Dynamical chiral symmetry breaking in NJL Model with a strong background magnetic field and Lorentz-violating extension of the Standard Model*

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Abstract: The Eigenstate Method has been developed to deduce the fermion propagator with a constant external magnetic field. In general, we find its result is equivalent to other methods and this new method is more convenient, especially when one evaluates the contribution from the infinitesimal imaginary term of the fermion propagator. Using the Eigenstate Method we try to discuss whether the infinitesimal imaginary frequency of the fermion propagator in a strong magnetic field and Lorentz-violating extension of the minimal $SU(3)\times SU(2)\times SU(1)$ Standard Model could have a significant influence on the dynamical mass. When the imaginary term of the fermion propagator in this model is not trivial ($\sqrt{(\alpha-1)eB/3}<\sigma<\sqrt{(\alpha-1)2eB/3}$), this model gives a correction to the dynamical mass. When one does not consider the influence from the imaginary term ($\sigma>\sqrt{(\alpha-1)2eB/3}$), there is another correction from the conventional term. Under both circumstances, chiral symmetry is broken.

Keywords: NJL, strong magnetic field, eigenstate method, chiral symmetry breaking, Lorentz-violation

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1 Introduction

In quantum field theory, the infinitesimal imaginary term of the fermion propagator can cause remarkable adjustment to the calculations in some cases. The method of Schwinger proper time [1, 2] is very useful in the NJL model, but the disadvantage with this method is that it is not easy to process the influence of the infinitesimal imaginary term. Song Shi, Hong-shi Zong et al. developed a new method, the Eigenstate Method [3], to deduce the NJL gap equation. This new method is very convenient for studying NJL problems with external magnetic field, because one does not need to first find the expression for $\langle x|\hat{S}|y\rangle$ (where $\hat{S}$ is the fermion propagator). It is also applicable for further study, such as the Schwinger-Dyson equations. For the finite chemical potential μ case, this method will continue giving the right fermion propagator. The starting point of developing this new Eigenstate Method is to evaluate how much influence the infinitesimal imaginary term of the fermion propagator has on the final results. However, for a conventional NJL model with finite temperature and chemical potential, the influence from the imaginary term is trivial, as shown in Ref. [3].

The $SU(3)\times SU(2)\times SU(1)$ Standard Model, although phenomenologically successful, leaves a variety of issues unresolved. Over the past few decades, many new theories have been proposed. Some studies showed that Lorentz violation may exist in loop quantum gravity, noncommutative field theories and M-theory, etc [4–6]. Kostelecký and Samuel considered the possibility that spontaneous breakdown of Lorentz symmetry is explored via covariant string field theory [7]. Colladay and Kostelecký then presented a general Lorentz-violating extension of the minimal $SU(3)\times SU(2)\times U(1)$ Standard Model including CPT-even and CPT-odd terms [8–10]. To identify signals between the new proposed fundamental theories and the Standard Model, one approach is to examine new proposed fundamental theories for effects that are qualitatively different from Standard Model physics. As this Lorentz-violating extension of the Standard Model may show, the signal differs from the Standard Model.

Strong magnetic fields could play an important role in astrophysics [11] and high energy physics [12]. The

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magnetic field of the early universe [13–15] and the inner superstrong field of some magnetars are estimated to be of order 10^{24}G and $10^{18}-10^{20}$G [11, 16, 17]. The magnetic fields produced at RHIC and LHC are estimated to $eB\sim 1.5m_\pi^2$ and even higher $15m_\pi^2$ [18], respectively. This has caused much research on the impact of strong magnetic fields on the nature of quark matter, on the QCD phase diagram and the color superconducting phase transition, etc. In the NJL model, due to the magnetic catalysis effect, the external magnetic field stimulates QCD condensate. In the chiral limit, however, when the temperature exceeds a critical value T_c, the system undergoes a phase transition, namely the broken chiral symmetry is restored. Hence, in this work we study the influence of the magnetic field and Lorentz-violating term on QCD condensate, and discuss whether the effects of the Lorentz-violating term are similar to those of the magnetic field or temperature on quark condensate.

Considering the above situation—strong magnetic field and Lorentz violation— we use the Eigenstate Method to discuss whether the infinitesimal imaginary frequency of the fermion propagator in a strong magnetic field and Lorentz-violating extension of the Standard Model will have a significant influence on the dynamical mass. This work is organized as follows. In Section 2, we introduce the Lorentz-violating term into the NJL model within a constant strong background magnetic field. In Section 3, by the Eigenstate Method, we discuss the dynamical mass and the imaginary term depending on the Lorentz-violating term and the constant strong magnetic field. The numerical results and analysis are given in Section 4.

