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Implications of the R_{K} and R_{K^*} anomalies*

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Abstract: We discuss the implications of the recently reported R_{K} and R_{K^*} anomalies, the lepton flavor non-universality in the $\mathrm{B}\to\mathrm{K}l^+l^-$ and $\mathrm{B}\to\mathrm{K}^*l^+l^-$ decay channels. Using two sets of hadronic inputs of form factors, we perform a fit of new physics to the R_{K} and R_{K^*} data, and significant new physics contributions are found. We suggest the study of lepton flavor universality in a number of related rare B,$\mathrm{B_s}$,$\mathrm{B_c}$ and Λ_{b} decay channels, and in particular we give predictions for the μ-to-e ratios of decay widths with different polarizations of the final state particles, and of the $\mathrm{b}\to\mathrm{d}l^+l^-$ processes, which are presumably more sensitive to the structure of the underlying new physics. With the new physics contributions embedded in the Wilson coefficients, we present theoretical predictions for lepton flavor non-universality in these processes.

Keywords: R_{K} and R^*_{K} anomalies, lepton flavor universality violation, rare B decay

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1 Introduction

The standard model (SM) of particle physics has now been completed by the discovery of the Higgs boson. The focus of particle physics has, therefore, gradually switched to the search for new physics (NP) beyond the SM. This can proceed in two distinct ways. One is direct searches at the high energy frontier, in which new particles beyond the SM are produced and detected directly. The other is indirect searches, at the high intensity frontier. The new particles will presumably manifest themselves as intermediate loop effects, and might be detectable by low-energy experiments with high precision.

In flavor physics, the $\mathrm{b}\to\mathrm{s}l^+l^-$ process is a flavor changing neutral current (FCNC) transition. This process is of special interest since it is induced by loop effects in the SM, which leads to tiny branching fractions. Many extensions of the SM can generate sizable effects that can be experimentally validated. In particular, the $\mathrm{B}\to\mathrm{K}^*(\to\mathrm{K}\pi)\mu^+\mu^-$ decay offers a large number of observables to test the SM, ranging from the differential decay widths and polarizations to a full analysis of angular distributions of the final state particles. For an incomplete list one can refer to Refs. [1–21] and many references therein.

In the past few years, quite a few observables in the channels mediated by the $\mathrm{b}\to\mathrm{s}l^+l^-$ transition have exhibited deviations from the SM expectations. The LHCb experiment first observed the so-called P_5' anomaly, a sizeable discrepancy at 3.7 σ between the measurement and the SM prediction in one bin for the angular observable P_5' [22]. This discrepancy was reproduced in a later LHCb analysis for the two adjacent bins at large K^* recoil [23]. To accommodate this discrepancy, considerable attention has been paid to explore new physics contributions (see Refs. [24–31] and references therein), while at the same time, this has also triggered the thought of revisiting the hadronic uncertainties [32, 33].

More strikingly, the LHCb measurement of the ratio [34]:

$$R_{\mathrm{K}}[q^2_{\mathrm{min}},q^2_{\mathrm{max}}]\equiv\frac{\displaystyle\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}^+\to\mathrm{K}^+\mu^+\mu^-)/\mathrm{d}q^2}{\displaystyle\int_{q^2_{\mathrm{min}}}^{q^2_{\mathrm{max}}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}^+\to\mathrm{K}^+\mathrm{e}^+\mathrm{e}^-)/\mathrm{d}q^2}, \quad (1)$$

gives a hint of lepton flavour universality violation (LFUV). A plausible speculation is that deviations from the SM are present in $\mathrm{b}\to\mathrm{s}\mu^+\mu^-$ transitions instead of in $\mathrm{b}\to\mathrm{s}\mathrm{e}^+\mathrm{e}^-$ ones. Very recently the LHCb collabora-

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tion has found sizable differences between $\mathrm{B}\to\mathrm{K}^*\mathrm{e}^+\mathrm{e}^-$ and $\mathrm{B}\to\mathrm{K}^*\mu^+\mu^-$ at both low q^2 region and central q^2 region [35]. Results for the ratios

$$R_{\mathrm{K}^*}[q^2_{\min},q^2_{\max}]\equiv\frac{\int_{q^2_{\min}}^{q^2_{\max}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{K}^*\mu^+\mu^-)/\mathrm{d}q^2}{\int_{q^2_{\min}}^{q^2_{\max}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{K}^*\mathrm{e}^+\mathrm{e}^-)/\mathrm{d}q^2},\quad(2)$$

are given in Table 1, from which we can see that the data show significant deviations from unity. These interesting results have subsequently attracted much theoretical attention [36–59].

The statistical significance in the data is low at this stage, about 3σ level. In order to obtain more conclusive results, one should measure the muon-versus-electron ratios in the $\mathrm{B}\to\mathrm{K}\mathrm{l}^+\mathrm{l}^-$ and $\mathrm{B}\to\mathrm{K}^*\mathrm{l}^+\mathrm{l}^-$ more precisely. One should also investigate more channels with better sensitivities to the structures of new physics contributions. In this paper, we will focus on the latter. To do so, we will first discuss the implications of the R_{K} and R_{K^*} anomalies in a model-independent way, where the new particle contributions are parameterized in terms of effective operators. Since there is not enough data, we analyze their impact on the Wilson coefficients of SM operators $O_{9,10}$. We then propose to study lepton flavor universality in a number of rare $\mathrm{B},\mathrm{B}_{\mathrm{s}},\mathrm{B}_{\mathrm{c}}$ and Λ_{b} decay channels. Incorporating the new physics contributions, we will present the predictions for the muon-versus-electron ratios in these channels, making use of various updates of form factors [61–66]. We will demonstrate that the measurements of lepton flavor non-universality with different polarizations of the final state hadron, and in the $\mathrm{b}\to\mathrm{d}\mathrm{l}^+\mathrm{l}^-$ processes, are of great value to decode the structure of the underlying new physics.

The rest of this paper is organized as follows. In the next section, we will use a model-independent approach and quantify the new physics effects in terms of the short-distance Wilson coefficients. In Section 3, we will study the LFUV in various FCNC channels. Our conclusion is given in the last section.

