are clarified by analogous doublets, for example, for $n=2$, the doublet $\left(\mathrm{P}^{\prime}, \mathrm{P}^{*^{\prime}}\right)$ has spin-parity $J_{\mathrm{s}_{1}}^{P}=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}}$ for $L=0$.

The $\mathrm{D}_{\mathrm{J}}(2580)^{0}$, $\mathrm{D}_{\mathrm{J}}(2740)^{0}$ and $\mathrm{D}_{\mathrm{J}}(3000)^{0}$ resonances have unnatural parity, and their possible spin-parity assignments are $J^{P}=0^{-}, 1^{+}, 2^{-}, 3^{+}, \cdots$. The $\mathrm{D}_{\mathrm{J}}^{*}(2650)^{0}$ and $\mathrm{D}_{\mathrm{J}}^{*}(2760)^{0}$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ resonances have natural parity, and their possible spin-parity assignments are $J^{P}=0^{+}, 1^{-}, 2^{+}, 3^{-}, \cdots$. The six lowlying states, $D, D^{*}, D_{0}(2400), D_{1}(2430), D_{1}(2420)$ and $\mathrm{D}_{2}(2460)$ have been established [24]. The newly observed charmed mesons $D_{J}(2580), D_{J}^{*}(2650), D_{J}(2740)$, $D_{J}^{*}(2760), D_{J}(3000), D_{J}^{*}(3000)$ can be tentatively identified as the missing states in the D meson family.

The mass is a fundamental parameter in describing a hadron. In Table 3, we present the predictions from some theoretical models, such as the relativized quark model based on a universal one-gluon exchange plus linear confinement potential (marked 'GI' in the table) [25],

Table 3. The masses of the charmed mesons from different quark models compared with experimental data, and the possible assignments of the newly observed charmed mesons. The N and U denote the natural parity and unnatural parity, respectively. All values in units of MeV .

|  | $n L \mathrm{~s}_{\mathrm{L}} J^{P}$ | Exp [1, 24] | GI [1, 25] | PE [26] | EFG [27] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| D | $1 S \frac{1}{2} 0^{-}$ | 1867 | 1864 | 1868 | 1871 |
| D* | $1 S \frac{1}{2} 1^{-}$ | 2008 | 2023 | 2005 | 2010 |
| $\mathrm{D}_{0}^{*}$ | $1 P \frac{1}{2} 0^{+}$ | 2400 | 2380 | 2377 | 2406 |
| $\mathrm{D}_{1}$ | $1 P \frac{1}{2} 1^{+}$ | 2427 | 2419 | 2490 | 2469 |
| $\mathrm{D}_{1}$ | $1 P \frac{3}{2} 1^{+}$ | 2420 | 2469 | 2417 | 2426 |
| $\mathrm{D}_{2}^{*}$ | $1 P \frac{3}{2} 2^{+}$ | 2460 | 2479 | 2460 | 2460 |
| $\mathrm{D}_{1}^{*}$ | $1 D \frac{3}{2} 1^{-}$ | ?2760(N) | 2796 | 2795 | 2788 |
| $\mathrm{D}_{2}$ | $1 D \frac{3}{2} 2^{-}$ | $? 2740$ (U) | 2801 | 2833 | 2850 |
| $\mathrm{D}_{2}$ | $1 D \frac{5}{2} 2^{-}$ | ?2740(U) | 2806 | 2775 | 2806 |
| $\mathrm{D}_{3}^{*}$ | $1 D \frac{5}{2} 3^{-}$ | ?2760(N) | 2806 | 2799 | 2863 |
| $\mathrm{D}_{2}^{*}$ | $1 F \frac{5}{2} 2^{+}$ | ?3000(N) | 3074 | 3101 | 3090 |
| $\mathrm{D}_{3}$ | $1 F \frac{5}{2} 3^{+}$ | ?3000(U) | 3074 | 3123 | 3145 |
| $\mathrm{D}_{3}$ | $1 F \frac{7}{2} 3^{+}$ | ?3000(U) | 3079 | 3074 | 3129 |
| $\mathrm{D}_{4}^{*}$ | $1 F \frac{7}{2} 4^{+}$ | ?3000(N) | 3084 | 3091 | 3187 |
| D | $2 S \frac{1}{2} 0^{-}$ | ?2580(U) | 2558 | 2589 | 2581 |
| D* | $2 S \frac{1}{2} 1^{-}$ | ?2650(N) | 2618 | 2692 | 2632 |
| $\mathrm{D}_{0}^{*}$ | $2 P \frac{1}{2} 0^{+}$ | $? 3000(\mathrm{~N})$ |  | 2949 | 2919 |
| $\mathrm{D}_{1}$ | $2 P \frac{1}{2} 1^{+}$ | ?3000(U) |  | 3045 | 3021 |
| $\mathrm{D}_{1}$ | $2 P \frac{3}{2} 1^{+}$ | ?3000(U) |  | 2995 | 2932 |
| $\mathrm{D}_{2}^{*}$ | $2 P \frac{3}{2} 2^{+}$ | ?3000(N) |  | 3035 | 3012 |
| D | $3 S \frac{1}{2} 0^{-}$ | ?3000(U) |  | 3141 | 3062 |
| D* | $3 S \frac{1}{2} 1^{-}$ | ?3000(N) |  | 3226 | 3096 |

the relativistic quark model including the leading order $1 / M_{\mathrm{h}}$ corrections (PE) [26], and the QCD-motivated relativistic quark model based on the quasipotential approach(EFG) [27]. We can identify the $\mathrm{D}_{\mathrm{J}}(2580)$, $D_{J}^{*}(2650), D_{J}(2740), D_{J}^{*}(2760), D_{J}(3000), D_{J}^{*}(3000)$ tentatively according to these masses.

In the following, we list out the possible assignments,

$$
\begin{aligned}
& \left(\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650)\right)=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}} \quad \text { with } \quad n=2, L=0, \\
& \left(\mathrm{D}_{\mathrm{J}}^{*}(2760), \mathrm{D}_{\mathrm{J}}(2740)\right)=\left(1^{-}, 2^{-}\right)_{\frac{3}{2}} \quad \text { with } \quad n=1, L=2, \\
& \left(\mathrm{D}_{\mathrm{J}}(2740), \mathrm{D}_{\mathrm{J}}^{*}(2760)\right)=\left(2^{-}, 3^{-}\right)_{\frac{5}{2}} \quad \text { with } \quad n=1, L=2, \\
& \left(\mathrm{D}_{\mathrm{J}}^{*}(3000), \mathrm{D}_{\mathrm{J}}(3000)\right)=\left(2^{+}, 3^{+}\right)_{\frac{5}{2}} \quad \text { with } \quad n=1, L=3, \\
& \left(\mathrm{D}_{\mathrm{J}}(3000), \mathrm{D}_{\mathrm{J}}^{*}(3000)\right)=\left(3^{+}, 4^{+}\right)_{\frac{7}{2}} \quad \text { with } \quad n=1, L=3, \\
& \left(\mathrm{D}_{\mathrm{J}}^{*}(3000), \mathrm{D}_{\mathrm{J}}(3000)\right)=\left(0^{+}, 1^{+}\right)_{\frac{1}{2}} \quad \text { with } \quad n=2, L=1, \\
& \left(\mathrm{D}_{\mathrm{J}}(3000), \mathrm{D}_{\mathrm{J}}^{*}(3000)\right)=\left(1^{+}, 2^{+}\right)_{\frac{3}{2}} \quad \text { with } \quad n=2, L=1, \\
& \left(\mathrm{D}_{\mathrm{J}}(3000), \mathrm{D}_{\mathrm{J}}^{*}(3000)\right)=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}} \quad \text { with } \quad n=3, L=0 .
\end{aligned}
$$

The article is arranged as follows. In Section 2, a brief review of the ${ }^{3} P_{0}$ decay model is given (for a detailed review see Refs. [9, 11, 12, 14]). In Section 3, we study the strong decays of the charmed mesons $\mathrm{D}_{\mathrm{J}}(2580)$,
$D_{J}^{*}(2650), D_{J}(2740), D_{J}^{*}(2760), D_{J}(3000), D_{J}^{*}(3000)$ with the ${ }^{3} P_{0}$ decay model, then in Section 4, we present our conclusions.

## 2 Method

### 2.1 The decay model

The main assumption of the ${ }^{3} P_{0}$ decay model is that strong decays take place via the creation of a ${ }^{3} P_{0}$ quarkantiquark pair from the vacuum. The newly produced quark-antiquark pair, together with the $q \bar{q}$ in the initial meson, regroup into two outgoing mesons in all possible quark rearrangement combinations, which correspond to the two Feynman diagrams as shown in Fig. 1 for the strong decay processes $\mathrm{A} \rightarrow \mathrm{B}+\mathrm{C}$.