2 NJL model in Lorentz-violating extension of the standard model

In 1+3 dimensional time-space, the bosonized two flavor NJL Lagrangian with external magnetic field is

$$\mathcal{L}=\mathrm{i}\bar{\psi}\gamma^\mu(\partial_\mu+\mathrm{i}qQ\otimes A)\psi-\bar{\psi}(\sigma+\mathrm{i}\gamma^5\otimes\vec{\pi}\cdot\vec{\tau})\psi-\frac{N}{2G}\Sigma^2, \tag{1}$$

where $\vec{\tau}$ are the Pauli matrices and

$$\Sigma^2=\sigma^2+\pi^2,\quad \ll\bar{\psi}\psi\gg=\sigma,\quad \ll\bar{\psi}\gamma_0\psi\gg=\pi,$$

$$Q_{11}=q_u=\frac{2}{3},\quad Q_{22}=q_d=-\frac{1}{3},\quad q=-e.$$

In the above Lagrangian, we have already assumed that the current mass of the fermion is zero, and the potential of the external magnetic field A_μ is present. A_μ can be defined as

$$(A_0,A_1,A_2,A_3)=\left(0,\frac{B}{2}x^2,-\frac{B}{2}x^1,0\right), \tag{2}$$

where x^i are the components of time-space coordinates (x^0,x^1,x^2,x^3). Depending on the Lagrangian (1), the fermion propagator within the magnetic field is

$$\hat{S}=\frac{1}{\gamma^\mu\hat{\Pi}_\mu-\sigma},$$

where

$$\hat{\Pi}_\mu=\mathrm{i}\partial_\mu+q_f eA_\mu,\quad q_f=q_{u,d}. \tag{3}$$

In the derivation above, we do not consider the effect of the imaginary term. For a free fermion propagator, the infinitesimal imaginary term is iϵ. It is well-known in quantum field theory that the infinitesimal imaginary term does not always play a role as a pointer of the integral path. When external fields or external elements interfere, it is not safe to claim that the imaginary term still remains iϵ. In some cases, it can cause remarkable adjustment to the calculations. A convincing example is the case when the system has finite chemical potential μ. In Minkowski space, the infinitesimal imaginary term is μ dependent [1, 19],

$$\mathcal{S}(k,\mu)=\frac{\tilde{\not{k}}+m}{\tilde{k}^2-m^2+i\epsilon(k_0+\mu)\mathrm{sgn}(k_0)},$$

therefore one needs to properly deduce the imaginary term with caution.

By a convenient trick for the path integral, the definition of the partition function of quantum field theory is

$$Z=\left\langle 0\left|T\exp\left\{-\mathrm{i}\int_{-\infty}^{+\infty}\hat{H}\,\mathrm{d}t\right\}\right|0\right\rangle.$$

Introducing a factor $(1-\mathrm{i}\epsilon)$ to change the expression of the partition function will derive a ϵ-dependent Lagrangian:

$$\begin{aligned}Z&=\lim_{\epsilon\to0^+}\left\langle 0\left|T\exp\left\{-\mathrm{i}(1-\mathrm{i}\epsilon)\int_{-\infty}^{+\infty}\hat{H}\,\mathrm{d}t\right\}\right|0\right\rangle\\&=\lim_{\epsilon\to0^+}\int\mathrm{d}\psi\left\langle \psi\left|T\exp\left\{-\mathrm{i}(1-\mathrm{i}\epsilon)\int_{-\infty}^{+\infty}\hat{H}\,\mathrm{d}t\right\}\right|\psi\right\rangle\\&=\lim_{\epsilon\to0^+}\int D\bar{\psi}D\psi\,\mathrm{e}^{\mathrm{i}\int\mathrm{d}x\,\mathcal{L}_\epsilon}.\end{aligned}$$

On the basis of the above, the improved fermion propagator in the NJL model with an external magnetic field becomes

$$\hat{S}=\frac{\not{\Pi}+\sigma}{(\not{\Pi})^2-\sigma^2+\mathrm{i}O(\epsilon)},$$
$$O(\epsilon)=\epsilon(|\vec{\Pi}|^2+\sigma^2-q_f eB\sigma^{12}), \tag{4}$$

where $\sigma^{12}=\mathrm{diag}(1,-1,1,-1)$.