Table 1. Ratios of decay widths with a pair of muons and electrons in $\mathrm{B}\to\mathrm{K}\mathrm{l}^+\mathrm{l}^-$ and $\mathrm{B}\to\mathrm{K}^*\mathrm{l}^+\mathrm{l}^-$.

observable	SM results	experimental data
R_{K}:q^2=[1,6]GeV^2	1.00±0.01 [60]	$0.745^{+0.090}_{-0.074}\pm0.036$ [34]
$R^{\mathrm{low}}_{\mathrm{K}^*}$:$q^2$=[0.045,1.1]GeV^2	$0.920^{+0.007}_{-0.006}$ [39]	$0.66^{+0.11}_{-0.07}\pm0.03$ [35]
$R^{\mathrm{central}}_{\mathrm{K}^*}$:$q^2$=[1.1,6]GeV^2	0.996±0.002 [39]	$0.69^{+0.11}_{-0.07}\pm0.05$ [35]

2 Implications from R_{K} and R_{K^*}

In this section, we will first study the impact of the R_{K} and R_{K^*} data. In the SM, the effective Hamiltonian for the transition $\mathrm{b}\to\mathrm{s}\mathrm{l}^+\mathrm{l}^-$

$$\mathcal{H}_{\mathrm{eff}}=-\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{\mathrm{tb}}V^*_{\mathrm{ts}}\sum_{i=1}^{10}C_i(\mu)O_i(\mu)$$

involves the four-quark and the magnetic penguin operators O_i. Here $C_i(\mu)$ are the Wilson coefficients for these local operators O_i. G_{F} is the Fermi constant, and V_{tb} and V_{ts} are CKM matrix elements. The dominant contributions to $\mathrm{b}\to\mathrm{s}\mathrm{l}^+\mathrm{l}^-$ come from the following operators:

$$\begin{aligned}O_7&=\frac{em_{\mathrm{b}}}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}+\frac{em_s}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1-\gamma_5)bF_{\mu\nu},\\ O_9&=\frac{\alpha_{\mathrm{em}}}{2\pi}(\bar{l}\gamma_\mu l)\bar{s}\gamma^\mu(1-\gamma_5)b,\\ O_{10}&=\frac{\alpha_{\mathrm{em}}}{2\pi}(\bar{l}\gamma_\mu\gamma_5 l)\bar{s}\gamma^\mu(1-\gamma_5)b.\end{aligned}\quad(3)$$

The above effective Hamiltonian gives the $\mathrm{B}\to\mathrm{K}\mathrm{l}^+\mathrm{l}^-$ decay width as:

$$\begin{aligned}\frac{\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{K}\mathrm{l}^+\mathrm{l}^-)}{\mathrm{d}q^2}&=\frac{G_{\mathrm{F}}^2\sqrt{\lambda}\alpha_{\mathrm{em}}^2\beta_{\mathrm{l}}}{1536m_{\mathrm{B}}^3\pi^5}|V_{\mathrm{tb}}V^*_{\mathrm{ts}}|^2\times\left[\lambda(1+2\hat{m}_{\mathrm{l}}^2)\left|C_9f_+(q^2)+C_7\frac{2m_{\mathrm{b}}f_{\mathrm{T}}(q^2)}{m_{\mathrm{B}}+m_{\mathrm{K}}}\right|^2\right.\\&\left.+\lambda\beta_{\mathrm{l}}^2|C_{10}|^2f_+^2(q^2)+6\hat{m}_{\mathrm{l}}^2|C_{10}|^2(m_{\mathrm{B}}^2-m_{\mathrm{K}}^2)^2f_0^2(q^2)\right],\end{aligned}\quad(4)$$

where $\hat{m}_{\mathrm{l}}=m_{\mathrm{l}}/\sqrt{q^2}$, $\beta_{\mathrm{l}}=\sqrt{1-\hat{m}_{\mathrm{l}}^2}$, $\lambda=(m_{\mathrm{B}}^2-m_{\mathrm{K}}^2-q^2)^2-4m_{\mathrm{K}}^2q^2$, and f_+, f_0 and f_{T} are the $\mathrm{B}\to\mathrm{K}$ form factors. In the above expression, we have neglected the non-factorizable contributions which are expected to be negligible for R_{K}.

The decay width for $\mathrm{B}\to\mathrm{K}^*\mathrm{l}^+\mathrm{l}^-$ can be derived in terms of the helicity amplitude [67–71]. The differential decay width is given as

$$\frac{\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{K}^*\mathrm{l}^+\mathrm{l}^-)}{\mathrm{d}q^2}=\frac{3}{4}\left(I_1^{\mathrm{c}}+2I_1^{\mathrm{s}}\right)-\frac{1}{4}\left(I_2^{\mathrm{c}}+2I_2^{\mathrm{s}}\right),\quad(5)$$

with

$$I_1^{\mathrm{c}}=(|A^1_{\mathrm{L0}}|^2+|A^1_{\mathrm{R0}}|^2)+8\hat{m}_{\mathrm{l}}^2\mathrm{Re}[A^1_{\mathrm{L0}}A^{1*}_{\mathrm{R0}}]+4\hat{m}_{\mathrm{l}}^2|A^1_{\mathrm{t}}|^2,$$

$$\begin{aligned}
I_1^{\rm s} &= (3/4-\hat{m}_{\rm l}^2)[|A^1_{\rm L\perp}|^2+|A^1_{\rm L||}|^2+|A^1_{\rm R\perp}|^2+|A^1_{\rm R||}|^2] \\
&\quad +4\hat{m}_{\rm l}^2{\rm Re}[A^1_{\rm L\perp}A^{1*}_{\rm R\perp}+A^1_{\rm L||}A^{1*}_{\rm R||}], \\
I_2^{\rm c} &= -\beta_{\rm l}^2(|A^1_{\rm L0}|^2+|A^1_{\rm R0}|^2), \\
I_2^{\rm s} &= \frac{1}{4}\beta_{\rm l}^2(|A^1_{\rm L\perp}|^2+|A^1_{\rm L||}|^2+|A^1_{\rm R\perp}|^2+|A^1_{\rm R||}|^2). \qquad (6)
\end{aligned}$$

The handedness label L or R corresponds to the chirality of the di-lepton system. Functions $A_{\rm L/Ri}$ can be expressed in terms of $\rm B\to K^*$ form factors

$$A^1_{\rm t} = 2\sqrt{N_{{\rm K}^*_J}}N_1C_{10}\frac{\sqrt{\lambda}}{\sqrt{q^2}}A_0(q^2), \qquad (7)$$

$$\begin{aligned}
A^1_{\rm L0} &= \frac{N_1\sqrt{N_{{\rm K}^*_J}}}{2m_{K^*_J}\sqrt{q^2}}\left[(C_9-C_{10})\left[(m_{\rm B}^2-m_{\rm K^*}^2-q^2)(m_{\rm B}+m_{\rm K^*})A_1-\frac{\lambda}{m_{\rm B}+m_{\rm K^*}}A_2\right]\right. \\
&\quad \left.+2m_{\rm b}C_7\left[(m_{\rm B}^2+3m_{\rm K^*}^2-q^2)T_2-\frac{\lambda}{m_{\rm B}^2-m_{\rm K^*}^2}T_3\right]\right], \qquad (8)
\end{aligned}$$

$$A^1_{\rm L\perp} = -\sqrt{2N_{{\rm K}^*_J}}N_1\left[(C_9-C_{10})\frac{\sqrt{\lambda}V}{m_{\rm B}+m_{\rm K^*}}+\frac{2m_{\rm b}C_7}{q^2}\sqrt{\lambda}T_1\right], \qquad (9)$$

