# Analysis of the charmonium－like states $\mathrm{X}^{*}(\mathbf{3 8 6 0}), \mathrm{X}(3872), \mathrm{X}(3915)$ ， $X(3930)$ and $X(3940)$ according to their strong decay behaviors＊ 

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#### Abstract

Inspired by the newly observed state $\mathrm{X}^{*}(3860)$ ，we analyze the strong decay behaviors of some charmonium－ like states， $\mathrm{X}^{*}(3860), \mathrm{X}(3872), \mathrm{X}(3915), \mathrm{X}(3930)$ and $\mathrm{X}(3940)$ ，with the ${ }^{3} P_{0}$ model．We carry out our work based on the hypothesis that all of these states are charmonium systems．Our analysis indicates that，as a $0^{++}$charmonium state， $\mathrm{X}^{*}(3860)$ can reproduce the experimental data．As for $\mathrm{X}(3872)$ ，it can tentatively be interpreted as the mixture of a c $\bar{c}$ system and a $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ molecular state．If we consider $\mathrm{X}(3940)$ as a $3^{1} S_{0}$ state，its total width in the present work is much lower than the experimental result．Thus，the $3^{1} S_{0}$ charmonium state seems not to be a good candidate for $\mathrm{X}(3940)$ ．Furthermore，our analysis implies that it is reasonable to assign $\mathrm{X}(3915)$ and $\mathrm{X}(3930)$ to be the same state， $2^{++}$．However，combining our analysis with that in Refs．［14，71］，we speculate that $\mathrm{X}(3915) / \mathrm{X}(3930)$ might also be the mixture of a cic system and a molecular state．


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## 1 Introduction

Very recently，the Belle collaboration observed a new charmonium－like state， $\mathrm{X}^{*}(3860)$ ，by performing a full amplitude analysis of the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{J} / \psi \mathrm{D} \overline{\mathrm{D}}[1]$ ． Its mass is $\left(3862_{-32-13}^{+26+40} \mathrm{MeV} / c^{2}\right)$ and its width is $\left(201_{-67-82}^{+154+88}\right) \mathrm{MeV}$ ．The assignment $J^{P C}=0^{++}$is favored over the assignment $2^{++}$at the level of $2.5 \sigma$ ．In Ref．［2］， this state was explained as a scalar $C \gamma_{5} \otimes \gamma_{5} C$ type tetraquark state by the QCD sum rules method．The $\mathrm{X}(3915)$ ，which was observed by the Belle and BaBar collaborations in the $\mathrm{B} \rightarrow \mathrm{J} / \psi \omega \mathrm{K}$ decay channel［4－8］， was assigned as the $0^{++}$charmonium state $[3,4]$ ．

After X（3915）was suggested to be the $\chi_{c 0}$ state，sev－ eral challenges were encountered［9－12］．For example， the decay $\chi_{c 0}(2 P) \rightarrow \mathrm{D} \overline{\mathrm{D}}$ ，which was expected to be a dominant decay mode，has not been observed experi－ mentally．In contrast，the decay mode $\mathrm{X}(3915) \rightarrow J \psi \omega$ ， which should be OZI（Okubo－Zweig－Iizuka）［13］sup－ pressed，has been observed in experiments．In addition， the mass splitting of $\chi_{c 2}(2 P)-\chi_{c 0}(2 P)$ is too small．A re－ analysis of the data from Ref．［4］，presented in Ref．［14］， showed that $\mathrm{X}(3915)$ could be the same state as $\mathrm{X}(3930)$ ，
whose quantum numbers are $2^{++}[17]$ ，due to the degen－ eracies of their masses and widths．Now，the observation of $\mathrm{X}^{*}(3860)$ ，which was assigned to be the $0^{++}$state，is an important verification of the result in Ref．［14］．