Unlike the free fermion propagator, in the propagator above the $\hat{\Pi}_1$ and $\hat{\Pi}_2$ are not commutable. Hence it is not possible to find a representation in which all four $\hat{\Pi}_\mu$ eigenstates exist simultaneously. Rather than employing Schwinger proper time regularization to rewrite the denominator, by the Eigenstate Method, we do not need to find the eigenstates for all $\hat{\Pi}_\mu$. Instead, we turn to find the eigenstates of $\hat{\Pi}^2$ to deal with this denominator. For operators $(\hat{\Pi}_0,\hat{\Pi}_3,\hat{\Pi}_\perp^2)$, we can use their eigenstates

$$|\hat{\Pi}_0,\hat{\Pi}_3;n,a\rangle=|\hat{\Pi}_0\rangle_0\otimes|\hat{\Pi}_3\rangle_3\otimes|\ n,a\rangle_{12} \tag{5}$$

as a set of complete basis tensors in four-dimensional Hilbert space, where

$$\begin{aligned}
&\hat{\Pi}_\perp^2=\hat{\Pi}_1^2+\hat{\Pi}_2^2,\\
&\hat{\Pi}_\perp^2|n,a\rangle=(2n+1)|q_f|eB|n,a\rangle, n\in\{0,1,2...\},\\
&\langle \Pi_0,\Pi_3|\Pi_0,\Pi_3\rangle=\frac{1}{(2\pi)^2}\int \mathrm{d}x_0\mathrm{d}x_3,\\
&\int \mathrm{d}a\,\langle n,a|n,a\rangle=\frac{|q_f|eB}{2\pi}\int \mathrm{d}x_1\mathrm{d}x_2.
\end{aligned} \tag{6}$$

From the discussion above, in Eqs. (4, 6), we can see that the imaginary term $O(\epsilon)$ is permanently positive. Since $\Pi_\perp^2$ is quantized to $(2n+1)|q_f|eB$, the imaginary term $O(\epsilon)$ is equivalent to $i\epsilon$, and the influence is trivial. As mentioned in the Introduction, in the NJL model with a magnetic field, the Eigenstate Method was developed to consider the influence of external interference on the infinitesimal imaginary term of the fermion propagator. However, in a conventional NJL model with finite temperature and finite chemical potential, the influence from the imaginary term is still trivial. This leads us to introduce a Lorentz-violating extension of the minimal Standard Model term into the NJL model within a constant external magnetic field.

Colladay and Kostelecký presented a general Lorentz-violating extension of the minimal $SU(3)\times SU(2)\times SU(1)$ Standard Model including CPT-even and CPT-odd terms [8]. This theory can be viewed as the low-energy limit of a physically relevant fundamental theory with Lorentz-covariant dynamics in which spontaneous Lorentz violation occurs. In this fermion sector of the Standard Model extension, the contribution to the Lagrangian can be divided into four parts according to whether the term is CPT even or odd and whether it involves leptons or quarks. The CPT-even and CPT-odd terms involving the quark fields are respectively

$$\begin{aligned}
\mathcal{L}_{\text{quark}}^{\text{CPT}-\text{odd}}&=-(a_q)_{\mu AB}\bar{\psi}_{q\,A}\gamma^\mu\psi_{qB}-(q\rightarrow u)-(q\rightarrow d)\\
\mathcal{L}_{\text{quark}}^{\text{CPT}-\text{even}}&=\frac{1}{2}\mathrm{i}(c_q)_{\mu\nu AB}\bar{\psi}_{q\,A}\gamma^\mu\overleftrightarrow{D}^\nu\psi_{qB}\\
&\quad+(q\rightarrow u)+(q\rightarrow d).
\end{aligned} \tag{7}$$

The Hermitian coefficients a_μ have the dimensions of mass. The dimensionless coupling coefficients $c_{\mu\nu}$ could in principle have both symmetric and antisymmetric space-time components but can be assumed traceless. A non-zero trace would not contribute to Lorentz violation and in any case can be absorbed by a conventional field normalization ensuring the usual kinetic operator for the matter fields.