$$A^1_{\rm L||} = \sqrt{2N_{{\rm K}^*_J}}N_1\left[(C_9-C_{10})(m_{\rm B}+m_{\rm K^*})A_1+\frac{2m_{\rm b}C_7}{q^2}(m_{\rm B}^2-m_{\rm K^*}^2)T_2\right], \qquad (10)$$

with $N_1=\frac{iG_{\rm F}}{4\sqrt{2}}\frac{\alpha_{\rm em}}{\pi}V_{\rm tb}V^*_{\rm ts}$, $N_{{\rm K}^*_J}=8/3\sqrt{\lambda}q^2\beta_{\rm l}/(256\pi^3m_{\rm B}^3)$ and $\lambda\equiv(m_{\rm B}^2-m_{\rm K^*}^2-q^2)^2-4m_{\rm K^*}^2q^2$. The right-handed decay amplitudes are obtained by reversing the sign of C_{10}:

$$A_{\rm Ri} = A_{\rm Li}|_{C_{10}\to -C_{10}}. \qquad (11)$$

Within the SM, one can easily find that results for $R_{\rm K}$ and $R_{\rm K^*}$ are extremely close to 1 and thus deviate from the experimental data. If new physics is indeed present, it can be in $\rm b\to s\mu^+\mu^-$ and/or $\rm b\to se^+e^-$transitions. In order to explain the $R_{\rm K}$ and $R_{\rm K^*}$ data, one can enhance the partial width for the electronic mode or reduce the one for the muonic mode. It seems that the SM result for the $\rm B\to Ke^+e^-$ is consistent with the data, and thus here we will adopt the strategy that the muonic decay width is reduced by new physics.

After integrating out the high scale intermediate states the new physics contributions can be incorporated into the effective operators. As there is not enough data that shows significant deviations from the SM, we will assume that NP contributions can be incorporated into Wilson coefficients C_9 and C_{10}. For this purpose, we define

$$\delta C_9^{\mu}=C_9^{\mu}-C_9^{\rm SM}, \delta C_{10}^{\mu}=C_{10}^{\mu}-C_{10}^{\rm SM}. \qquad (12)$$

The O_7 contribution to $\rm b\to sl^+l^-$ arises from the coupling of a photon with the lepton pair. This coupling is highly constrained by the $\rm b\to s\gamma$ data. Furthermore, this coefficient is flavor blinded and thus even if NP affects C_7, the μ-to-e will not be affected.

For the analysis, we adopt three scenarios,

1. Only C_9 is affected, with $\delta C_9^{\mu}\neq 0$.
2. Only C_{10} is affected, with $\delta C_{10}^{\mu}\neq 0$.
3. Both C_9 and C_{10} are affected, in the form: $\delta C_9^{\mu}=-\delta C_{10}^{\mu}\neq 0$.

Using the $R_{\rm K}$ and $R_{\rm K^*}$ data, we show our results in Fig. 1. Figure 1(a) corresponds to scenario 1, (b) corresponds to scenario 2, and (c) corresponds to scenario 3 with a nonzero $\delta C_9^{\mu}-\delta C_{10}^{\mu}$. In this analysis, we have used two sets of $\rm B\to K$ and $\rm B\to K^*$ form factors. One is from the light-cone sum rules (LCSR) [72–74], corresponding to the dashed curves. The other is from lattice QCD (LQCD) [65, 75], which gives the solid curves. As one can see clearly from the figure, the results are not sensitive to the form factors, and this also partly validates the neglect of other hadronic uncertainties like non-factorizable contributions. Using the LQCD set of form factors [65, 75] and the data in Table 1, we find the best-fitted central value and the 1σ range for δC_9^{μ} in scenario 1 as

$$\delta C_9^{\mu}=-1.83, -2.63<\delta C_9^{\mu}<-1.25. \qquad (13)$$

For scenario 2, we have

$$\delta C_{10}^{\mu}=1.43, 1.04<\delta C_{10}^{\mu}<1.89, \qquad (14)$$

while for $\delta C_9^{\mu}=-\delta C_{10}^{\mu}$, we obtain

$$\delta C_9^{\mu}-\delta C_{10}^{\mu}=-1.47, -1.89<\delta C_9^{\mu}-\delta C_{10}^{\mu}<-1.08. \qquad (15)$$

A few remarks are in order.

1) Since the Wilson coefficient in the electron channel is unchanged, δC_9^{μ} and δC_{10}^{μ} could be viewed as the difference between the Wilson coefficients for the lepton and muon case.

2) We have found the largest deviation between the fitted results and the data comes from the low-q^2 region. Removing this data, we show the χ^2 in Fig. 1 as dotted and dot-dashed curves, where the χ^2 has been greatly reduced. The reason is that in low-q^2 region, the dominant contribution to $R_{\rm K^*}$ arises from the transverse

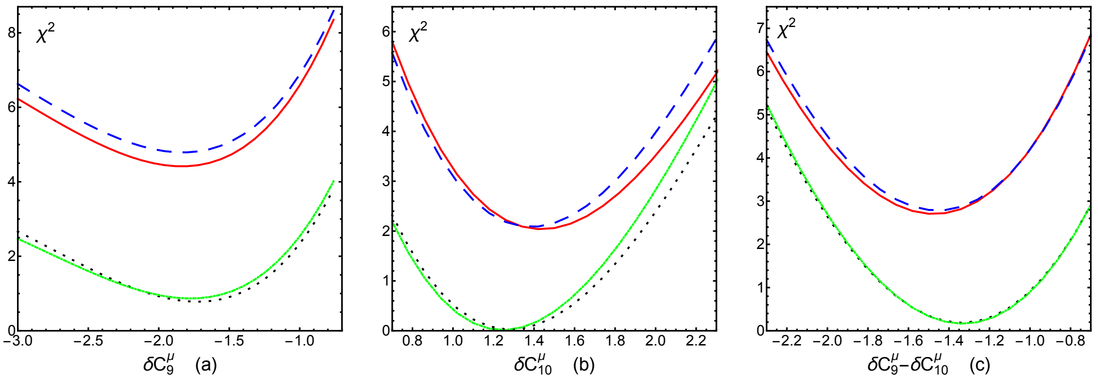

Fig. 1. (color online) Impact of R_{K} and R_{K^*} data on the δC_9^μ (a), δC_{10}^μ (b), and $\delta C_9^\mu - \delta C_{10}^\mu$ (c). The dependence of the total χ^2 for all data in Table 1 on the Wilson coefficients is shown as the solid (red) and dashed (blue) curves, which correspond to the form factors from LQCD [65, 75] and LCSR [72, 73], respectively. Removing the low-q^2 data for $\mathrm{B} \to \mathrm{K}^* \mathrm{l}^+ \mathrm{l}^-$, the results are shown as the dotted (black) and and dot-dashed (green) curves.