The mass of this newly observed $\mathrm{X}^{*}(3860)$ is close to that of the charmonium－like state $\mathrm{X}(3872)$ ．However， these two hadrons cannot be the same state because of their different decay modes and widths（see Table 1）． After the $\mathrm{X}(3872)$ was discovered by the Belle collabo－ ration［18］and confirmed by the BaBar［19］，CDF［20］， D0［21］and Belle［22］collaborations，its nature has re－ mained very controversial．It is usually explained by structures such as a molecular state［27－36］，a hybrid charmonium state［37－39］，a tetraquark state［40－44］，or a mixture of charmonium and molecular $\mathrm{DD}^{*}$ compo－ nents［45，46］．Another important explanation is that it is a charmonium state with quanta of $1^{++}[47,48]$ ，which has a dominant decay mode $\mathrm{D}^{0} \overline{\mathrm{D}}^{* 0}$ ．

The Belle collaboration reported another charmonium－like state $\mathrm{X}(3940)$ from the inclusive pro－ cess $\mathrm{e}^{+} \mathrm{x}^{-} \rightarrow \mathrm{J} / \psi+\mathrm{cc}$ at a mass of $M=(3.943 \pm 0.006 \pm$ $0.006) \mathrm{GeV} / c^{2}[25]$ ．Later，its decay width was con－ firmed to be $\Gamma=\left(37_{-15}^{+26} \pm 8\right) \mathrm{MeV}[26]$ ．Its structure has

[^0]Table 1. Experimental information about the $X$ states in this paper.

| state | mass/( $\mathrm{MeV} / c^{2}$ ) | width/MeV | $J^{P C}$ | decay channels |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{X}^{*}(3860)$ [1] | $3862_{-32-13}^{+26+40}$ | $201_{-67-82}{ }^{+154+88}$ | $0^{++}\left(2^{3} P_{0}\right)$ | D $\overline{\mathrm{D}}$ |
| $\mathrm{X}(3915)$ [4] | $3919.4 \pm 2.2 \pm 1.6$ | $13 \pm 6 \pm 3$ | $0^{++}\left(2^{3} P_{0}\right), 2^{++}\left(2^{3} P_{2}\right)$ | $J / \psi \omega$ |
| X (3930) | $3929 \pm 5 \pm 2$ [15] | $29 \pm 10 \pm 2$ | $2^{++}\left(2^{3} P_{2}\right)$ | D $\bar{D}$ |
|  | $3926.7 \pm 2.7 \pm 1.1$ [16] | $21.3 \pm 6.8 \pm 3.6$ | $2^{++}\left(2^{3} P_{2}\right)$ | D $\overline{\text { D }}$ |
|  | $3872 \pm 0.6 \pm 0.5$ [18] |  |  | $\mathrm{J} / 4 \pi^{+} \pi^{-}$ |
|  | $3871.3 \pm 0.7 \pm 0.4$ [20] |  |  | $\mathrm{J} / 4 \pi^{+} \pi^{-}$ |
| $\mathrm{X}(3872)$ | $3871.8 \pm 3.1 \pm 3.0$ [21] | $<2.3$ | $1^{++}\left(2^{3} P_{1}\right)$ | $\mathrm{J} / \psi \pi^{+} \pi^{-}$ |
|  | $3873.4 \pm 1.4$ [19] |  |  | $\mathrm{J} / \psi \pi^{+} \pi^{-}$ |
|  | $3875.4 \pm 0.7_{-2.0}^{+1.2}[22]$ |  |  | $\mathrm{D}^{0} \overline{\mathrm{D}}^{0} \pi^{0}, \mathrm{~J} / \psi \omega$ |
|  |  |  |  | $\mathrm{J} / \psi \pi^{+} \pi^{-} \pi^{0}, \mathrm{~J} / \mathrm{/} \gamma[23]$ |
|  | $3875.6 \pm 0.7_{1}^{+1.4}[24]$ |  |  | $\mathrm{D}^{0} \overline{\mathrm{D}}^{* 0}+$ h.c.., $\mathrm{J} / \psi \rho$ |
| $\underline{\mathrm{X}(3940)[25,26]}$ | $3942_{-6}^{+7} \pm 6$ | $37_{-15}^{+26} \pm 8$ | $0^{-+}\left(3^{1} S_{0}\right)$ | D $\bar{D}^{*}$ |