The transition operator $T$ of the decay $\mathrm{A} \rightarrow \mathrm{B}+\mathrm{C}$ in the ${ }^{3} P_{0}$ decay model is given by

$$
\begin{align*}
T= & -3 \gamma \sum_{m}\langle 1 m 1-m \mid 00\rangle \int \mathrm{d}^{3} \vec{p}_{3} \mathrm{~d}^{3} \vec{p}_{4} \delta^{3}\left(\vec{p}_{3}+\vec{p}_{4}\right) \mathcal{Y}_{1}^{m} \\
& \times\left(\frac{\vec{p}_{3}-\vec{p}_{4}}{2}\right) \chi_{1-m}^{34} \phi_{0}^{34} \omega_{0}^{34} b_{3}^{\dagger}\left(\vec{p}_{3}\right) d_{4}^{\dagger}\left(\vec{p}_{4}\right) \tag{1}
\end{align*}
$$

where $\gamma$ is a dimensionless parameter representing the
probability of the quark-antiquark pair $\mathrm{q}_{3} \overline{\mathrm{q}}_{4}$ with $J^{P C}=$ $0^{++}$being created from the vacuum, and $\vec{p}_{3}$ and $\vec{p}_{4}$ are the momenta of the created quark $\mathrm{q}_{3}$ and antiquark $\overline{\mathrm{q}}_{4}$, respectively. $\phi_{0}^{34}, \omega_{0}^{34}$, and $\chi_{1-m}^{34}$ are the flavor, color, and spin wave functions, respectively, of $\mathrm{q}_{3} \overline{\mathrm{q}}_{4}$. The solid harmonic polynomial $\mathcal{Y}_{1}^{m}(\vec{p}) \equiv|\vec{p}|^{1} Y_{1}^{m}\left(\theta_{\mathrm{p}}, \phi_{\mathrm{p}}\right)$ reflects the momentum-space distribution of $\mathrm{q}_{3} \overline{\mathrm{q}}_{4}$.


Fig. 1. The two possible diagrams contributing to $\mathrm{A} \rightarrow \mathrm{B}+\mathrm{C}$ in the ${ }^{3} P_{0}$ decay model.

For the meson wave function, we adopt the mock meson state $\left|A\left(n_{\mathrm{A}}^{2 S_{\mathrm{A}}+1} L_{\mathrm{AJ}_{\mathrm{A}} \mathrm{M}_{\mathrm{A}}}\right)\left(\vec{P}_{\mathrm{A}}\right)\right\rangle$ defined by [28]

$$
\begin{align*}
& \left|A\left(n_{\mathrm{A}}^{2 S_{\mathrm{A}}+1} L_{\mathrm{AJ}_{\mathrm{A}} \mathrm{M}_{\mathrm{A}}}\right)\left(\vec{P}_{\mathrm{A}}\right)\right\rangle \\
\equiv & \sqrt{2 E_{\mathrm{A}}} \sum_{M_{\mathrm{L}_{\mathrm{A}}} M_{\mathrm{S}_{\mathrm{A}}}}\left\langle L_{\mathrm{A}} M_{\mathrm{L}_{\mathrm{A}}} S_{\mathrm{A}} M_{\mathrm{S}_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{\mathrm{J}_{\mathrm{A}}}\right\rangle \\
& \times \int \mathrm{d}^{3} \vec{p}_{\mathrm{A}} \psi_{\mathrm{n}_{\mathrm{A}} \mathrm{~L}_{\mathrm{A}} \mathrm{M}_{\mathrm{L}_{\mathrm{A}}}}\left(\vec{p}_{\mathrm{A}}\right) \chi_{\mathrm{S}_{\mathrm{A}} \mathrm{M}_{\mathrm{S}_{\mathrm{A}}}}^{12} \phi_{\mathrm{A}}^{12} \omega_{\mathrm{A}}^{12} \\
& \times\left|q_{1}\left(\frac{m_{1}}{m_{1}+m_{2}} \vec{P}_{\mathrm{A}}+\vec{p}_{\mathrm{A}}\right) q_{2}\left(\frac{m_{2}}{m_{1}+m_{2}} \vec{P}_{\mathrm{A}}-\vec{p}_{\mathrm{A}}\right)\right\rangle, \tag{2}
\end{align*}
$$

where $m_{1}$ and $m_{2}$ are the masses of quark $\mathrm{q}_{1}$ with momentum $\vec{p}_{1}$ and antiquark $\overline{\mathrm{q}}_{2}$ with momentum $\vec{p}_{2}$, respectively, $n_{\mathrm{A}}$ is the radial quantum number of the meson A composed of $\mathrm{q}_{1} \overline{\mathrm{q}}_{2}, \vec{S}_{\mathrm{A}}=\vec{s}_{\mathrm{q}_{1}}+\vec{s}_{\mathrm{q}_{2}}, \vec{J}_{\mathrm{A}}=\vec{L}_{\mathrm{A}}+\vec{S}_{\mathrm{A}}$, $\vec{s}_{\mathrm{q}_{1}}\left(\vec{s}_{\mathrm{q}_{2}}\right)$ is the spin of $\mathrm{q}_{1}\left(\overline{\mathrm{q}}_{2}\right), \vec{L}_{\mathrm{A}}$ is the relative orbital angular momentum between $\mathrm{q}_{1}$ and $\overline{\mathrm{q}}_{2}, \vec{P}_{\mathrm{A}}=\vec{p}_{1}+\vec{p}_{2}$, $\vec{p}_{\mathrm{A}}=\frac{m_{2} \vec{p}_{1}-m_{1} \vec{p}_{2}}{m_{1}+m_{2}},\left\langle L_{\mathrm{A}} M_{\mathrm{L}_{\mathrm{A}}} S_{\mathrm{A}} M_{\mathrm{S}_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{\mathrm{J}_{\mathrm{A}}}\right\rangle$ is a ClebschGordan coefficient, $E_{\mathrm{A}}$ is the total energy of meson A, $\chi_{\mathrm{S}_{\mathrm{A}} \mathrm{M}_{\mathrm{S}_{\mathrm{A}}}}^{12}, \phi_{\mathrm{A}}^{12}, \omega_{\mathrm{A}}^{12}$, and $\psi_{\mathrm{n}_{\mathrm{A}} \mathrm{L}_{\mathrm{A}} \mathrm{M}_{\mathrm{L}_{\mathrm{A}}}}\left(\vec{p}_{\mathrm{A}}\right)$ are the spin, flavor, color, and space wave functions of meson A, respectively.

The $S$-matrix of the process $\mathrm{A} \rightarrow \mathrm{BC}$ is defined by

$$
\begin{equation*}
\langle B C| S|A\rangle=I-2 \pi \mathrm{i} \delta\left(E_{\mathrm{A}}-E_{\mathrm{B}}-E_{\mathrm{C}}\right)\langle B C| T|A\rangle \tag{3}
\end{equation*}
$$

with

$$
\begin{equation*}
\langle B C| T|A\rangle=\delta^{3}\left(\vec{P}_{\mathrm{A}}-\vec{P}_{\mathrm{B}}-\vec{P}_{\mathrm{C}}\right) \mathcal{M}^{M_{\mathrm{J}_{\mathrm{A}}} M_{\mathrm{J}_{\mathrm{B}}} M_{\mathrm{J}_{\mathrm{C}}}}, \tag{4}
\end{equation*}
$$

where $\mathcal{M}^{M_{\mathrm{J}_{\mathrm{A}}}}{ }^{M_{\mathrm{J}_{\mathrm{B}}}}{ }^{M_{\mathrm{J}_{\mathrm{C}}}}$ is the helicity amplitude of $\mathrm{A} \rightarrow \mathrm{BC}$. In the center of mass frame of meson $\mathrm{A}, \mathcal{M}^{M_{\mathrm{J}_{\mathrm{A}}}}{ }^{M_{\mathrm{J}_{\mathrm{B}}} M_{\mathrm{J}_{\mathrm{C}}}}$ can be written as