Considering the above situation, we can apply the Lorentz-violating extension of the minimal Standard Model to the NJL Model. Under normal circumstances, we need to consider the influence of the effects of both CPT-odd and CPT-even terms on the infinitesimal imaginary term. However, as a general rule, the more complex the theoretical structure becomes, the less likely it is that a useful field redefinition exists. For the sake of simplicity, we do not discuss the CPT-odd term in this work. When only considering the CPT-even term, the Lorentz-violating extension of the NJL model with a constant background field is

$$\begin{aligned}
\mathcal{L}_{\mathrm{LV}}=&\mathrm{i}\bar{\psi}\gamma^\mu(\partial_\mu+\mathrm{i}qQ\otimes A)\psi-\bar{\psi}(\sigma+\mathrm{i}\gamma^5\otimes\vec{\pi}\cdot\vec{\tau})\psi\\
&+\mathrm{i}\eta^{\mu\beta}(L)\bar{\psi}\gamma_\mu D_\beta\psi-\frac{N}{2G}\Sigma^2.
\end{aligned} \tag{8}$$

The coupling coefficients $\eta^{\mu\beta}$ which are equivalent to $c_{\mu\nu}$ in Eq. (7) above, could in principle be Hermitian, dimensionless and can be assumed to be traceless. They contribute to both Lorentz violation and CPT-evenness. Because the existence of the pion condensate would violate parity, throughout this paper it is considered to be zero.

Based on Eq. (8), we can obtain the propagator

$$\hat{S}(k,\mu)=\frac{\gamma^\mu(\hat{\Pi}_\mu+{\eta_\mu}^\nu\hat{\Pi}_\nu)+\sigma}{[\gamma^\mu(\Pi_\mu+{\eta_\mu}^\nu\Pi_\nu)]^2-\sigma^2+\mathrm{i}\epsilon}, \tag{9}$$

and the following expression

$$\begin{aligned}
[\gamma^\mu(\Pi_\mu+{\eta_\mu}^\nu\Pi_\nu)]^2=&\frac{q_fe}{2}\sigma^{\mu\rho}[F_{\mu\rho}+({\eta_\rho}^\sigma F_{\mu\sigma}+{\eta_\mu}^\nu F_{\nu\rho})\\
&+{\eta_\mu}^\nu{\eta_\rho}^\sigma F_{\nu\sigma}]+\Pi^2+(\eta^{\mu\rho}+\eta^{\rho\mu})\Pi_\mu\Pi_\rho+{\eta_\mu}^\nu\eta^{\mu\rho}\Pi_\nu\Pi_\rho,
\end{aligned} \tag{10}$$

where $\sigma^{\mu\rho}=\mathrm{i}[\gamma^\mu,\gamma^\rho]/2$. By the Eigenstate Method we have the following relationships

$$\begin{aligned}
&\Pi_0=P_0,\quad [\Pi_1,\Pi_2]=-\mathrm{i}q_feB,\quad \Pi_3=P_3,\\
&\Pi_1=-p^1+\frac{1}{2}q_feBx^2, \Pi_2=-p^2-\frac{1}{2}q_feBx^1.
\end{aligned} \tag{11}$$

Using relationship (11) and $\eta^{\mu\nu}(L)$ anti-symmetry, Eq. (10) can be converted to

$$\begin{aligned}
[\gamma^\mu(\Pi_\mu+{\eta_\mu}^\nu\Pi_\nu)]^2=&\Pi^2-q_feB\sigma^{12}-[\sigma^{13}{\eta_3}^2(L)\\
&-\sigma^{23}{\eta_3}^1(L)]q_feB.
\end{aligned} \tag{12}$$

Using Eqs. (9, 12), we obtain the imaginary term from a strong background magnetic field and Lorentz-violating term as follows

$$O(\epsilon)=\Pi_\perp^2+\Pi_3^2+q_f eB[\sigma^{12}+\sigma^{13}{\eta_3}^2(L) -\sigma^{23}{\eta_3}^1(L)]+\sigma^2. \tag{13}$$

By using $\Pi_\perp^2=(2n+1)|q_f|eB$, the imaginary term $O(\epsilon)$ may be negative permanently, so when $\eta^{\mu\nu}(L)$ is of appropriate value, the influence is no longer trivial.

3 Gap equations

Now we discuss the gap equation within the Lorentz-violating extension. The general gap equation in the two flavor NJL model is

$$\frac{\sigma}{G}\int \mathrm{d}^4x=\mathrm{i}\sum_f \mathrm{Tr}\hat{S}. \tag{14}$$

The rigorous deduction of the gap equation is shown in Ref. [3]. Depending on Eq. (12), we simplify the propagator (9) to be

$$\hat{S}(k,\mu)=\frac{\gamma^\mu(\hat{\Pi}_\mu+{\eta_\mu}^\nu\hat{\Pi}_\nu)+\sigma}{\hat{\Pi}_0^2-\hat{\Pi}_3^2-\hat{\Pi}_\perp^2-\sigma^2+q_f eBb_1+\mathrm{i}O(\epsilon)}, \tag{15}$$

where $b_1=({\eta_3}^2\sigma^{31}+{\eta_3}^1\sigma^{23}-\sigma^{12})$.