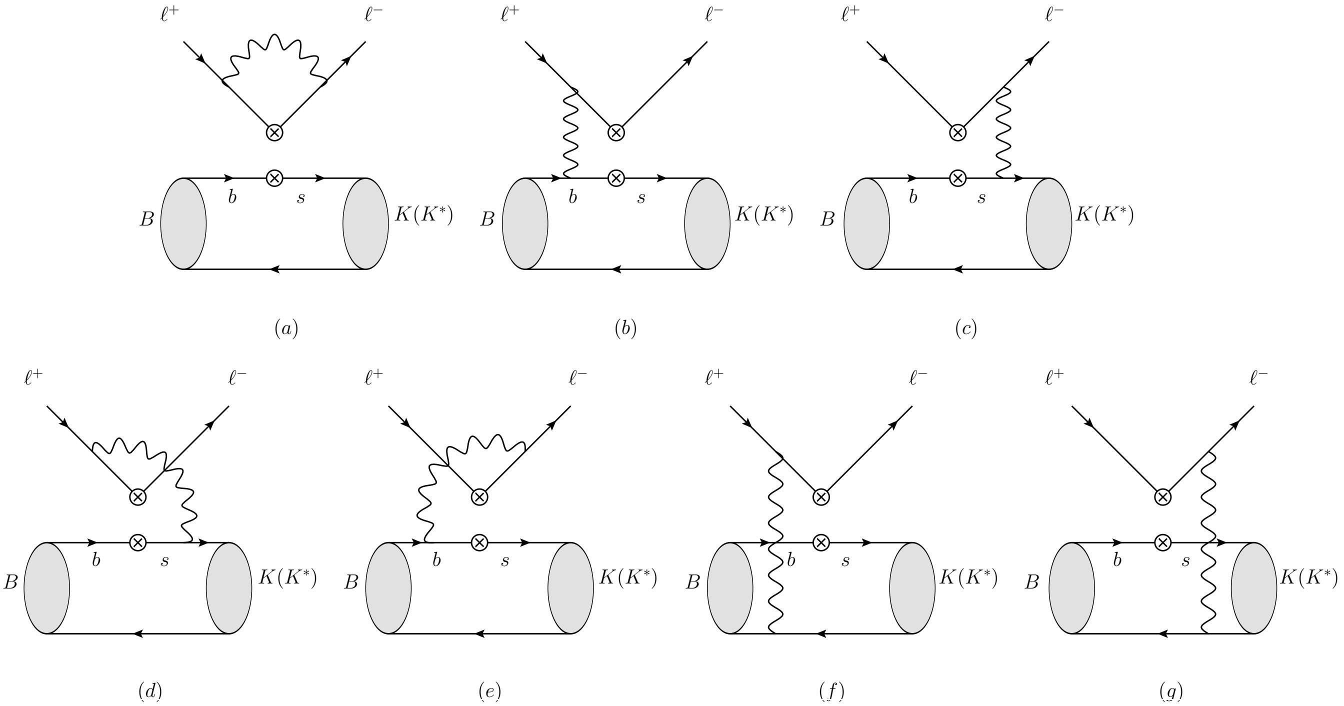

Fig. 2. The electromagnetic corrections to $\mathrm{B} \to \mathrm{K} \mathrm{l}^- \mathrm{l}^+$ and $\mathrm{B} \to \mathrm{K}^* \mathrm{l}^- \mathrm{l}^+$.

polarization of K^*. From Eqs. (9) and (10), this contribution is dominated by O_7 and is less sensitive to $O_{9,10}$. A light mediator that only couples to the $\mu^+\mu^-$ is explored, for instance, in Refs. [47, 52, 54].

3) For the R_{K} and R_{K^*} predictions in Refs. [39, 60], the theoretical errors are typically less than one percent, while Ref. [76] gives the prediction with even smaller uncertainty $R_{\mathrm{K}} = 1.0003 \pm 0.0001$. However, it is necessary to stress that these results did not consider the electromagnetic corrections properly. We give the Feynman diagrams in Fig. 2. Figure 2(a) is the typical Sudakov form factor, which usually introduces a double logarithm in terms of $\alpha/\pi \ln(q^2/m_{\mathrm{l}}^2)$. The difference between the double logarithms for the electron and muon modes is about 3%. A complete analysis requires the detailed calculation of all the diagrams in Fig. 2, and analyses can be found in Ref. [77]. The nonfactorizable corrections to the amplitude can be found in Ref. [78].

4) There are a number of observables in $\mathrm{B} \to \mathrm{K} \mu^+ \mu^-$ and $\mathrm{B} \to \mathrm{K}^* \mu^+ \mu^-$ that have been experimentally measured. These observables are of great values to provide very stringent constraints on the Wilson coefficients in the factorization approach. However, most of these observables in $\mathrm{B} \to \mathrm{K} \mu^+ \mu^-$ and $\mathrm{B} \to \mathrm{K}^* \mu^+ \mu^-$ are not sen-

sitive to the flavor non-universality coupling, since only the mu lepton is involved. The exploration of the μ-to-e ratios will be able to detect the difference in the new physics couplings to fermions. It is always meaningful to conduct a comprehensive global analysis and incorporate as many observables as possible. At this stage, the study of flavor non-universality in flavor physics is just beginning, and we believe measuring more μ to e ratios (for instance the ones in Table 2, shown in the following section) will be helpful.

5) For a more comprehensive analysis, one may combine various experimental data on the flavor changing neutral current processes, for instance as in Refs. [36–40]. We quote the results in scenario I in Ref. [36],

$$\delta C_9^{\mu} = -1.58\pm0.28,\ \delta C_9^{e} = -0.10\pm0.45, \tag{16}$$

from which we can see that the results are close to our scenario 1. This implies that for the determination of flavor dependent Wilson coefficient, $R_{\rm K}$ and $R_{\rm K^*}$ are dominant. From a practical viewpoint, since the main purpose of this paper is to explore the implications of large lepton flavor non-universality, we will use our fitted results to predict lepton flavor non-universality for a number of other channels.

Explicit models which can realize these scenarios include the flavor non-universal Z' model, leptoquark model, and vector-like models, see, e.g., Refs. [79–108] and many references therein. Their generic contributions are shown in Fig. 3. Taking the Z' model as an example, the SM can be extended by including an additional U(1)′ symmetry, which leads to the Lagrangian of $Z'\bar{b}s$ couplings

$$\mathcal{L}^{Z'}_{\rm FCNC} = -g'(B^{\rm L}_{\rm sb}\bar{s}_{\rm L}\gamma_\mu b_{\rm L} + B^{\rm R}_{\rm sb}\bar{s}_{\rm R}\gamma_\mu b_{\rm R})Z'^{\mu} + \text{h.c.}. \tag{17}$$