$$
\begin{align*}
\mathcal{M}^{M_{\mathrm{J}_{\mathrm{A}}} M_{\mathrm{J}_{\mathrm{B}}} M_{\mathrm{J}_{\mathrm{C}}}}(\vec{P})= & \gamma \sqrt{8 E_{\mathrm{A}} E_{\mathrm{B}} E_{\mathrm{C}}} \sum_{\substack{M_{\mathrm{L}_{\mathrm{A}}}, M_{\mathrm{S}_{\mathrm{A}}} \\
M_{\mathrm{L}_{\mathrm{B}}}, M_{\mathrm{S}_{\mathrm{B}}}, M_{\mathrm{L}_{\mathrm{C}}}, M_{\mathrm{S}_{\mathrm{C}}}, m}}\left\langle L_{\mathrm{A}} M_{\mathrm{L}_{\mathrm{A}}} S_{\mathrm{A}} M_{\mathrm{S}_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{\mathrm{J}_{\mathrm{A}}}\right\rangle \\
& \times\left\langle L_{\mathrm{B}} M_{\mathrm{L}_{\mathrm{B}}} S_{\mathrm{B}} M_{\mathrm{S}_{\mathrm{B}}} \mid J_{\mathrm{B}} M_{\mathrm{J}_{\mathrm{B}}}\right\rangle\left\langle L_{\mathrm{C}} M_{\mathrm{L}_{\mathrm{C}}} S_{\mathrm{C}} M_{\mathrm{S}_{\mathrm{C}}} \mid J_{\mathrm{C}} M_{\mathrm{J}_{\mathrm{C}}}\right\rangle \\
& \times\langle 1 m 1-m \mid 00\rangle\left\langle\chi_{\mathrm{S}_{\mathrm{B}} M_{\mathrm{S}_{\mathrm{B}}}}^{14} \chi_{\mathrm{S}_{\mathrm{C}} \mathrm{M}_{\mathrm{S}_{\mathrm{C}}}}^{32} \mid \chi_{\mathrm{S}_{\mathrm{A}} \mathrm{M}_{\mathrm{S}_{\mathrm{A}}}}^{12} \chi_{1-m}^{34}\right\rangle \\
& \times\left[\left\langle\phi_{\mathrm{B}}^{14} \phi_{\mathrm{C}}^{32} \mid \phi_{\mathrm{A}}^{12} \phi_{0}^{34}\right\rangle I\left(\vec{P}, m_{1}, m_{2}, m_{3}\right)\right. \\
& \left.+(-1)^{1+S_{\mathrm{A}}+S_{\mathrm{B}}+S_{\mathrm{C}}}\left\langle\phi_{\mathrm{B}}^{32} \phi_{\mathrm{C}}^{14} \mid \phi_{\mathrm{A}}^{12} \phi_{0}^{34}\right\rangle I\left(-\vec{P}, m_{2}, m_{1}, m_{3}\right)\right],
\end{align*}
$$

where the two terms in the bracket [ ] correspond to the two possible diagrams in Fig. 1(a) and 1(b), respectively, and the spatial integral is defined as

$$
\begin{align*}
I\left(\vec{P}, m_{1}, m_{2}, m_{3}\right)= & \int \mathrm{d}^{3} \vec{p} \psi_{\mathrm{n}_{\mathrm{B}} \mathrm{~L}_{\mathrm{B}} \mathrm{M}_{\mathrm{L}_{\mathrm{B}}}}^{*}\left(\frac{m_{3}}{m_{1}+m_{2}} \overrightarrow{P_{\mathrm{B}}}+\vec{p}\right) \\
& \times \psi_{\mathrm{n}_{\mathrm{C}} \mathrm{~L}_{\mathrm{C}} \mathrm{M}_{\mathrm{L}_{\mathrm{C}}}}^{*}\left(\frac{m_{3}}{m_{2}+m_{3}} \overrightarrow{P_{\mathrm{B}}}+\vec{p}\right) \\
& \times \psi_{\mathrm{n}_{\mathrm{A}} \mathrm{~L}_{\mathrm{A}} \mathrm{M}_{\mathrm{L}_{\mathrm{A}}}}\left(\vec{P}_{\mathrm{B}}+\vec{p}\right) \mathcal{Y}_{1}^{m}(\vec{p}), \tag{6}
\end{align*}
$$

where $\vec{P}=\vec{P}_{\mathrm{B}}=-\vec{P}_{\mathrm{C}}, \vec{p}=\vec{p}_{3}, m_{3}$ is the mass of the cre-
ated quark $\mathrm{q}_{3}$, and the SHO approximation is used for the meson's radial wave functions. In momentum-space, the SHO wave function is

$$
\begin{align*}
\Psi_{\mathrm{nLM}_{\mathrm{L}}}(\vec{p})= & (-1)^{n}(-\mathrm{i})^{L} R^{L+\frac{3}{2}} \sqrt{\frac{2 n!}{\Gamma\left(n+L+\frac{3}{2}\right)}} \\
& \times \exp \left(-\frac{R^{2} p^{2}}{2}\right) L_{\mathrm{n}}^{L+\frac{1}{2}}\left(R^{2} p^{2}\right) \mathcal{Y}_{\mathrm{LM}_{\mathrm{L}}}(\vec{p}), \tag{7}
\end{align*}
$$

where $\mathcal{Y}_{\mathrm{LM}_{\mathrm{L}}}(\vec{p})=|\vec{p}|^{\mathrm{L}} Y_{\mathrm{LM}_{\mathrm{L}}}\left(\Omega_{\mathrm{p}}\right)$, and $L_{n}^{L+\frac{1}{2}}\left(R^{2} p^{2}\right)$ is an as-
sociated Laguerre polynomial. The overlaps of the flavor and spin wave functions of the mesons and the created pair in Eq. (5) can be calculated according to the method in Ref. [14].

Using the Jacob-Wick formula the helicity amplitude can be converted into the partial wave amplitude [29]

$$
\begin{align*}
\mathcal{M}^{\mathrm{JL}}(\vec{P})= & \frac{\sqrt{4 \pi(2 L+1)}}{2 J_{\mathrm{A}}+1} \sum_{M_{\mathrm{J}_{\mathrm{B}}} M_{\mathrm{J}_{\mathrm{C}}}}\left\langle L 0 J M_{\mathrm{J}_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{\mathrm{J}_{\mathrm{A}}}\right\rangle \\
& \times\left\langle J_{\mathrm{B}} M_{\mathrm{J}_{\mathrm{B}}} J_{\mathrm{C}} M_{\mathrm{J}_{\mathrm{C}}} \mid J M_{\mathrm{J}_{\mathrm{A}}}\right\rangle \mathcal{M}^{M_{\mathrm{J}_{\mathrm{A}}} M_{\mathrm{J}_{\mathrm{B}}} M_{\mathrm{J}_{\mathrm{C}}}}(\vec{P}) \tag{8}
\end{align*}
$$

The decay width in terms of the partial wave amplitude using relativistic phase space is

$$
\begin{equation*}
\Gamma=\frac{\pi}{4} \frac{|\vec{P}|}{M_{\mathrm{A}}^{2}} \sum_{\mathrm{JL}}\left|\mathcal{M}^{\mathrm{JL}}\right|^{2} \tag{9}
\end{equation*}
$$

where

$$
|\vec{P}|=\frac{\sqrt{\left[M_{\mathrm{A}}^{2}-\left(M_{\mathrm{B}}+M_{\mathrm{C}}\right)^{2}\right]\left[M_{\mathrm{A}}^{2}-\left(M_{\mathrm{B}}-M_{\mathrm{C}}\right)^{2}\right]}}{2 M_{\mathrm{A}}},
$$

and $M_{\mathrm{A}}, M_{\mathrm{B}}$, and $M_{\mathrm{C}}$ are the masses of the mesons A, B , and C , respectively.

### 2.2 Mixed states

The heavy-light mesons are not charge conjugation eigenstates and so mixing can occur between states with $J=L$ and $S=1$ or 0 . A general relation between the heavy quark symmetric states and the non-relativistic states ${ }^{3} L_{\mathrm{L}}$ and ${ }^{1} L_{\mathrm{L}}$ can be written as [30]

$$
\begin{align*}
\binom{\left\lvert\, s_{1}=L-\frac{1}{2}\right., L^{\mathrm{P}}}{\left|s_{1}=L+\frac{1}{2}, L^{\mathrm{P}}\right\rangle}= & \frac{1}{\sqrt{2 L+1}}\left(\begin{array}{cc}
\sqrt{L+1} & -\sqrt{L} \\
\sqrt{L} & \sqrt{L+1}
\end{array}\right) \\
& \times\binom{\left|{ }^{3} L_{\mathrm{L}}\right\rangle}{\left|{ }^{1} L_{\mathrm{L}}\right\rangle}, P=(-1)^{L+1} . \tag{10}
\end{align*}
$$

Commonly, we express this relation as a mixture. When $J=L=1$, the corresponding mixing angle is $\theta=-54.7^{\circ}$
or $\theta=35.3^{\circ}$, thus Eq. (10) transforms into

$$
\binom{\left|\frac{1}{2}, 1^{+}\right\rangle}{\left|\frac{3}{2}, 1^{+}\right\rangle}=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{11}\\
\sin \theta & \cos \theta
\end{array}\right)\binom{\left|{ }^{3} P_{1}\right\rangle}{\left|{ }^{1} P_{1}\right\rangle} .
$$

In our calculation, the final states are related to $\mathrm{D}(2420) / \mathrm{D}(2430)$ and $\mathrm{D}_{\mathrm{s}}(2460) / \mathrm{D}_{\mathrm{s}}(2536)$, which are the $1^{+}$states in the D and $\mathrm{D}_{\mathrm{s}}$ meson families, respectively. The $\mathrm{D}(2420) / \mathrm{D}(2430)$ and $\mathrm{D}_{\mathrm{s}}(2460) / \mathrm{D}_{\mathrm{s}}(2536)$ are mixtures of the ${ }^{3} P_{1}$ and ${ }^{1} P_{1}$ states, which satisfies Eq. (11). In addition, the initial states of $1^{+}$are also mixtures of the ${ }^{3} P_{1}$ and ${ }^{1} P_{1}$ states. As far as the

$$
1 F \frac{5}{2} 3^{+} / 1 F \frac{7}{2} 3^{+}
$$

and

$$
1 D \frac{3}{2} 2^{-} / 1 D \frac{5}{2} 2^{-}
$$

states are concerned, they are mixtures of the ${ }^{3} F_{3} /{ }^{1} F_{3}$ and ${ }^{3} D_{2} /{ }^{1} D_{2}$ states respectively, and the mixing angle can be determined from Eq. (10).