Normally, we need to calculate $\mathrm{Tr}\hat{S}$. By a set of complete basis tensors in four-dimensional Hilbert space (5), $\mathrm{Tr}\hat{S}$ can be treated as

$$\begin{aligned}\mathrm{Tr}\hat{S}&=\int \mathrm{d}\Pi_0\mathrm{d}\Pi_3\int \mathrm{d}a\sum_{n=0}^{+\infty}\langle\Pi_0,\Pi_3;n,a|\mathrm{tr}\hat{S}|\Pi_0,\Pi_3;n,a\rangle\\ &=\sum_{n=0}^{+\infty}\int \mathrm{d}\Pi_0\mathrm{d}\Pi_3\,\langle\Pi_0,\Pi_3|\Pi_0,\Pi_3\rangle\int \mathrm{d}a\langle n,a|n,a\rangle\\ &\quad\times\mathrm{tr}\frac{\sigma}{\Pi_0^2-\Pi_3^2-\Pi_\perp^2-\sigma^2+q_f eBb_1+\mathrm{i}O(\epsilon)}.\end{aligned} \tag{16}$$

In order to further simplify the relations above, we can employ the relations (6) to cancel $\int \mathrm{d}x^4$ on the LHS of Eq. (14), then let Π_0 have a Wick rotation. Firstly, we insert the relations

$$\begin{aligned}\langle\Pi_0,\Pi_3|\Pi_0,\Pi_3\rangle&=\frac{1}{(2\pi)^2}\int \mathrm{d}x_0\mathrm{d}x_3,\\ \int \mathrm{d}a\,\langle n,a|n,a\rangle&=\frac{|q_f|eB}{2\pi}\int \mathrm{d}x_1\mathrm{d}x_2,\end{aligned} \tag{17}$$

into Eq. (16) and calculate the trace. Then Eq. (16) will be simplified to

$$\begin{aligned}\mathrm{Tr}\hat{S}&=\frac{|q_f|eB\sigma}{4\pi^3}\int \mathrm{d}\Pi_0\mathrm{d}\Pi_3\int \mathrm{d}x^4\Bigg(\sum_{n=0}^{+\infty}\frac{1}{a+\alpha|q_f|eB}\\ &\quad+\sum_{n=1}^{+\infty}\frac{1}{a-|q_f|eB\alpha}+A\Bigg)\\ &=\mathrm{Tr}\hat{S}_1+\hat{A},\end{aligned} \tag{18}$$

where

$$\begin{aligned}&\alpha=\sqrt{{\eta_3}^2+{\eta_3}^1+1},\quad a=\Pi_0^2-\Pi_3^2-(2n+1)|q_f|eB,\\ &\hat{A}=\frac{|q_f|eB\sigma}{4\pi^3}\int \mathrm{d}\Pi_0\mathrm{d}\Pi_3\int \mathrm{d}x^4\frac{1}{a-\alpha|q_f|eB+\mathrm{i}\epsilon}.\end{aligned} \tag{19}$$

When $\sigma>\sqrt{c}=\sqrt{(\alpha-1)|q_f|eB}$, there is no influence of the imaginary term, and the Wick rotation is regular.

Now we discuss the case of $\sigma<\sqrt{c}$. Let Π_0 have a Wick rotation. By making a cutoff $\frac{1}{\Lambda^2}$ to the lower limit of the integral of s, eventually we get $\mathrm{Tr}\hat{S}_1$ and $\hat{A}$ that are suitable for numerical calculation, as shown in Eq. (20). We employ the above results, replace Eq. (20) into Eq. (14), and simplify the gap equation. Then we have Eq. (21).