It contributes to the $\rm b\to sl^+l^-$ decay at tree level

$$\begin{aligned}\mathcal{H}^{Z'}_{\rm eff} &= \frac{8G_{\rm F}}{\sqrt{2}}(\rho^{\rm L}_{\rm sb}\bar{s}_{\rm L}\gamma_\mu b_{\rm L} + \rho^{\rm R}_{\rm sb}\bar{s}_{\rm R}\gamma_\mu b_{\rm R}) \\ &\quad \times(\rho^{\rm L}_{ll}\bar{l}_{\rm L}\gamma^\mu l_{\rm L} + \rho^{\rm R}_{ll}\bar{l}_{\rm R}\gamma^\mu l_{\rm R}),\end{aligned} \tag{18}$$

where the coupling is

$$\rho^{\rm L,R}_{\rm ff'} \equiv \frac{g' M_Z}{g M_{Z'}} B^{\rm L,R}_{\rm ff'}, \tag{19}$$

where g is the standard model $SU(2)_{\rm L}$ coupling. For simplicity, one can assume that the FCNC couplings of the Z' and quarks only occur in the left-handed sector: $\rho^{\rm R}_{\rm sb}=0$. Thus in this case the effects of the Z' will modify the Wilson coefficients C_9 and C_{10}:

$$\begin{aligned}C_9^{Z'} &= C_9 - \frac{4\pi}{\alpha_{\rm em}}\frac{\rho^{\rm L}_{\rm sb}(\rho^{\rm L}_{ll}+\rho^{\rm R}_{ll})}{V_{\rm tb}V^*_{\rm ts}}, \\ C_{10}^{Z'} &= C_{10} + \frac{4\pi}{\alpha_{\rm em}}\frac{\rho^{\rm L}_{\rm sb}(\rho^{\rm L}_{ll}-\rho^{\rm R}_{ll})}{V_{\rm tb}V^*_{\rm ts}}.\end{aligned} \tag{20}$$

From this expression, we can see that the δC_9^{μ} and δC_{10}^{μ} are not entirely correlated. This corresponds to scenarios 1 and 2 in our previous analysis.

The impact in a leptoquark model has been discussed, for instance, in Ref. [43], where the NP contribution satisfies

$$\delta C_9^{{\rm L}Q,\mu} = -\delta C_{10}^{{\rm L}Q,\mu}. \tag{21}$$

This corresponds to our scenario 3.

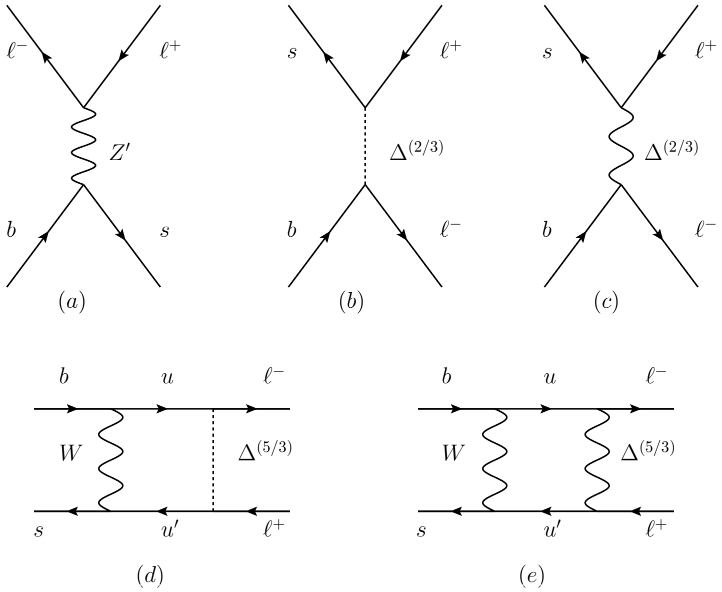

Fig. 3. New physics scenarios that can contribute to $\rm b\to s\mu^+\mu^-$. (a) shows a Z', and in the other four diagrams Δ denotes a leptoquark with different spins and charges.

3 Lepton flavor universality in FCNC channels

In this section, we will study the μ-to-e ratios of decay widths in various FCNC channels. Since the three scenarios considered in the last section describe the data equally well, we will choose the first one for illustration in the following. We follow a similar definition

$$R_{\mathrm{B,M}}[q^2_{\min},q^2_{\max}]\equiv\frac{\displaystyle\int_{q^2_{\min}}^{q^2_{\max}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{M}\mu^+\mu^-)/\mathrm{d}q^2}{\displaystyle\int_{q^2_{\min}}^{q^2_{\max}}\mathrm{d}q^2\mathrm{d}\Gamma(\mathrm{B}\to\mathrm{M}\mathrm{e}^+\mathrm{e}^-)/\mathrm{d}q^2},\quad(22)$$

where B denotes a heavy particle and M denotes a final state. The channels to be studied include $\mathrm{B}\to\mathrm{K}^*_{0,2}(1430)\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B_s}\to\mathrm{f_0}(980)\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B}\to\mathrm{K_1}(1270)\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B_s}\to\mathrm{f_2}(1525)\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B_s}\to\phi\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B_c}\to\mathrm{D_s}\mathrm{l}^+\mathrm{l}^-$, $\mathrm{B_c}\to\mathrm{D_s^*}\mathrm{l}^+\mathrm{l}^-$. The expressions for their decay widths have been given in the last section. In addition, we will also analyze the R ratio for the baryonic decay $\Lambda_\mathrm{b}\to\Lambda\mathrm{l}^+\mathrm{l}^-$. The differential decay width for $\Lambda_\mathrm{b}\to\Lambda\mathrm{l}^+\mathrm{l}^-$ is given as [109]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}[\Lambda_\mathrm{b}\to\Lambda\mathrm{l}^+\mathrm{l}^-]=2K_{1\mathrm{ss}}+K_{1\mathrm{cc}},\quad(23)$$

where

$$\begin{aligned}K_{1\mathrm{ss}}(q^2)&=\frac{1}{4}\Big[|A^\mathrm{R}_{\perp_1}|^2+|A^\mathrm{R}_{\|_1}|^2+2|A^\mathrm{R}_{\perp_0}|^2+2|A^\mathrm{R}_{\|_0}|^2+(\mathrm{R}\leftrightarrow\mathrm{L})\Big],\\ K_{1\mathrm{cc}}(q^2)&=\frac{1}{2}\Big[|A^\mathrm{R}_{\perp_1}|^2+|A^\mathrm{R}_{\|_1}|^2+(\mathrm{R}\leftrightarrow\mathrm{L})\Big].\end{aligned}\quad(24)$$

The functions A are defined as

$$\begin{aligned}A^{\mathrm{L(R)}}_{\perp_1}&=\sqrt{2}N\left[(C_9\mp C_{10})H^V_+-\frac{2m_\mathrm{b}C_7}{q^2}H^\mathrm{T}_+\right],\\ A^{\mathrm{L(R)}}_{\|_1}&=-\sqrt{2}N\left[(C_9\mp C_{10})H^A_++\frac{2m_\mathrm{b}C_7}{q^2}H^\mathrm{T5}_+\right],\\ A^{\mathrm{L(R)}}_{\perp_0}&=\sqrt{2}N\left[(C_9\mp C_{10})H^V_0-\frac{2m_\mathrm{b}C_7}{q^2}H^\mathrm{T}_0\right],\\ A^{\mathrm{L(R)}}_{\|_0}&=-\sqrt{2}N\left[(C_9\mp C_{10})H^A_0+\frac{2m_\mathrm{b}C_7}{q^2}H^\mathrm{T5}_0\right],\end{aligned}\quad(25)$$

where the normalization factor N is

$$N=G_\mathrm{F}V_\mathrm{tb}V^*_\mathrm{ts}\alpha_\mathrm{em}\sqrt{\frac{q^2\sqrt{\lambda(m^2_{\Lambda_\mathrm{b}},m^2_\Lambda,q^2)}}{3\cdot2^{11}m^3_{\Lambda_\mathrm{b}}\pi^5}}.\quad(26)$$