In order to determine $L$ from Eq. (8), we choose $l$ as the orbital angular momentum of the $D$ mesons in the following three Eqs. (12-14). If the initial states $\mathrm{A}\left(l^{\mathrm{P}}\right)$ are mixtures, the partial wave amplitude can be deduced as

$$
\binom{\mathcal{M}_{\left|l-\frac{1}{2}, l^{\mathrm{P}}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}{\mathcal{M}_{\left|l+\frac{1}{2}, l^{\mathrm{P}}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{12}\\
\sin \theta & \cos \theta
\end{array}\right)\binom{\mathcal{M}_{\left|{ }^{3} l_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}{\mathcal{M}_{\left.\left.\right|^{1} l_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}},
$$

in the case of the mixings of the final states $\mathrm{B}\left(l^{\prime} \mathrm{P}^{\prime}\right)$

$$
\binom{\mathcal{M}_{\mathrm{A} \rightarrow \left\lvert\, l^{\prime}-\frac{1}{2}\right., l^{\prime} \mathrm{P}^{\prime}}^{\mathrm{JL}}}{\mathcal{M}_{\mathrm{A} \rightarrow\left|l^{\prime}+\frac{1}{2}, l^{\prime} \mathrm{P}^{\prime}\right\rangle \mathrm{C}}^{\mathrm{JL}}}=\left(\begin{array}{cc}
\cos \theta^{\prime} & -\sin \theta^{\prime}  \tag{13}\\
\sin \theta^{\prime} & \cos \theta^{\prime}
\end{array}\right)\binom{\mathcal{M}_{\left.\mathrm{A} \rightarrow| |^{3} l_{1}^{\prime}{ }_{1}\right\rangle \mathrm{C}}^{\mathrm{JL}}}{\mathcal{M}_{\left.\mathrm{A} \rightarrow| |_{1^{\prime}}{ }^{\prime}\right\rangle \mathrm{C}}^{\mathrm{JL}}} .
$$

When the initial and the final states ( A and B ) are both mixtures, we can get a similar relation:
where $\theta$ and $\theta^{\prime}$ are the mixtures of the initial and final states, respectively. Thus the decay width can also be deduced from the general relations of (12-14). For example, in the case of the mixtures of the initial states of $1^{+}$,

$$
\binom{\mathcal{M}_{\left|\frac{1}{2}, 1+\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}{\mathcal{M}_{\left|\frac{3}{2}, 1^{+}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}=\left(\begin{array}{cc}
\cos \theta & -\sin \theta  \tag{15}\\
\sin \theta & \cos \theta
\end{array}\right)\binom{\mathcal{M}_{\left.\left.\right|^{3} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}}{\mathcal{M}_{\left.\left.\right|^{1} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}},
$$

and the decay width can be expressed as

$$
\begin{align*}
\Gamma\left(\left|\frac{1}{2}, 1^{+}\right\rangle \rightarrow \mathrm{BC}\right)= & \left.\frac{\pi}{4} \frac{|\vec{P}|}{M_{\mathrm{A}}^{2}} \sum_{\mathrm{JL}} \right\rvert\, \cos \theta \mathcal{M}_{\left.\left.\right|^{3} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}} \\
& -\left.\sin \theta \mathcal{M}_{\left.\left.\right|^{1} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}\right|^{2} \\
\Gamma\left(\left|\frac{3}{2}, 1^{+}\right\rangle \rightarrow \mathrm{BC}\right)= & \left.\frac{\pi}{4} \frac{|\vec{P}|}{M_{\mathrm{A}}^{2}} \sum_{\mathrm{JL}} \right\rvert\, \sin \theta \mathcal{M}_{\left.\left.\right|^{3} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}} \\
& +\left.\cos \theta \mathcal{M}_{\left.\left.\right|^{1} P_{1}\right\rangle \rightarrow \mathrm{BC}}^{\mathrm{JL}}\right|^{2} \tag{16}
\end{align*}
$$

## 3 Numerical results

The parameters involved in the ${ }^{3} P_{0}$ decay model include the light quark pair ( $\mathrm{q} \overline{\mathrm{q}}$ ) creation strength $\gamma$, the SHO wave function scale parameter $R$, and the masses of the mesons and the constituent quarks. Based on Ref. [14], we adopt the SHO wave functions with the common oscillator parameter $R$ whose value is chosen to be $2.5 \mathrm{GeV}^{-1}$. Correspondingly, the value of the $\gamma$ is chosen to be 6.25 for the creation of the $u / d$ quark $[14,19]$. As for the strange quark pair ( $\mathrm{s} \overline{\mathrm{s}}$ ), its creation strength can be related by $\gamma_{\mathrm{ss}}=\gamma / \sqrt{3}$ [10]. The adopted masses of the mesons are listed in Table 4, and $m_{\mathrm{u}}=m_{\mathrm{d}}=0.22$
$\mathrm{GeV}, m_{\mathrm{s}}=0.419 \mathrm{GeV}$ and $m_{\mathrm{c}}=1.65 \mathrm{GeV}$.
The numerical values of the widths of the strong decays of the charmed mesons $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650)$, $\mathrm{D}_{\mathrm{J}}(2740), \mathrm{D}_{\mathrm{J}}^{*}(2760), \mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ observed by the LHCb collaboration are presented in Tables 5-8. The measurements of the LHCb collaboration favor the assignment $\left(\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650)\right)=\left(0^{-}, 1^{-}\right)_{\frac{1}{2}}$ with $n=2$. They also favor the following two possible assignments

$$
\begin{array}{rll}
\left(\mathrm{D}_{\mathrm{J}}^{*}(2760), \mathrm{D}_{\mathrm{J}}(2740)\right) & =\left(1^{-}, 2^{-}\right)_{\frac{3}{2}} & \text { with } \\
\left(\mathrm{D}_{\mathrm{J}}(2740), \mathrm{D}_{\mathrm{J}}^{*}(2760)\right)=\left(2^{-}, 3^{-}\right)_{\frac{5}{2}} & \text { with } & n=1, L=2,
\end{array}
$$

The partial and total decay widths in the above assignments are listed in Table 5. Comparing with the experimental data from the LHCb and BaBar collaborations, our results are of the same order of magnitude. We can see that, except for the $\mathrm{D}_{\mathrm{J}}(2580)$, the predicted total widths of the $\mathrm{D}_{\mathrm{J}}(2740)$ and $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ are somewhat bigger than the experimental values, and the width of the $D_{J}^{*}(2650)$ is roughly in agreement with the total width measured by the BaBar collaboration. In addition, the $1 D \frac{5}{2} 3^{-}$state may be the optimal assignment of the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ since the corresponding total width is close

Table 4. The adopted masses of the mesons used in our calculation.

| states | $M_{\pi^{+}}$ | $M_{\pi^{0}}$ | $M_{\mathrm{K}+}$ | $M_{\mathrm{K}^{*}}$ | $M_{\eta}$ | $M_{\eta^{\prime}}$ | $M_{\mathrm{D}^{+}}$ | $M_{\mathrm{D}^{0}}$ | $M_{\mathrm{D}^{*+}}$ | $M_{\mathrm{D}^{* 0}}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| mass $/ \mathrm{MeV}$ | 139.57 | 134.98 | 493.68 | 891.66 | 547.85 | 957.78 | 1869.6 | 1864.83 | 2010.25 | 2006.96 |
| states | $M_{\mathrm{D}_{\mathrm{s}}^{*+}}$ | $M_{\mathrm{D}_{\mathrm{s}}^{+}}$ | $M_{\mathrm{D}(2400)}$ | $M_{\mathrm{D}(2430)}$ | $M_{\mathrm{D}(2420)}$ | $M_{\mathrm{D}(2460)}$ | $M_{\mathrm{D}_{\mathrm{s}}(2317)}$ | $M_{\rho}$ | $M_{\omega}$ |  |
| mass $/ \mathrm{MeV}$ | 212.3 | 1968.47 | 2318 | 2427 | 2421.3 | 2464.4 | 2317.8 | 770 | 782 |  |

Table 5. The strong decay widths $\Gamma_{\mathrm{p}}$ and the ratios of partial decay widths $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ of the newly observed charmed mesons $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740)$ and $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ with possible assignments. If the corresponding decay channel is forbidden, we mark it by "-". All values of $\Gamma_{\mathrm{p}}$ are in units of MeV .