Up to now, by using this Eigenstate Method, we have deduced gap equations such as those in the NJL model with magnetic field and Lorentz-violating extension Standard Model. This method is very convenient for studying NJL problems with external magnetic field, especially when one evaluates the contribution from the infinitesimal imaginary term of the fermion propagator, because one does not need to first find an expression for $\langle x|\hat{S}|y\rangle$.

$$\begin{aligned}\mathrm{Tr}\hat{S}_1=&-\mathrm{i}\frac{\pi|q_f|eB\sigma}{4(\pi)^3}\int \mathrm{d}x^4\int_0^{+\infty}\mathrm{d}s\frac{\mathrm{e}^{-\sigma^2 s}}{s}\times\Bigg(\frac{1}{\mathrm{e}^{3|q_f|eBs}-\mathrm{e}^{|q_f|eBs}}\mathrm{e}^{seB\sqrt{({\eta_3}^2)^2+({\eta_3}^1)^2+1}}\\ &+\frac{1}{2}\mathrm{Csch}(|q_f|eBs)\mathrm{e}^{-seB\sqrt{({\eta_3}^2)^2+({\eta_3}^1)^2+1}}\Bigg).\end{aligned}$$

$$\hat{A}=\frac{|q_f|eB\sigma\sqrt{\pi}}{4\pi^3}\int \mathrm{d}x^4\int_{\frac{1}{\Lambda^2}}^{\infty}\left[\frac{\mathrm{e}^{-(c-\sigma^2)s}}{\sqrt{s}}\int_0^{\sqrt{c-\sigma^2}}\mathrm{e}^{\Pi_3^2 s}\mathrm{d}\Pi_3\mathrm{d}s-\mathrm{i}\int_{\frac{1}{\Lambda^2}}^{\infty}\int_{\sqrt{c-\sigma^2}}^{\infty}\mathrm{d}\frac{\mathrm{e}^{-(\Pi_3^2+\sigma^2-c)s}}{\sqrt{s}}\Pi_3\mathrm{d}s\right]. \quad (20)$$

$$\frac{4\pi^3}{G}=\sum_f |q_f|eB\pi\int_0^{+\infty}\mathrm{d}s\frac{\mathrm{e}^{-\sigma^2}s}{s}\left(\frac{1}{\mathrm{e}^{3|q_f|eBs}-\mathrm{e}^{|q_f|eBs}}\mathrm{e}^{seB\sqrt{({\eta_3}^2)^2+({\eta_3}^1)^2+1}}+\frac{1}{2}\mathrm{Csch}(|q_f|eBs)\mathrm{e}^{-seB\sqrt{({\eta_3}^2)^2+({\eta_3}^1)^2+1}}\right)$$

$$+\mathrm{i}\sum_f |q_f|eB\sqrt{\pi}\int_{\frac{1}{\Lambda^2}}^{\infty}\mathrm{d}s\left(\frac{\mathrm{e}^{(\sigma^2-c)s}}{\sqrt{s}}\int_0^{\sqrt{c-\sigma^2}}\mathrm{e}^{\Pi_3^2 s}\mathrm{d}\Pi_3-\mathrm{i}\frac{\mathrm{e}^{(c-\sigma^2)s}}{\sqrt{s}}\int_{\sqrt{c-\sigma^2}}^{\infty}\mathrm{d}\mathrm{e}^{-\Pi_3^2 s}\right). \quad (21)$$

4 Numerical results and analysis

In the analysis above, we get a gap equation (21) that corresponds to a Lorentz-violating Standard Model. Now we discuss the influence of the imaginary term from this model, as shown in Fig. 1. When $\sigma <= \sqrt{(\alpha-1)eB/3}$, as in region Ⅰ, there is no influence from the imaginary term, since it does not meet the minimum free energy. Based on the same reasons, the influence from the imaginary term mainly exists in region Ⅱ, except for the right-hand side of region Ⅱ. In region III, where $\sigma > \sqrt{(\alpha-1)2eB/3}$, there is only the conventional influence from the Lorentz-violating term α, and no influence from the imaginary term of the fermion propagator.

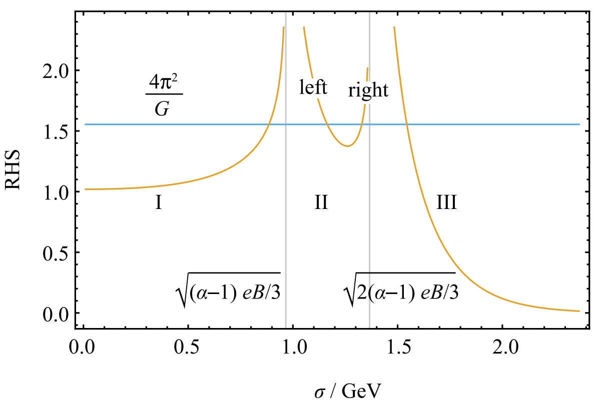

Fig. 1. (color online) Influence of the imaginary term from Lorentz-violating minimal Standard Model.