The helicity amplitudes are given by

$$\begin{aligned}H^\mathrm{V}_0&=f^\mathrm{V}_0(q^2)\frac{m_{\Lambda_\mathrm{b}}+m_\Lambda}{\sqrt{q^2}}\sqrt{s_-},\ H^\mathrm{V}_+=-f^\mathrm{V}_\perp(q^2)\sqrt{2s_-},\\ H^\mathrm{A}_0&=f^\mathrm{A}_0(q^2)\frac{m_{\Lambda_\mathrm{b}}-m_\Lambda}{\sqrt{q^2}}\sqrt{s_+},\ H^\mathrm{A}_+=-f^\mathrm{A}_\perp(q^2)\sqrt{2s_+}\\ H^\mathrm{T}_0&=-f^\mathrm{T}_0(q^2)\sqrt{q^2}\sqrt{s_-},\\ H^\mathrm{T}_+&=f^\mathrm{T}_\perp(q^2)(m_{\Lambda_\mathrm{b}}+m_\Lambda)\sqrt{2s_-},\\ H^\mathrm{T5}_0&=f^\mathrm{T5}_0(q^2)\sqrt{q^2}\sqrt{s_+},\\ H^\mathrm{T5}_+&=-f^\mathrm{T5}_\perp(q^2)(m_{\Lambda_\mathrm{b}}-m_\Lambda)\sqrt{2s_+},\end{aligned}\quad(27)$$

where $s_\pm\equiv(m_{\Lambda_\mathrm{b}}\pm m_\Lambda)^2-q^2$. The $f^\mathrm{i}_{0/\perp}$ with i=V,A,T,T5 are the $\Lambda_\mathrm{b}\to\Lambda$ form factors.

The $\mathrm{B_s}\to\phi\mathrm{l}^+\mathrm{l}^-$ and $\Lambda_\mathrm{b}\to\Lambda$ form factors used are from LQCD calculations in Refs. [65, 110], respectively. The $\mathrm{B}\to\mathrm{K}^*_0(1430)$ and $\mathrm{B_s}\to\mathrm{f_0}(980)$ form factors are taken from Refs. [61, 111]. The $\mathrm{B}\to\mathrm{K_1}(1270)$ form factors are calculated in the perturbative QCD approach [63], and the mixing angle between $\mathrm{K_1}(1^{++})$ and $\mathrm{K_1}(1^{+-})$ is set to be approximately 45°. In this case the $\mathrm{B}\to\mathrm{K_1}(1400)\mathrm{l}^+\mathrm{l}^-$ is greatly suppresed [112]. The $\mathrm{B}\to\mathrm{K_2}$ and $\mathrm{B_s}\to\mathrm{f_2}(1525)$ form factors are taken from Ref. [64]. The $\mathrm{B_c}\to\mathrm{D_s}/\mathrm{D^*_s}$ form factors are provided in the light-front quark model [62], and in this work we have calculated the previously-missing tensor form factors. Using the Wilson coefficient δC^μ_9 in Eq. (13), we present our numerical results for $R_\mathrm{B,M}$ in Table 2. Three kinematics regions are chosen in the analysis: low q^2 with [0.045, 1] GeV^2, central q^2 with [1, 6] GeV^2, and the high q^2 region with [14 GeV^2, $q^2_{\max}=(m_\mathrm{B}-m_\mathrm{M})^2$]. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as L and T, respectively. For $\Lambda_\mathrm{b}\to\Lambda\mathrm{l}^+\mathrm{l}^-$, a similar decomposition is used, in which the superscript 0 means the Λ_b and Λ have the same polarization while 1 corresponds to different polarizations. The SM predictions for these ratios are listed in Table 3.

A few remarks are in order.

1) From the decay widths for $\mathrm{B}\to\mathrm{K}^*\mathrm{l}^+\mathrm{l}^-$, we can see that in the transverse polarization, the contribution from O_7 is enhanced at low q^2, and thus the $R^\mathrm{T}_\mathrm{B,M}$ is less sensitive to the NP in $O_{9,10}$. Measurements of the μ-to-e ratio in the transverse polarization of $\mathrm{B}\to\mathrm{V}\mathrm{l}^+\mathrm{l}^-$ at low q^2 can show whether the NP is from the q^2 independent contribution in $C_{9,10}$ or the q^2 dependent contribution in C_7.

2) In the central q^2 region, the operators O_7 and $O_{9,10}$ will contribute destructively to the transverse polarization of $\mathrm{B}\to\mathrm{V}\mathrm{l}^+\mathrm{l}^-$. Reducing C_9 with $\delta C^\mu_9<0$ will affect the cancellation, and as a result the decay width for the muonic decay mode will be enhanced. Thus instead of having a ratio smaller than 1, one will obtain a surplus for this ratio.

3) Results for $\Lambda_\mathrm{b}\to\Lambda$ with different polarizations are similar, but the differential decay widths in Eq. (23) have neglected the kinematic lepton mass corrections. Thus, the results in the low q^2 region are not accurate.

4) For $\mathrm{B}\to\mathrm{K}_{0,2}(1430)\mathrm{l}^+\mathrm{l}^-$ and $\mathrm{B_c}\to\mathrm{D^*_s}$, the high q^2 region has limited kinematics, and thus the results are difficult to measure.

5) Among the decay processes involved in Table 2, a few of them have been experimentally investigated: the branching fractions of $\mathrm{B_s}\to\phi\mathrm{l}^+\mathrm{l}^-$ [113, 114], $\Lambda_\mathrm{b}\to\Lambda\mathrm{l}^+\mathrm{l}^-$ [115] and $\mathrm{B_s}\to\mathrm{f_0}(980)\mathrm{l}^+\mathrm{l}^-$ [116] have been

Table 2. Theoretical results for the μ-to-e ratio $R_{\mathrm{B,M}}$ of decay widths as defined in Eq. (22) in various $\mathrm{b}\to\mathrm{s}\mathrm{l}^+\mathrm{l}^-$ channels. Three kinematics regions are chosen: low, central and high q^2 regions. Wilson coefficient C_9 is used as in Eq. (13), based on the analysis of R_{K} and R_{K^*}. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as L and T, respectively. For $\Lambda_{\mathrm{b}}\to\Lambda\mathrm{l}^+\mathrm{l}^-$, a similar decomposition is used: the superscript 0 means that the Λ_{b} and Λ have the same polarization, while 1 corresponds to different polarizations.