|  | $\begin{gathered} \hline \mathrm{D}_{\mathrm{J}}(2580) \\ 2 S \frac{1}{2} 0^{-} \\ \hline \end{gathered}$ |  | $\begin{gathered} \hline \mathrm{D}_{\mathrm{J}}^{*}(2650) \\ 2 S \frac{1}{2} 1^{-} \\ \hline \end{gathered}$ |  | $\begin{gathered} \mathrm{D}_{\mathrm{J}}^{*}(2760) \\ 1 D \frac{3}{2} 1^{-} \\ \hline \end{gathered}$ |  | $\begin{gathered} \hline \mathrm{D}_{\mathrm{J}}(2740) \\ 1 D \frac{3}{2} 2^{-} \\ \hline \end{gathered}$ |  | $\begin{gathered} \hline \mathrm{D}_{\mathrm{J}}(2740) \\ 1 D \frac{5}{2} 2^{-} \\ \hline \end{gathered}$ |  | $\begin{gathered} \mathrm{D}_{\mathrm{J}}^{*}(2760) \\ 1 D \frac{5}{2} 3^{-} \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ |
| $\mathrm{D}^{*+} \pi^{-}$ | 49.80 | 0.66 | 34.72 | 0.38 | 16.46 | 0.09 | 17.04 | 0.15 | 48.63 | 0.44 | 10.35 | 0.11 |
| $\mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{-}$ | - | - | 2.02 | 0.02 | 2.86 | 0.02 | 0.38 | 0.003 | 6.95 | 0.06 | 0.09 | 0.0009 |
| $\mathrm{D}^{* 0} \pi^{0}$ | 25.00 | 0.33 | 17.32 | 0.19 | 8.19 | 0.04 | 8.74 | 0.08 | 24.24 | 0.22 | 5.32 | 0.06 |
| $\mathrm{D}^{* 0} \eta$ | 0.81 | 0.01 | 3.26 | 0.04 | 2.80 | 0.02 | 0.59 | 0.005 | 7.41 | 0.07 | 0.24 | 0.003 |
| $\mathrm{D}^{0}(2400) \pi^{0}$ | 0.35 | 0.004 | - | - | - | - | 0.06 | 0.0005 | 0.00028 | 0 | - | - |
| $\mathrm{D}^{0}(2460) \pi^{0}$ | - |  | 0.024 | 0.0003 | 0.23 | 0.001 | 0.97 | 0.009 | 23.26 | 0.21 | 0.17 | 0.002 |
| $\mathrm{D}(2420) \pi^{0}$ | - |  | 0.12 | 0.001 | 23.92 | 0.13 | 0.26 | 0.002 | 0.089 | 0.0008 | 0.024 | 0.0002 |
| $\mathrm{D}(2427) \pi^{0}$ | - |  | 0.30 | 0.003 | 3.10 | 0.02 | 13.48 | 0.12 | 0.0057 | 0.0001 | 0.065 | 0.0007 |
| $\mathrm{D}^{+} \pi^{-}$ | - |  | 13.57 | 0.15 | 25.15 | 0.14 | - | - | - | - | 17.02 | 0.18 |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{-}$ | - |  | 6.56 | 0.07 | 10.34 | 0.06 | - | - | - | - | 0.70 | 0.007 |
| $\mathrm{D}^{0} \pi^{0}$ | - |  | 6.65 | 0.07 | 12.35 | 0.07 | - | - | - | - | 8.73 | 0.09 |
| $\mathrm{D}^{0} \eta$ | - |  | 3.53 | 0.04 | 6.78 | 0.04 | - | - | - | - | 0.99 | 0.10 |
| $\mathrm{D}^{+} \rho$ | - |  | 1.59 | 0.02 | 37.00 | 0.20 | 36.02 | 0.32 | 0.50 | 0.005 | 26.44 | 0.27 |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{*}$ | - |  | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}^{0} \rho$ | - |  | 1.37 | 0.02 | 19.10 | 0.10 | 18.90 | 0.17 | 0.29 | 0.003 | 13.72 | 0.14 |
| $\mathrm{D}^{0} \omega$ | - |  | 0.14 | 0.002 | 17.80 | 0.10 | 16.90 | 0.15 | 0.20 | 0.002 | 12.62 | 0.13 |
| $\Gamma_{\mathrm{T}} / \mathrm{MeV}$ | 75.96 |  | 91.18 |  | 186.06 |  | 113.34 |  | 111.56 |  | 96.49 |  |

Table 6. The strong decay widths $\Gamma_{\mathrm{p}}$ and the ratios of partial decay widths $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ of the newly observed charmed meson $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ with possible assignments. If the corresponding decay channel is forbidden, we mark it by "-".

|  | $1 F \frac{5}{2} 2^{+}$ |  | $1 F \frac{7}{2} 4^{+}$ |  | $2 P \frac{1}{2} 0^{+}$ |  | $2 P \frac{3}{2} 2^{+}$ |  | $3 S \frac{1}{2} 1^{-}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ |
| $\mathrm{D}^{*+} \pi^{-}$ | 10.46 | 0.09 | 9.36 | 0.09 | - | - | 11.91 | 0.07 | 3.19 | 0.04 |
| $\mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{-}$ | 1.80 | 0.02 | 0.27 | 0.003 | - | - | 6.38 | 0.04 | 1.50 | 0.02 |
| $\mathrm{D}^{* 0} \pi^{0}$ | 5.21 | 0.04 | 4.78 | 0.05 | - | - | 5.86 | 0.03 | 1.66 | 0.02 |
| $\mathrm{D}^{* 0} \eta$ | 1.87 | 0.02 | 0.54 | 0.005 | - | - | 3.16 | 0.02 | 0 | 0 |
| $\mathrm{D}^{* 0} \eta^{\prime}$ | 0.10 | 0.0009 | - | - | - | - | 0.25 | 0.001 | 1.84 | 0.02 |
| $\mathrm{D}^{+} \pi^{-}$ | 12.61 | 0.11 | 14.08 | 0.14 | 23.94 | 0.11 | 3.40 | 0.02 | 3.61 | 0.04 |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{-}$ | 3.71 | 0.03 | 0.81 | 0.008 | 2.85 | 0.01 | 5.47 | 0.03 | 0.081 | 0.0009 |
| $\mathrm{D}^{0} \pi^{0}$ | 6.22 | 0.05 | 7.19 | 0.07 | 11.97 | 0.05 | 1.61 | 0.01 | 1.84 | 0.02 |
| $\mathrm{D}^{0} \eta$ | 2.94 | 0.03 | 1.21 | 0.01 | 4.26 | 0.02 | 1.6 | 0.01 | 0.23 | 0.003 |
| $\mathrm{D}^{0} \eta^{\prime}$ | 4.39 | 0.04 | 0.11 | 0.001 | 1.07 | 0.005 | 4.31 | 0.02 | 1.69 | 0.02 |
| $\mathrm{D}^{*+} \rho$ | 6.74 | 0.06 | 28.23 | 0.28 | 62.01 | 0.28 | 25.22 | 0.14 | 18.01 | 0.21 |
| $\mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{*}$ | 0.0057 | 0 | 0.032 | 0.0003 | 3.06 | 0.01 | 12.21 | 0.07 | 0.57 | 0.007 |
| $\mathrm{D}^{* 0} \rho$ | 3.49 | 0.03 | 14.45 | 0.14 | 31.60 | 0.14 | 12.78 | 0.07 | 8.73 | 0.10 |
| $\mathrm{D}^{* 0} \omega$ | 3.16 | 0.03 | 13.46 | 0.13 | 29.91 | 0.13 | 12.38 | 0.07 | 9.65 | 0.11 |
| $\mathrm{D}^{+} \rho$ | 11.69 | 0.10 | 2.78 | 0.03 | - | - | 20.71 | 0.12 | 0.14 | 0.002 |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{*}$ | 0.61 | 0.005 | 0.016 | 0.0002 | - | - | 2.29 | 0.01 | 3.53 | 0.04 |
| $\mathrm{D}^{0} \rho$ | 5.90 | 0.05 | 1.46 | 0.01 | - | - | 10.37 | 0.06 | 0.049 | 0.0006 |
| $\mathrm{D}^{0} \omega$ | 5.81 | 0.05 | 1.33 | 0.01 | - | - | 10.38 | 0.06 | 0.11 | 0.001 |
| $\mathrm{D}(2420) \pi^{0}$ | 15.01 | 0.13 | 0.04 | 0.0004 | 26.20 | 0.12 | 3.91 | 0.02 | 7.10 | 0.08 |
| $\mathrm{D}(2420) \eta$ | 0.61 | 0.005 | $1.82 \times 10^{-7}$ | 0 | 1.37 | 0.006 | $1.62 \times 10^{-3}$ | 0 | 0.11 | 0.001 |
| $\mathrm{D}(2427) \pi^{0}$ | 2.06 | 0.02 | 0.41 | 0.004 | 6.69 | 0.03 | 1.95 | 0.01 | 0.91 | 0.01 |
| $\mathrm{D}(2427) \eta$ | 0.04 | 0.0003 | $1.56 \times 10^{-4}$ | 0 | 0.35 | 0.002 | 0.13 | 0.0007 | 0.77 | 0.009 |
| $\mathrm{D}(2400) \pi^{0}$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}(2400) \eta$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}_{\mathrm{S}}(2460) \mathrm{K}^{-}$ | 3.07 | 0.03 | 0.016 | 0.0002 | 12.81 | 0.06 | 1.03 | 0.006 | 0.98 | 0.01 |
| $\mathrm{D}_{\mathrm{S}}(2536) \mathrm{K}^{-}$ | 1.54 | 0.01 | 0.0081 | 0.0001 | 6.40 | 0.03 | 0.64 | 0.004 | 0.49 | 0.006 |
| $\mathrm{D}^{+}(2460) \pi^{-}$ | 4.85 | 0.04 | 1.14 | 0.01 | - | - | 11.08 | 0.06 | 13.57 | 0.16 |
| $\mathrm{D}^{0}(2460) \pi^{0}$ | 2.45 | 0.02 | 0.59 | 0.006 | - | - | 5.59 | 0.03 | 6.87 | 0.08 |
| $\mathrm{D}^{0}(2460) \eta$ | 0.09 | 0.0008 | - | - | - | - | 0.05 | 0.0003 | 0.008 | 0.0001 |
| $\mathrm{D}_{\mathrm{S}}^{+}(2317) \mathrm{K}^{-}$ | - | - | - | - | - | - | - | - | - | - |
| $\Gamma_{\mathrm{T}} / \mathrm{MeV}$ |  |  | 102. |  | 224 |  | 174 |  |  |  |