been explored with different kinds of methods such as the light-cone formalism [49], the NRQCD factorization formula $[50,51]$, and the QCD sum rules $[52,53]$. According to these studies, there seems to be no doubt that the assignment of $\mathrm{X}(3940)$ is $3^{1} S_{0}$. However, its structure is still controversial, and has been explained as different states such as a charmonium state [49], a molecular state [52-54], and a mixed charmonium-molecule state $[55,56]$.

In summary, these newly discovered charmonium-like states have inspired much interest about their physical natures. In order to further study their inner structures, we perform an analysis of the strong decay behaviors of $\mathrm{X}^{*}(3860), \mathrm{X}(3872), \mathrm{X}(3915), \mathrm{X}(3930)$ and $\mathrm{X}(3940)$ with the ${ }^{3} P_{0}$ decay model. The experimental information about these states is listed in Table 1. Since these $X$ states cannot be completely ruled out from the $c \bar{c}$ systems at present, we carry out our calculations by assuming them to be the charmonia. The results of this work will be helpful in revealing the inner structures of these $X$ states and confirming their quantum numbers.

We can employ several methods to study the strong decay behaviors of the hadrons, including the constituent quark model [57], heavy-quark symmetry theory [58], heavy meson effective theory [59], and the ${ }^{3} P_{0}$ decay model. An effective and simple method, the ${ }^{3} P_{0}$ model was first introduced by Micu in 1969 [60] and further developed by other collaborations [61, 62]. This method
can give a good description of the decay behaviors of many hadrons [63-67]. To date, it has been extensively applied to evaluating the strong decays of the heavy mesons in the charmonium [68-72] and bottomonium systems [73-75], the baryons [76], and even the tetraquark states [77]. This article is arranged as follows. In Section 2, we give a brief review of the ${ }^{3} P_{0}$ decay model; in Section 3 we study the strong decays of $\mathrm{X}^{*}(3860), \mathrm{X}(3872), \mathrm{X}(3915), \mathrm{X}(3930)$ and $\mathrm{X}(3940)$; and in Section 4, we present our conclusions.

## 2 The decay model

The principle of the ${ }^{3} P_{0}$ decay model is illustrated in Fig. 1, where a quark-antiquark pair $\left(\mathrm{q}_{3} \overline{\mathrm{q}}_{4}\right)$ is created from the vacuum with $0^{++}$quantum numbers. With the $\mathrm{q}_{1} \overline{\mathrm{q}}_{2}$ within the initial meson, this quark system regroups into two outgoing mesons via quark rearrangement for a meson decay process $\mathrm{A} \rightarrow \mathrm{BC}$. Its transition operator in the nonrelativistic limit reads

$$
\begin{align*}
T= & -3 \gamma \sum_{m}\langle 1 m 1-m \mid 00\rangle \int \mathrm{d}^{3} \vec{p}_{3} \mathrm{~d}^{3} \vec{p}_{4} \delta^{3}\left(\vec{p}_{3}+\vec{p}_{4}\right) \mathcal{Y}_{1}^{m} \\
& \times\left(\frac{\vec{p}_{3}-\vec{p}_{4}}{2}\right) \chi_{1-m}^{34} \varphi_{0}^{34} \omega_{0}^{34} b_{3}^{\dagger}\left(\vec{p}_{3}\right) d_{4}^{\dagger}\left(\vec{p}_{4}\right) \tag{1}
\end{align*}
$$

where $b_{3}^{\dagger}$ and $d_{4}^{\dagger}$ are the creation operators in mo-mentum-space for the quark-antiquark $\mathrm{q}_{3} \overline{\mathrm{q}}_{4}$ pair. $\gamma$ is a