Based on the gap equation above and the data from Ref. [20] (for $f_\pi = 93$ MeV, $m_\pi = 138$ MeV, $G = 25.4$ GeV^{-2}, the current mass $m_0 = 5.5$MeV and the cutoffs $\Lambda = 0.99$ GeV), we are able to draw the relation of the Lorentz-violating terms and eB as shown in Fig. 2. It shows the dependence of external field eB with Lorentz-violating terms $\alpha-1$. When the Lorentz-violating term $\alpha-1$ is non-zero, it turns out there is a critical eB. As long as the system's magnetic field exceeds the critical eB, the imaginary term will be non-trivial. In fact, it is well known that the Lorentz-violating term should be a minimal value. This shows that the influence from the imaginary term is still very small under the current observed magnetic field in nature. In order to show this dependency better, here we let the Lorentz-violating term expand to 0.01 magnitude.

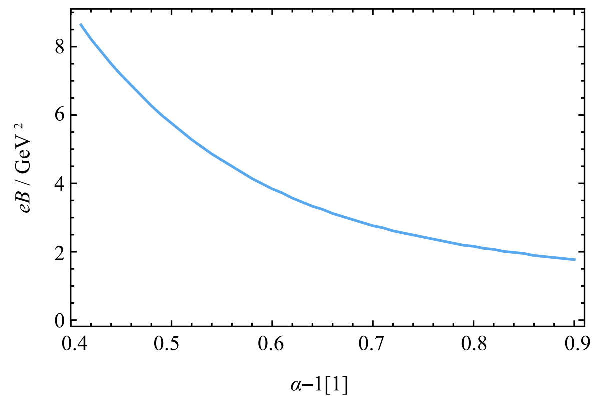

Fig. 2. (color online) The external field eB dependence of Lorentz-violating terms $\alpha-1$, when the influence of the imaginary term from the Lorentz-violating minimal Standard Model is not trivial.

When the influence from imaginary term is not trivial (and $\sqrt{(\alpha-1)eB/3} < \sigma < \sqrt{(\alpha-1)2eB/3}$), it is clear that when the strength of the magnetic field eB and Lorentz-violating terms $\alpha-1$ increases, the dynamical mass σ will increase with it, and the chiral symmetry is always broken. When eB and $\alpha-1$ are strong enough, the dynamical mass σ has a nearly linear response to the magnetic field and Lorentz-violating terms, as shown in Fig. 3(a) and 3(b).

When one does not consider the influence from the imaginary term ($\sigma > \sqrt{(\alpha-1)2eB/3}$), Fig. 4(a) and Fig. 4(b) show the relations between dynamical mass σ and Lorentz-violating terms $\alpha-1$ (or external field eB), and the chiral symmetry is always broken. In Fig. 4(b), when Lorentz-violating terms $\alpha-1\rightarrow 0$, the (σ, eB) curves restore to the conventional NJL model. When $eB\rightarrow 0$, there are lower $[\sigma,(\alpha-1)]$ curves, then the bigger the strength of eB, the higher the $[\sigma,(\alpha-1)]$ curves. With a fixed Lorentz-violating term, when the external field is strong enough, the dynamical mass has a nearly linear response to the magnetic field.

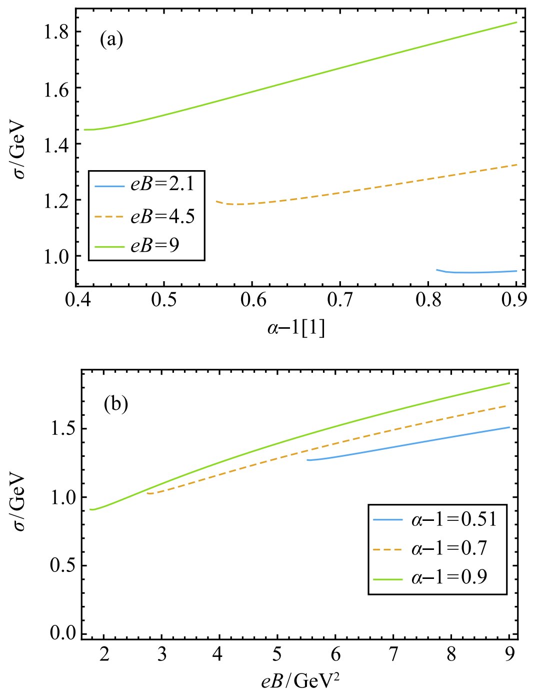

Fig. 3. (color online) σ dependence of (a) $\alpha-1$ with different eB and (b) of eB with different $\alpha-1$, when the influence from the imaginary term is not trivial.