to the experimental value of the LHCb collaboration. However, the LHCb collaboration identify the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ as the $1 D \frac{3}{2} 1^{-}$state, which is incompatible with our results. From Table 5, we can see that, if the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ is the $1 D \frac{5}{2} 3^{-}$state, the main decay channels are $\mathrm{D}^{+} \rho$, $\mathrm{D}^{+} \pi^{-}, \mathrm{D}^{0} \rho$ and $\mathrm{D}^{0} \omega$. The decay behavior of the $1 D \frac{3}{2} 1^{-}$ state is very similar to that of the $1 D \frac{5}{2} 3^{-}$state except for the decay channel $\mathrm{D}(2420) \pi^{0}$. This difference can be used to further identify the assignment of the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$ in the future. Furthermore, we tentatively identify the $\mathrm{D}_{\mathrm{J}}(2740)$ as the $1 D$ state with $J^{P}=2^{-}$, and we can see that the total widths of the $1 D \frac{3}{2} 2^{-}$and $1 D \frac{5}{2} 2^{-}$states are of the same order. However, the decay behaviors of these two states are different from each other. The main decay modes of the $1 D \frac{3}{2} 2^{-}$state are $\mathrm{D}^{+} \rho, \mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{0} \rho$,
$\mathrm{D}^{0} \omega$ and $\mathrm{D}(2427) \pi^{0}$, while the $1 D \frac{5}{2} 2^{-}$state mainly decays into $\mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{* 0} \pi^{0}$ and $\mathrm{D}^{0}(2460) \pi^{0}$.

As discussed at the end of Section 1, the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ is a natural parity state. Thus, we study its decay behavior with the $1 F \frac{5}{2} 2^{+}, 1 F \frac{7}{2} 4^{+}, 2 P \frac{1}{2} 0^{+}, 2 P \frac{3}{2} 2^{+}$and $3 S \frac{1}{2} 1^{-}$ assignments. We can see from Table 6 that the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ is most likely to be the $1 F \frac{5}{2} 2^{+}$state or $1 F \frac{7}{2} 4^{+}$state, since the total widths are in good agreement with the experimental data. However, these two assignments lead to different decay modes, which can be used to further identify its quantum numbers. If the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ is the $1 F \frac{5}{2} 2^{+}$ state, the $\mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{+} \pi^{-}, \mathrm{D}^{+} \rho$ and $\mathrm{D}(2420) \pi^{0}$ are the main decay modes, on the other hand, if the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ is the $1 F \frac{7}{2} 4^{+}$state, the $\mathrm{D}^{*+} \rho, \mathrm{D}^{* 0} \rho, \mathrm{D}^{* 0} \omega, \mathrm{D}^{+} \pi^{-}$and $\mathrm{D}^{*+} \pi^{-}$are the main decay modes. Our results show that

Table 7. The strong decay widths $\Gamma_{\mathrm{p}}$ and the ratios of partial decay widths $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ of the newly observed charmed meson $\mathrm{D}_{\mathrm{J}}(3000)$ with possible assignments. If the corresponding decay channel is forbidden, we mark it by "-".

|  | $1 F \frac{5}{2} 3^{+}$ |  | $1 F \frac{7}{2} 3^{+}$ |  | $2 P \frac{1}{2} 1^{+}$ |  | $2 P \frac{3}{2} 1^{+}$ |  | $3 S \frac{1}{2} 0^{-}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ | $\Gamma_{\mathrm{p}} / \mathrm{MeV}$ | $\Gamma_{\mathrm{p}} / \Gamma_{\mathrm{T}}$ |
| $\mathrm{D}^{*+} \pi^{-}$ | 17.02 | 0.15 | 25.95 | 0.14 | 20.32 | 0.11 | 23.62 | 0.08 | 4.78 | 0.05 |
| $\mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{-}$ | 0.57 | 0.005 | 4.40 | 0.02 | 9.45 | 0.05 | 1.22 | 0.004 | 2.25 | 0.02 |
| $\mathrm{D}^{* 0} \pi^{0}$ | 8.69 | 0.08 | 12.93 | 0.07 | 10.03 | 0.06 | 11.85 | 0.04 | 2.47 | 0.03 |
| $\mathrm{D}^{* 0} \eta$ | 1.05 | 0.01 | 4.59 | 0.02 | 4.92 | 0.03 | 2.48 | 0.009 | 0 | 0 |
| $\mathrm{D}^{* 0} \eta^{\prime}$ | 0.0057 | 0 | 0.25 | 0.001 | 2.71 | 0.02 | 18.72 | 0.06 | 2.75 | 0.03 |
| $\mathrm{D}^{+} \pi^{-}$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{-}$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}^{0} \pi^{0}$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}^{0} \eta$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}^{0} \eta^{\prime}$ | - | - | - | - | - | - | - | - | - | - |
| $\mathrm{D}^{*+}{ }_{\rho}$ | 12.16 | 0.11 | 12.79 | 0.07 | 41.34 | 0.23 | 36.26 | 0.12 | 15.44 | 0.17 |
| $\mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{*}$ | 0.0081 | 0.0001 | 0.016 | 0.0001 | 2.05 | 0.01 | 4.08 | 0.01 | 0.49 | 0.005 |
| $\mathrm{D}^{* 0} \rho$ | 6.24 | 0.05 | 6.56 | 0.04 | 21.07 | 0.12 | 18.46 | 0.06 | 7.48 | 0.08 |
| $\mathrm{D}^{* 0} \omega$ | 5.77 | 0.05 | 6.07 | 0.03 | 19.93 | 0.11 | 17.53 | 0.06 | 8.26 | 0.09 |
| $\mathrm{D}^{+} \rho$ | 29.22 | 0.26 | 5.01 | 0.03 | 10.59 | 0.06 | 34.52 | 0.12 | 0.21 | 0.002 |
| $\mathrm{D}_{\mathrm{S}}^{+} \mathrm{K}^{*}$ | 1.50 | 0.01 | 0.024 | 0.0001 | 7.13 | 0.04 | 3.82 | 0.01 | 5.31 | 0.06 |
| $\mathrm{D}^{0} \rho$ | 14.74 | 0.13 | 2.63 | 0.01 | 5.61 | 0.03 | 17.27 | 0.06 | 0.073 | 0.0008 |
| $\mathrm{D}^{0} \omega$ | 14.52 | 0.13 | 2.39 | 0.01 | 4.99 | 0.03 | 17.30 | 0.06 | 0.15 | 0.002 |
| $\mathrm{D}(2420) \pi^{0}$ | 0.99 | 0.009 | 0.40 | 0.002 | 0.0081 | 0 | 0.024 | 0.0001 | - | - |
| $\mathrm{D}(2420) \eta$ | $1.7 \times 10^{-3}$ | 0 | $6.3 \times 10^{-4}$ | 0 | $3.0 \times 10^{-3}$ | 0 | $6.1 \times 10^{-3}$ | 0 | - | - |
| $\mathrm{D}(2427) \pi^{0}$ | 0.0081 | 0.0001 | $9.7 \times 10^{-3}$ | 0.0001 | $9.9 \times 10^{-3}$ | 0.0001 | 0.0081 | 0 | - |  |
| $\mathrm{D}(2427) \eta$ | $5.9 \times 10^{-4}$ | 0 | $1.8 \times 10^{-4}$ | 0 | $1.5 \times 10^{-3}$ | 0 | $3.0 \times 10^{-3}$ | 0 | - | - |
| $\mathrm{D}(2400) \pi^{0}$ | 0.32 | 0.003 | 0.057 | 0.0003 | 0.24 | 0.0001 | 0.17 | 0.0006 | 0.51 | 0.006 |
| $\mathrm{D}(2400) \eta$ | 0.011 | 0.0001 | 0.0027 | 0 | 0.27 | 0.0002 | 0.30 | 0.001 | 0.28 | 0.003 |
| $\mathrm{D}_{\mathrm{S}}(2460) \mathrm{K}^{-}$ | $1.9 \times 10^{-3}$ | 0 | $2.5 \times 10^{-3}$ | 0 | 0.0081 | 0 | 0.024 | 0.0001 | - | - |
| $\mathrm{D}_{\mathrm{S}}(2536) \mathrm{K}^{-}$ | $3.7 \times 10^{-3}$ | 0 | $4.9 \times 10^{-3}$ | 0 | 0.024 | 0.0001 | 0.049 | 0.0002 | - | - |
| $\mathrm{D}^{+}(2460) \pi^{-}$ | 0.99 | 0.009 | 36.52 | 0.19 | 10.52 | 0.06 | 56.21 | 0.19 | 27.15 | 0.30 |
| $\mathrm{D}^{0}(2460) \pi^{0}$ | 0.50 | 0.004 | 18.32 | 0.10 | 5.39 | 0.03 | 28.05 | 0.10 | 13.74 | 0.15 |
| $\mathrm{D}^{0}(2460) \eta$ | 0.019 | 0.0002 | 0.85 | 0.005 | 0.024 | 0.0001 | 0.56 | 0.002 | 0.013 | 0.0001 |
| $\mathrm{D}_{\mathrm{S}}^{+}(2317) \mathrm{K}^{-}$ | 0.049 | 0.0004 | 0.016 | 0.0001 | 0.83 | 0.005 | 0.52 | 0.002 | 0.27 | 0.003 |
| $\Gamma_{\mathrm{T}} / \mathrm{MeV}$ | 114 |  | 187 |  | 177 |  | 293 |  |  |  |