(a)

(b)

Fig. 1. The two possible diagrams contributing to $\mathrm{A} \rightarrow \mathrm{BC}$ in the ${ }^{3} P_{0}$ model.
dimensionless parameter reflecting its creation strength. The momenta of this quark-antiquark pair are written as $\vec{p}_{3}$ and $\vec{p}_{4} . \varphi_{0}^{34}, \omega_{0}^{34}$ and $\chi_{1-m}^{34}$ represent its flavor, color and spin wave functions. The solid harmonic polynomial
$\mathcal{Y}_{1}^{m}(\vec{p}) \equiv|\vec{p}|^{1} Y_{1}^{m}\left(\theta_{p}, \phi_{p}\right)$ reflects the momentum-space distribution of the quark-antiquark pair.

The helicity amplitude of a decay process $\mathrm{A} \rightarrow \mathrm{BC}$ in the parent meson A center of mass frame is

$$
\begin{align*}
\mathcal{M}^{M_{J_{\mathrm{A}}} M_{J_{\mathrm{B}}} M_{J_{\mathrm{C}}}(\vec{P})=} & \gamma \sqrt{8 E_{\mathrm{A}} E_{\mathrm{B}} E_{\mathrm{C}}} \sum_{\substack{M_{L_{\mathrm{A}}}, M_{S_{\mathrm{A}}}, M_{L_{\mathrm{B}}}, M_{S_{\mathrm{B}}}, M_{L_{\mathrm{C}}}, M_{S_{\mathrm{C}}}, m}}\left\langle L_{\mathrm{A}} M_{L_{\mathrm{A}}} S_{\mathrm{A}} M_{S_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{J_{\mathrm{A}}}\right\rangle\left\langle L_{\mathrm{B}} M_{L_{\mathrm{B}}} S_{\mathrm{B}} M_{S_{\mathrm{B}}} \mid J_{\mathrm{B}} M_{J_{\mathrm{B}}}\right\rangle \\
& \times\left\langle L_{\mathrm{C}} M_{L_{\mathrm{C}}} S_{\mathrm{C}} M_{S_{\mathrm{C}}} \mid J_{\mathrm{C}} M_{J_{\mathrm{C}}}\right\rangle\langle 1 m 1-m \mid 00\rangle\left\langle\chi_{S_{\mathrm{B}} M_{S_{\mathrm{B}}}}^{14} \chi_{S_{\mathrm{C}} M_{S_{\mathrm{C}}}}^{32} \mid \chi_{S_{\mathrm{A}} M_{S_{\mathrm{A}}}}^{12} \chi_{1-m}^{34}\right\rangle \\
& \times\left[\left\langle\phi_{\mathrm{B}}^{14} \phi_{\mathrm{C}}^{32} \mid \phi_{\mathrm{A}}^{12} \phi_{0}^{34}\right\rangle I\left(\vec{P}, m_{1}, m_{2}, m_{3}\right)+(-1)^{1+S_{\mathrm{A}}+S_{\mathrm{B}}+S_{\mathrm{C}}}\left\langle\phi_{\mathrm{B}}^{32} \phi_{\mathrm{C}}^{14} \mid \phi_{\mathrm{A}}^{12} \phi_{0}^{34}\right\rangle I\left(-\vec{P}, m_{2}, m_{1}, m_{3}\right)\right], \tag{2}
\end{align*}
$$

where the two terms in the bracket [] correspond to the two possible diagrams in Fig. 1(a) and 1(b), respectively. $I\left(\vec{P}, m_{1}, m_{2}, m_{3}\right)$ is the spatial integral, which is defined as