observable	low q^2:[0.045,1]/GeV^2	central q^2:[1,6]/GeV^2	high q^2:[14GeV^2,$q^2_{\max}$]
$R_{\mathrm{B,K}_0^*(1430)}$	$0.688^{+0.075}_{-0.073}$	$0.702^{+0.076}_{-0.075}$	$0.721^{+0.074}_{-0.074}$
$R_{\mathrm{B_s,f_0(980)}}$	$0.687^{+0.074}_{-0.074}$	$0.700^{+0.076}_{-0.076}$	$0.707^{+0.075}_{-0.074}$
$R_{\mathrm{B_c,D_s}}$	$0.686^{+0.075}_{-0.075}$	$0.699^{+0.077}_{-0.077}$	$0.706^{+0.076}_{-0.076}$
$R_{\mathrm{B_s},\phi}$	$0.863^{+0.016}_{-0.010}$	$0.772^{+0.051}_{-0.040}$	$0.710^{+0.071}_{-0.067}$
$R^{\mathrm{L}}_{\mathrm{B_s},\phi}$	$0.697^{+0.074}_{-0.074}$	$0.701^{+0.076}_{-0.076}$	$0.706^{+0.073}_{-0.071}$
$R^{\mathrm{T}}_{\mathrm{B_s},\phi}$	$0.975^{-0.024}_{+0.034}$	$1.059^{-0.049}_{+0.108}$	$0.712^{+0.070}_{-0.065}$
$R_{\mathrm{B_c,D_s^*}}$	$0.926^{-0.006}_{+0.012}$	$0.940^{-0.003}_{+0.034}$	$0.749^{+0.056}_{-0.041}$
$R^{\mathrm{L}}_{\mathrm{B_c,D_s^*}}$	$0.704^{+0.066}_{-0.059}$	$0.719^{+0.067}_{-0.060}$	$0.736^{+0.060}_{-0.049}$
$R^{\mathrm{T}}_{\mathrm{B_c,D_s^*}}$	$0.956^{-0.015}_{+0.021}$	$1.289^{-0.113}_{+0.182}$	$0.756^{+0.053}_{-0.037}$
$R_{\mathrm{B,K_2^*}}$	$0.851^{+0.017}_{-0.011}$	$0.759^{+0.055}_{-0.044}$	$0.718^{+0.068}_{-0.062}$
$R^{\mathrm{L}}_{\mathrm{B,K_2^*}}$	$0.675^{+0.075}_{-0.076}$	$0.696^{+0.077}_{-0.077}$	$0.713^{+0.070}_{-0.065}$
$R^{\mathrm{T}}_{\mathrm{B,K_2^*}}$	$0.983^{-0.026}_{+0.038}$	$1.051^{-0.049}_{+0.109}$	$0.721^{+0.066}_{-0.059}$
$R_{\mathrm{B_s,f_2}}$	$0.858^{+0.014}_{-0.008}$	$0.767^{+0.052}_{-0.040}$	$0.720^{+0.067}_{-0.060}$
$R^{\mathrm{L}}_{\mathrm{B_s,f_2}}$	$0.675^{+0.075}_{-0.075}$	$0.697^{+0.076}_{-0.076}$	$0.716^{+0.069}_{-0.063}$
$R^{\mathrm{T}}_{\mathrm{B_s,f_2}}$	$0.982^{-0.026}_{+0.037}$	$1.063^{-0.052}_{+0.114}$	$0.723^{+0.065}_{-0.058}$
$R_{\mathrm{B,K_1(1270)}}$	$0.909^{+0.008}_{-0.004}$	$0.880^{+0.002}_{-0.002}$	$0.714^{+0.069}_{-0.065}$
$R^{\mathrm{L}}_{\mathrm{B,K_1(1270)}}$	$0.751^{+0.085}_{-0.094}$	$0.717^{+0.088}_{-0.100}$	$0.712^{+0.071}_{-0.067}$
$R^{\mathrm{T}}_{\mathrm{B,K_1(1270)}}$	$0.978^{-0.025}_{+0.036}$	$1.078^{-0.056}_{+0.118}$	$0.714^{+0.069}_{-0.064}$
$R_{\Lambda_{\mathrm{b}},\Lambda}$	$0.931^{+0.014}_{-0.007}$	$0.773^{+0.051}_{-0.039}$	$0.712^{+0.071}_{-0.068}$
$R^{0}_{\Lambda_{\mathrm{b}},\Lambda}$	$0.708^{+0.073}_{-0.070}$	$0.705^{+0.074}_{-0.072}$	$0.707^{+0.073}_{-0.072}$
$R^{1}_{\Lambda_{\mathrm{b}},\Lambda}$	$1.071^{-0.023}_{+0.032}$	$1.104^{-0.060}_{+0.124}$	$0.715^{+0.070}_{-0.065}$

Table 3. Theoretical results for the μ-to-e ratio $R_{\mathrm{B,M}}$ of decay widths as defined in Eq. (22) in various $\mathrm{b}\to\mathrm{s}\mathrm{l}^+\mathrm{l}^-$ channels in the SM. Three kinematics regions are chosen: low, central and high q^2 regions. For a vector final state, the longitudinal and transverse polarizations are separated and labeled as L and T, respectively. We do not present the results $\Lambda_{\mathrm{b}}\to\Lambda\mathrm{l}^+\mathrm{l}^-$ since the lepton mass effects are not included in Eq. (25).

observable	low q^2:[0.045,1]/GeV^2	central q^2:[1,6]/GeV^2	high q^2:[14GeV^2,$q^2_{\max}$]
$R_{\mathrm{B,K}_0^*(1430)}$	0.980	1.001	1.029
$R_{\mathrm{B_s,f_0(980)}}$	0.980	1.000	1.004
$R_{\mathrm{B_c,D_s}}$	0.981	1.001	1.006
$R_{\mathrm{B_s},\phi}$	0.937	0.998	0.998
$R^{\mathrm{L}}_{\mathrm{B_s},\phi}$	0.991	1.001	0.999
$R^{\mathrm{T}}_{\mathrm{B_s},\phi}$	0.902	0.985	0.997
$R_{\mathrm{B_c,D_s^*}}$	0.917	0.995	0.997
$R^{\mathrm{L}}_{\mathrm{B_c,D_s^*}}$	0.978	0.997	0.997
$R^{\mathrm{T}}_{\mathrm{B_c,D_s^*}}$	0.908	0.990	0.997
$R_{\mathrm{B,K_2^*}}$	0.932	0.996	0.997
$R^{\mathrm{L}}_{\mathrm{B,K_2^*}}$	0.971	0.998	0.998
$R^{\mathrm{T}}_{\mathrm{B,K_2^*}}$	0.902	0.985	0.997
$R_{\mathrm{B_s,f_2}}$	0.930	0.995	0.998
$R^{\mathrm{L}}_{\mathrm{B_s,f_2}}$	0.971	0.998	0.998
$R^{\mathrm{T}}_{\mathrm{B_s,f_2}}$	0.902	0.985	0.997
$R_{\mathrm{B,K_1(1270)}}$	0.950	1.015	0.998
$R^{\mathrm{L}}_{\mathrm{B,K_1(1270)}}$	1.064	1.039	0.999
$R^{\mathrm{T}}_{\mathrm{B,K_1(1270)}}$	0.901	0.985	0.997

measured. So for these channels, the measurement of the μ-to-e ratio will be straightforward when enough statistical luminosity is accumulated.