the assignments of the $2 P \frac{1}{2} 0^{+}, 2 P \frac{3}{2} 2^{+}$and $3 S \frac{1}{2} 0^{-}$states can be excluded since the corresponding total widths are quite different from the experimental values. Nevertheless, this information is valuable in searching for the partners of the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$. In Ref. [22], Y. Sun et al. identify the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ as the $2^{3} P_{0}$ state with the effective oscillator parameter $R$. In their studies, it is proposed that the main decay channels of the $2^{3} P_{0}$ state are $\mathrm{D}^{*} \rho, \mathrm{D}(2420) \pi$, $\mathrm{D}(2427) \pi, \mathrm{D} \eta, \mathrm{D}_{\mathrm{S}} \mathrm{K}$, and $\mathrm{D}^{*} \omega$.

As for the $D_{J}(3000)$, the possible assignments are the $3 S \frac{1}{2} 0^{-}, 2 P \frac{1}{2} 1^{+}, 2 P \frac{3}{2} 1^{+}, 1 F \frac{5}{2} 3^{+}$and $1 F \frac{7}{2} 3^{+}$states. In Table 7, the partial and total decay widths of the $\mathrm{D}_{\mathrm{J}}(3000)$ in those possible assignments are given. We can easily see from the table that both the widths of the
$1 F \frac{7}{2} 3^{+}$and $2 P \frac{1}{2} 1^{+}$states are in good agreement with the experimental data. So the $\mathrm{D}_{\mathrm{J}}(3000)$ is most likely to be the $1 F \frac{7}{2} 3^{+}$or $2 P \frac{1}{2} 1^{+}$state. If the $\mathrm{D}_{\mathrm{J}}(3000)$ is the $1 F \frac{7}{2} 3^{+}$state, it decays dominantly to $\mathrm{D}^{+}(2460) \pi^{-}$, $\mathrm{D}^{0}(2460) \pi^{0}, \mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{* 0} \pi^{0}$ and $\mathrm{D}^{*+} \rho$, on the other hand, if the $\mathrm{D}_{\mathrm{J}}(3000)$ is the $2 P \frac{1}{2} 1^{+}$state, it decays dominantly to $\mathrm{D}^{*+} \rho, \mathrm{D}^{* 0} \rho, \mathrm{D}^{* 0} \omega, \mathrm{D}^{*+} \pi^{-}, \mathrm{D}^{+} \rho$ and $\mathrm{D}^{* 0} \pi^{0}$. These conclusions are consistent with the experimental observation [1], where the $\mathrm{D}_{\mathrm{J}}(3000)$ was firstly observed in the $\mathrm{D}^{*+} \pi^{-}$decay channel. As for the other three assignments $3 S \frac{1}{2} 0^{-}, 2 P \frac{3}{2} 1^{+}$and $1 F \frac{5}{2} 3^{+}$, we can also see the main decay modes from Table 7 , which are valuable in
searching for these states experimentally in the future. In Ref. [22], Y. Sun et al. also suggest that the $2 P 1^{+}$ state is the mostly probable assignment of the $\mathrm{D}_{\mathrm{J}}(3000)$, which is compatible with our observation. However, the $1 F 3^{+}$assignment is excluded in their studies as the width deviates from the experimental value. The differences between the results of Y. Sun et al. and ours are mainly due to the influence of the input parameter $R$. We will give a short discussion about the dependence on $R$ at the end of this section.

From Tables 5-7, we can also see that most of the ratios of different decay channels are roughly consistent with the results in Ref. [7]. For example, the ra$\operatorname{tios} \frac{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}_{\mathrm{S}}^{*+} \mathrm{K}^{-}\right)}{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}, \frac{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{* 0} \pi^{0}\right)}{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$and $\frac{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{* 0} \eta\right)}{\Gamma\left(\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$from the heavy meson effective theory are $0,0.51$ and 0.02 , respectively [7], which are consistent with the present results $0,0.52$, and 0.02 , respectively. In Table 8, we also present the experimental value of the ratio $\frac{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{+} \pi^{-}\right)}{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$for the well established meson $\mathrm{D}_{2}^{*}(2460)$ from the BaBar [2], CLEO [31, 32], ARGUS [33], and ZEUS [34] collaborations. The result based on heavy meson effective theory in the leading order approximation is also listed in Table 8 . The present prediction of 2.29 based on the ${ }^{3} P_{0}$ decay model is in excellent agreement with the average experimental value of 2.35 [7]. Furthermore, this result is consistent with the prediction based on the heavy meson effective theory. Finally, it should be noted that the ratios of the decay widths of different charmed mesons based on the ${ }^{3} P_{0}$ decay model are roughly consistent with the experimental data. For example, for the predicted ratio $\frac{\Gamma_{\mathrm{D}_{\mathrm{J}}(2580)}}{\Gamma_{\mathrm{D}_{\mathrm{J}}^{*}(2650)}} \approx 0.83$ based on the ${ }^{3} P_{0}$ decay model, the corresponding experimental value is 1.27 ; for the predicted ratio $\frac{\Gamma_{\mathrm{D}_{\mathrm{J}}^{*}(2760)}}{\Gamma_{\mathrm{D}_{\mathrm{J}}(2740)}} \approx 0.86$, the corresponding experimental value is 1.0 .

We now give a short discussion of the uncertainties of the results based on the ${ }^{3} P_{0}$ decay model. Since this model is a simplified model of a complicated theory, it is not surprising that the prediction is not very accurate. Especially, the input parameter $R$ has a significant influence on the shapes of the radial wave functions. The spatial integral in Eq. (6) is sensitive to the parameter $R$, therefore the decay width based on the ${ }^{3} P_{0}$ decay model is sensitive to the parameter $R$. We take the decay $D_{J}(2580) \rightarrow D^{*+} \pi^{-}$as an example, and plot the decay width versus the input parameter $R$ in Figs. 2 and 3 . From these two figures, we can see easily the dependence of the decay width on the input parameter $R$. If $R_{\mathrm{D}^{*+}}$ and $R_{\pi^{-}}$are fixed to be $2.5 \mathrm{GeV}^{-1}$, the de-
cay width of the $\mathrm{D}_{\mathrm{J}}(2580)$ changes several times with the value of $R_{\mathrm{D}_{\mathrm{J}}(2580)}$ changing from $1.5 \mathrm{GeV}^{-1}$ to $3.0 \mathrm{GeV}^{-1}$. Similarly, the decay width changes $2-3$ times, when the $R_{\mathrm{D}_{\mathrm{J}}(2580)}$ and $R_{\pi^{-}}$(or the $R_{\mathrm{D}_{\mathrm{J}}(2580)}$ and $R_{\mathrm{D}^{*+}}$ ) are fixed to be $2.5 \mathrm{GeV}^{-1}$ while the $R_{\mathrm{D}^{*+}}$ (or $R_{\pi^{-}}$) changes. In Ref. [14], H. G. Blundel et al. carry out a series of least squares fits of the model predictions to the decay widths of 28 of the best known meson decays, and the common oscillator parameter $R$ with a value of $2.5 \mathrm{GeV}^{-1}$ is suggested to be the optimal value [14]. As for the factor $\gamma$, it describes the strength of quark-antiquark pair creation from the vacuum, which also needs to be fitted according to experimental data, giving a fitted value of $6.25[14,19]$. Once the optimal values of $\gamma$ and $R$ are determined, the best predictions based on the ${ }^{3} P_{0}$ decay model are expected to be within a factor of 2 . More detailed analysis of the uncertainties of the results in the ${ }^{3} P_{0}$ decay model can be found in Ref. [14].