$$
\begin{align*}
I\left(\vec{P}, m_{1}, m_{2}, m_{3}\right)= & \int \mathrm{d}^{3} \vec{p} \psi_{n_{\mathrm{B}} L_{\mathrm{B}} M_{L_{\mathrm{B}}}}^{*}\left(\frac{m_{3}}{m_{1}+m_{2}} \vec{P}_{\mathrm{B}}+\vec{p}\right) \\
& \times \psi_{n_{\mathrm{C}} L_{\mathrm{C}} M_{L_{\mathrm{C}}}}^{*}\left(\frac{m_{3}}{m_{2}+m_{3}} \vec{P}_{\mathrm{B}}+\vec{p}\right) \\
& \times \psi_{n_{\mathrm{A}} L_{\mathrm{A}} M_{L_{\mathrm{A}}}}\left(\vec{P}_{\mathrm{B}}+\vec{p}\right) \mathcal{Y}_{1}^{m}(\vec{p}), \tag{3}
\end{align*}
$$

where $\vec{P}=\vec{P}_{\mathrm{B}}=-\vec{P}_{\mathrm{C}}, \vec{p}=\vec{p}_{3}, m_{3}$ is the mass of the created quark $\mathrm{q}_{3}$. We employ the simple harmonic oscillator ( $\mathrm{SHO} \mathrm{)} \mathrm{approximation} \mathrm{as} \mathrm{the} \mathrm{meson} \mathrm{space} \mathrm{wave} \mathrm{functions}$ in Eq. (3), giving

$$
\begin{align*}
\Psi_{n L M_{L}}(\vec{p})= & (-1)^{n}(-i)^{L} R^{L+\frac{3}{2}} \sqrt{\frac{2 n!}{\Gamma\left(n+L+\frac{3}{2}\right)}} \\
& \times \exp \left(-\frac{R^{2} p^{2}}{2}\right) L_{n}^{L+\frac{1}{2}}\left(R^{2} p^{2}\right) \mathcal{Y}_{L M_{L}}(\vec{p}), \tag{4}
\end{align*}
$$

where $R$ is the scale parameter of the SHO. With the Jacob-Wick formula, the helicity amplitude can be converted into the partial wave amplitude

$$
\begin{align*}
\mathcal{M}^{J L}(\vec{P})= & \frac{\sqrt{4 \pi(2 L+1)}}{2 J_{\mathrm{A}}+1} \sum_{M_{J_{\mathrm{B}}} M_{J_{\mathrm{C}}}}\left\langle L 0 J M_{J_{\mathrm{A}}} \mid J_{\mathrm{A}} M_{J_{\mathrm{A}}}\right\rangle \\
& \times\left\langle J_{\mathrm{B}} M_{J_{\mathrm{B}}} J_{\mathrm{C}} M_{J_{\mathrm{C}}} \mid J M_{J_{\mathrm{A}}}\right\rangle \mathcal{M}^{M_{J_{\mathrm{A}}} M_{J_{\mathrm{B}}} M_{J_{\mathrm{C}}}}(\vec{P}), \tag{5}
\end{align*}
$$

where $M_{J_{\mathrm{A}}}=M_{J_{\mathrm{B}}}+M_{J_{\mathrm{C}}}, \boldsymbol{J}_{\mathrm{A}}=\boldsymbol{J}_{\mathrm{B}}+\boldsymbol{J}_{\mathrm{C}}$ and $\boldsymbol{J}_{\mathrm{A}}+\boldsymbol{J}_{\mathrm{P}}=$ $J_{\mathrm{B}}+J_{\mathrm{C}}+J_{\mathrm{L}}$.

Finally, the decay width in terms of partial wave amplitudes is

$$
\begin{equation*}
\Gamma=\frac{\pi}{4} \frac{|\vec{P}|}{M_{\mathrm{A}}^{2}} \sum_{J L}\left|\mathcal{M}^{J L}\right|^{2} \tag{6}
\end{equation*}
$$

where $P=|\vec{P}|=\frac{\sqrt{\left[M_{\mathrm{A}}^{2}-\left(M_{\mathrm{B}}+M_{\mathrm{C}}\right)^{2}\right]\left[M_{\mathrm{A}}^{2}-\left(M_{\mathrm{B}}-M_{\mathrm{C}}\right)^{2}\right]}}{2 M_{\mathrm{A}}}$, and $M_{\mathrm{A}}$,
$M_{\mathrm{B}}$, and $M_{\mathrm{C}}$ are the masses of the mesons $\mathrm{A}, \mathrm{B}$, and C , respectively.