For the other channels, we believe most of them, except the B_c decay, might also be experimentally measurable, especially at the Belle-II with the designed 50 ab^{-1} data, and the high luminosity upgrade of the LHC.

6) In Fig. 3, a new particle like Z' or a leptoquark can contribute to R_K and R_{K^*}. The coupling strength is unknown, and in principle it could be different from the CKM pattern. In the SM, the $B\to\pi l^+l^-$ and $B_s\to K l^+l^-$ have smaller CKM matrix elements. Thus if the NP contributions had the same magnitude as in $b\to s l^+l^-$, their impact in $B\to\pi l^+l^-$ and $B_s\to K l^+l^-$ would be much larger. But in many frameworks, the new physics in $b\to d l^+l^-$ is suppressed compared to that in $b\to s l^+l^-$. For recent discussions see Ref. [117]. This can be resolved by experiments in the future.

7) The weak phases from Z' and leptoquarks can be different from that in $b\to s\mu^+\mu^-$ or $b\to d\mu^+\mu^-$, which may induce direct CP violations. In the $b\to d\mu^+\mu^-$ process, the current data on $B\to\pi\mu^+\mu^-$ contains a large uncertainty [118]

$$\mathcal{A}_{\rm CP}(B^\pm\to\pi^\pm\mu^+\mu^-)=(-0.12\pm0.12\pm0.01). \tag{28}$$

This can certainly be refined in the future. The SM contribution may also contain a CP violation source [119, 120] since the up-type quark loop contributions are sizable.

4 Conclusions

Due to their small branching fractions in the SM, rare decays of heavy mesons can provide a rich laboratory to search for the effects of physics beyond the SM. To date, quite a few quantities in B decays have exhibited moderate deviations from the SM. This happens in both tree operator and penguin operator induced processes. The so-called $R_{D(D^*)}$ anomaly gives a hint that the tau lepton might have a different interaction with the light leptons. The V_{ub} and V_{cb} puzzles refer to the differences between the CKM matrix elements extracted from the exclusive and inclusive decay modes. In the $b\to s l^+l^-$ mode, the P_5' in $B\to K^* l^+l^-$ has received considerable attention in both the reliable estimates of hadronic uncertainties and new physics effects. In addition, LHCb has also observed a systematic deficit with respect to SM predictions for the branching ratios of several decay modes, such as $B_s\to\phi\mu^+\mu^-$ [113, 114]. Though the statistical significance is low, all these anomalies indicate that NP particles could be detected in flavor physics.

In this work, we have presented an analysis of the recently observed R_K and R_{K^*} anomalies. In terms of the effective operators, we have performed a model-independent fit to the R_K and R_{K^*} data. In the analysis, we have used two sets of form factors and found the results are rather stable against these hadronic inputs. Since the statistical significance in R_K and R_{K^*} is rather low, we propose to study a number of related rare B, B_s, B_c and Λ_b decay channels, and in particular we have pointed out that the μ-to-e ratios of decay widths with different polarizations of the final state particles, and in the $b\to d l^+l^-$ processes, are likely more sensitive to the structure of the underlying new physics. After taking into account the new physics contributions, we made theoretical predictions on lepton flavor non-universality in these processes, which can be stringently examined by experiments in future.

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Appendices A

Definitions of $R^{L,T}$ and $R^{0,1}$

For B decays to vector final state, we define the longitudinal and transverse ratios R^L and R^T as

$$R_V^{L,T}[q^2_{\rm min},q^2_{\rm max}]\equiv\frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 d\Gamma^{L,T}(B\to V\mu^+\mu^-)/dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 d\Gamma^{L,T}(B\to Ve^+e^-)/dq^2}, \tag{A1}$$

where the longitudinal and transverse differential widths are defined by

$$d\Gamma^L(B\to V\mu^+\mu^-)/dq^2 = \frac{3}{4}I_1^c-\frac{1}{4}I_2^c, \tag{A2}$$

$$d\Gamma^T(B\to V\mu^+\mu^-)/dq^2 = \frac{3}{2}I_1^s-\frac{1}{2}I_2^s, \tag{A3}$$

where V denotes a vector final state. The expressions for $I_1^{c,s}$ and $I_2^{c,s}$ are given by Eq. (6).

For $\Lambda_b\to\Lambda l^+l^-$ decays, we define ratios with equal or different polarization as [109]

$$R^{0,1}[q^2_{\rm min},q^2_{\rm max}]\equiv\frac{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 d\Gamma^{0,1}(\Lambda_b\to\Lambda\mu^+\mu^-)/dq^2}{\int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 d\Gamma^{0,1}(\Lambda_b\to\Lambda e^+e^-)/dq^2}. \tag{A4}$$

The superscript 0 means that the $\Lambda_{\rm b}$ and Λ have the same polarization, while 1 corresponds to different polarizations. The expressions for $\mathrm{d}\Gamma^{0,1}/\mathrm{d}q^2$ are

$$\mathrm{d}\Gamma^0(\Lambda_{\rm b}\to\Lambda\mu^+\mu^-)/\mathrm{d}q^2 = 2K^0_{\rm 1ss}, \tag{A5}$$

$$\mathrm{d}\Gamma^1(\Lambda_{\rm b}\to\Lambda\mu^+\mu^-)/\mathrm{d}q^2 = 2K^1_{\rm 1ss}+K^1_{\rm 1cc}, \tag{A6}$$

$K^{0,1}_{\rm 1ss}$ and $K^1_{\rm 1cc}$ are defined by

$$K^0_{\rm 1ss} = \frac{1}{2}(|A^{\rm R}_{\perp 0}|^2+|A^{\rm R}_{\parallel 0}|^2+|A^{\rm L}_{\perp 0}|^2+|A^{\rm L}_{\parallel 0}|^2), \tag{A7}$$

$$K^1_{\rm 1ss} = \frac{1}{4}(|A^{\rm R}_{\perp 1}|^2+|A^{\rm R}_{\parallel 1}|^2+|A^{\rm L}_{\perp 1}|^2+|A^{\rm L}_{\parallel 1}|^2), \tag{A8}$$

$$K^1_{\rm 1cc} = \frac{1}{2}(|A^{\rm R}_{\perp 1}|^2+|A^{\rm R}_{\parallel 1}|^2+|A^{\rm L}_{\perp 1}|^2+|A^{\rm L}_{\parallel 1}|^2). \tag{A9}$$

The A functions have already been defined in Eq. (25).

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