Fig. 2. The strong decay of $\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{*+} \pi^{-}$ with $R_{\pi^{-}}=2.5 \mathrm{GeV}^{-1}$.


Fig. 3. The strong decay of $\mathrm{D}_{\mathrm{J}}(2580) \rightarrow \mathrm{D}^{*+} \pi^{-}$ with $R_{\mathrm{D}^{*+}}=2.5 \mathrm{GeV}^{-1}$.

Table 8. Experimental values and numerical results based on the leading order heavy meson effective theory (HMET) for the ratio $\frac{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{+} \pi^{-}\right)}{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$compared to our numerical result based on the ${ }^{3} P_{0}$ decay model.

| BaBar [2] | CLEO [31] | CLEO [32] | ARGUS [33] | ZEUS [34] | HMET [7] | this work |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $1.47 \pm 0.03 \pm 0.16$ | $2.2 \pm 0.7 \pm 0.6$ | $2.3 \pm 0.8$ | $3.0 \pm 1.1 \pm 1.5$ | $2.8 \pm 0.8_{-0.6}^{+0.5}$ | 2.29 | 2.29 |

## 4 Conclusion

In this article, we study the properties of the charmed mesons $\quad \mathrm{D}_{\mathrm{J}}(2580), \quad \mathrm{D}_{\mathrm{J}}^{*}(2650), \quad \mathrm{D}_{\mathrm{J}}(2740), \quad \mathrm{D}_{\mathrm{J}}^{*}(2760)$, $\mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ with the ${ }^{3} P_{0}$ decay model. Our results support the $1 D \frac{5}{2} 3^{-}$assignment of the $\mathrm{D}_{\mathrm{J}}^{*}(2760)$, but more experimental data are still needed to identify it. Furthermore, both the mass spectra of the $D$ mesons and the two-body decay behaviors indicate that the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ may be the $1 F \frac{5}{2} 2^{+}$state or the $1 F \frac{7}{2} 4^{+}$ state, since the widths in these two assignments are both in good agreement with the experimental data. We ten-
tatively identify the $\mathrm{D}_{\mathrm{J}}(3000)$ as the $1 F \frac{7}{2} 3^{+}$state and $2 P \frac{1}{2} 1^{+}$state based on the decay widths. It is noted that the $\mathrm{D}_{\mathrm{J}}^{*}(3000)$ and $\mathrm{D}_{\mathrm{J}}(3000)$ states are strongly correlated with the background parameters as shown in Ref. [1]. Thus, more experimental data are still needed to draw a more clear conclusion on the existence of these two states. In studying the $\mathrm{D}_{\mathrm{J}}(2580), \mathrm{D}_{\mathrm{J}}^{*}(2650), \mathrm{D}_{\mathrm{J}}(2740)$, $\mathrm{D}_{\mathrm{J}}^{*}(2760), \mathrm{D}_{\mathrm{J}}(3000)$ and $\mathrm{D}_{\mathrm{J}}^{*}(3000)$, we have also obtained their partial decay widths in different channels in the assignments $2 P \frac{1}{2} 0^{+}, 2 P \frac{3}{2} 2^{+}, 3 S \frac{1}{2} 1^{-}, 3 S \frac{1}{2} 0^{-}$, etc, which can be used to confirm or reject the assignments of the newly observed charmed mesons in the future.

## References

1 Aaij R et al. (LHCb collaboration). JHEP, 2013, 1309: 145
2 del Amo Sanchez P et al. Phys. Rev. D, 2010, 82: 111101
3 WANG Z G. Phys. Rev. D, 2011, 83: 014009
4 ZHONG X H. Phys. Rev. D, 2010, 82: 114014
5 CHEN B, YUAN L, ZHANG A L. Phys. Rev. D, 2011, 83: 114025
6 Colangelo P, De Fazio F, Giannuzzi F, Nicotri S. Phys. Rev. D, 2012, 86: 054024
7 WANG Z G. Phys. Rev. D, 2013, 88: 114003
8 Micu L. Nucl. Phys. B, 1969, 10: 521
9 Le Yaouanc A, Oliver L, Pene O, Raynal J-C. Phys. Rev. D, 1973, 8: 2223; Phys. Rev. D, 1974, 9: 1415; Phys. Rev. D, 1975, 11: 1272; Phys. Lett. B, 1977, 71: 397
10 Le Yaouanc A, Oliver L, Pene O, Raynal J C. Phys. Lett. B, 1977, 72: 57
11 Le Yaouanc A, Oliver L, Pene O, Raynal J C. Hadron Transitons in the Quark Model, New York: Gordon and Breach Science Publishers, 1988
12 Roberts W, Brac B S. Few-Body Syst., 1992, 11: 171
13 Capstick S, Isgur N. Phys. Rev. D, 1986, 34: 2809; Capstick S, Roberts W. Phys. Rev. D, 1994, 49: 4570
14 Blundell H G. hep-ph/9608473; Blundell H G, Godfrey S. Phys. Rev. D, 1996, 53: 3700; Blundell H G, Godfrey S, Phelps B. Phys. Rev. D, 1996, 53: 3712
15 Ackleh E S, Barnes T, Swanson E S. Phys. Rev. D, 1996, 54: 6811; Barnes T, Black N, Page P R. Phys. Rev. D, 2003, 68: 054014; Close F E, Swanson E S. Phys. Rev. D, 2005, 72 : 094004; Close F E, Thomas C E, Lakhina O, Swanson E S.

Phys. Lett. B, 2007, 647: 159
16 ZHOU H Q, PING R G, ZOU B S. Phys. Lett. B, 2005, 611: 123; DING G J, YAN M L. Phys. Lett. B, 2007, 657: 49
17 CHEN B, WANG D X, ZHANG A L. Phys. Rev. D, 2009, 80: 071502
18 LI D M, ZHOU S. Phys.Rev. D, 2008, 78: 054013; LI D M, JI P F, MA B. Eur. Phys. J. C, 2011, 71: 1582
19 LI D M, MA B. Phys. Rev. D, 2010, 81: 014021
20 ZHANG B, LIU X, DENG W Z, ZHU S L. Eur. Phys. J. C, 2007, 50: 617; SUN Y, SONG Q T, CHEN D Y, LIU X, ZHU
S L. arXiv:1401.1595 [hep-ph]
21 YANG Y C, XIA Z R, PING J L. Phys. Rev. D, 2010, 81: 094003
22 SUN Y, LIU X, Matsuki T. Phys. Rev. D, 2013, 88: 094020
23 Manohar A V, Wise M B. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol, 2000, 10: 1; Neubert M. Phys. Rept., 1994, 245: 259
24 Beringer J et al. Phys. Rev. D, 2012, 86: 010001
25 Godfrey S, Isgur N. Phys. Rev. D, 1985, 32: 189
26 Pierro M D, Eichten E. Phys. Rev. D, 2001, 64: 114004
27 Ebert D, Faustov R N, Galkin V O. Eur. Phys. J. C, 2010, 66: 197
28 Hayne C, Isgur N. Phys. Rev. D, 1982, 25: 1944
29 Jacob M, Wick G C. Ann. Phys., 1959, 7: 404
30 Matsuki T, Morii T, Seo K. Prog. Theor. Phys., 2010, 124: 285
31 Avery P et al. Phys. Lett. B, 1994, 331: 236
32 Avery P et al. Phys. Rev. D, 1990, 41: 774
33 Albrecht H et al. Phys. Lett. B, 1989, 232: 398
34 Chekanov S et al. Eur. Phys. J. C, 2009, 60: 25


[^0]:    Received 28 September 2014，Revised 13 January 2015
    ＊Supported by National Natural Science Foundation of China（11375063）and Fundamental Research Funds for the Central Universities （13QN59，2014MS165）

    1）E－mail：yuguoliang2011＠163．com
    2）E－mail：zgwang＠aliyun．com
    

    Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence．Any further distribution of this work must maintain attribution to the author（s）and the title of the work，journal citation and DOI．Article funded by SCOAP ${ }^{3}$ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