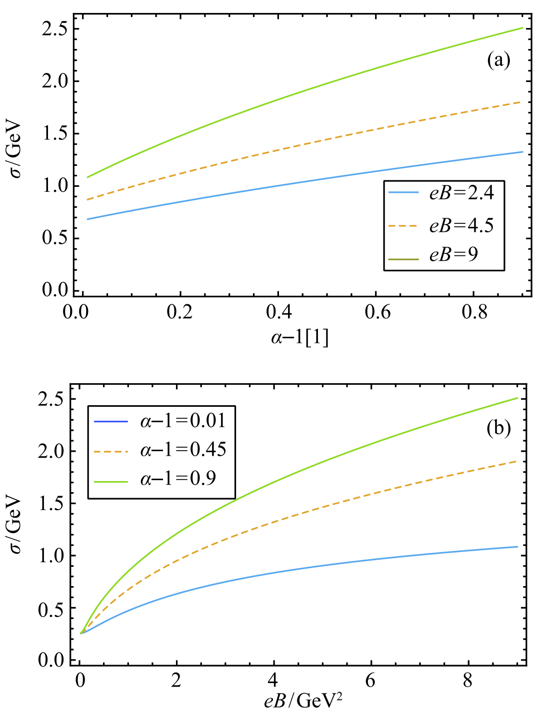

Fig. 4. (color online) σ dependence of (a) $\alpha-1$ with different fixed eB and (b) eB with different fixed $\alpha-1$, when the influence from the imaginary term is not considered.

5 Discussion

In this paper, we have introduced the Lorentz-violating extension of the minimal $SU(3)\times SU(2)\times SU(1)$ Standard Model into the NJL model within a constant strong magnetic field. Then, using the Eigenstate Method (which is different from the Schwinger proper time method) to deduce the fermion propagator, we discuss the influence from the Lorentz-violating term α and constant strong magnetic field eB on dynamical mass σ.

The Lagrangian of this model gives Lorentz-violation terms $\eta^{\mu\nu}$. To maintain the influence of the imaginary term from the Lorentz-violation extension minimal Standard Model, the external eB and Lorentz-violation term α must maintain a certain relationship as shown in Fig. 2. When the imaginary term of the fermion propagator in this model is not trivial ($\sqrt{(\alpha-1)eB/3} < \sigma < \sqrt{(\alpha-1)2eB/3}$), the dynamical mass σ will increase as eB and the Lorentz-violating term α increase, as shown in Fig. 3(a) and Fig. 3(b), and chiral symmetry is always broken. When we do not consider the influence from the imaginary term ($\sigma > \sqrt{(\alpha-1)2eB/3}$), the Lorentz-violation terms also influence the dynamical mass, as shown in Fig. 4(a) and Fig. 4(b). Under the two kinds of circumstances considered here (whether the influence of the imaginary term is trivial or not), the chiral symmetry is always broken.

The effect from finite temperature can also be discussed, and it is also possible to include this effect from Lorentz violation on the QCD condensate and dynamical chiral symmetry breaking. That is because the imaginary term $O(\epsilon)$ within both finite temperature and Lorentz violation may be permanently negative, so when $\eta^{\mu\nu}(L)$ is an appropriate value, the influence is no longer trivial. In addition, in this model we also can study various susceptibilities. We need to enhance the NJL model of Eq. (8) with the current mass m. Correspondingly, the new gap equation can be achieved simply by replacing σ with $\sigma+m$ on the RHS of the gap equation. Then, treating σ as an implicit function of m,T,α (the CPT term) and eB, namely $\sigma(m,T,eB,\alpha)$, we can make partial differentiations of m,T,α,eB and get the corresponding equations for the susceptibilities. We would like to discuss this situation in our next work.

The Eigenstate Method has been developed to deduce the fermion propagator with a constant external magnetic field. We find its result is equivalent to other methods, when there is only an external constant magnetic field rather than an external electric field or an external electromagnetic field. This new method is more convenient than other methods, especially when evaluating the contribution from the infinitesimal imaginary term of the fermion propagator.

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