## 3 Results and discussion

The decay width based on the ${ }^{3} P_{0}$ model depends on the following input parameters: the light quark pair ( $q \bar{q}$ ) creation strength $\gamma$, the SHO wave function scale parameter $R$, and the masses of the mesons and the constituent quarks. The masses used for the hadrons are listed in Table 2; $m_{\mathrm{u}}=m_{\mathrm{d}}=0.22 \mathrm{GeV}, m_{\mathrm{s}}=0.419 \mathrm{GeV}$, and $m_{\mathrm{c}}=1.65 \mathrm{GeV}$ [78].

Table 2. Hadron masses used in our calculations.

| state | mass $/ \mathrm{MeV}$ | state | mass $/ \mathrm{MeV}[78]$ |
| :---: | :---: | :---: | :---: |
| $M_{\mathrm{X}^{*}(3860)}$ | $3862[1]$ | $M_{\mathrm{D}^{ \pm}}$ | 1869.6 |
| $M_{\mathrm{X}(3872)}$ | $3872[18]$ | $M_{\mathrm{D}^{0}}$ | 1864.83 |
| $M_{\mathrm{X}(3915)}$ | $3919[4]$ | $M_{\mathrm{D}^{* \pm}}$ | 2010 |
| $M_{\mathrm{X}(3930)}$ | $3927[16]$ | $M_{\mathrm{D}^{* 0}}$ | 2007 |
| $M_{\mathrm{X}(3940)}$ | $3942[25,26]$ |  |  |

For the scale parameter $R$, there are two main choices, the common value and the effective value. The effective value can be fixed to reproduce the realistic root mean square radius by solving the Schrödinger equation with a linear potential [79, 80]. For a c $\overline{\mathrm{c}}$ system, the $R$ value of the $2 P$ state is estimated to be $2.3 \sim 2.5 \mathrm{GeV}^{-1}[81]$. For the mesons D and $\mathrm{D}^{*}$, their values are taken to be $R_{\mathrm{D}^{0}\left[\mathrm{D}^{ \pm}\right]}=1.52 \mathrm{GeV}^{-1}$ and $R_{\mathrm{D}^{* 0}\left[\mathrm{D}^{* \pm}\right]}=1.85 \mathrm{GeV}^{-1}[79,81,82]$ in this work. In Ref. [63], H.G.Blundel et al. carried out a series of least squares fits of the model predictions to the decay widths of 28 of the best known meson decays. The fitted $\gamma$ value, 6.25 , was suggested to be optimal for the creation of $u / d$ quarks [63].

As a simple test, we calculate the decay ratio $\frac{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{+} \pi^{-}\right)}{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$of the $D_{2}^{*}(2460)$ meson with $R_{\mathrm{D}_{2}^{*}(2460)}=2.22 \mathrm{GeV}^{-1}$ [82] and $R_{\pi^{-}}=1.41 \mathrm{GeV}^{-1}$ [82]. The experimental data from the BaBar [83], CLEO [84, 85], ARGUS [86], and ZEUS [87] collaborations are listed

Table 3. Experimental values and numerical result based on the ${ }^{3} P_{0}$ decay model of the ratio $\frac{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{+} \pi^{-}\right)}{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$.

| BaBar $[83]$ | CLEO [84] | CLEO [85] | ARGUS [86] | ZEUS [87] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1.47 \pm 0.03 \pm 0.16$ | $2.2 \pm 0.7 \pm 0.6$ | $2.3 \pm 0.8$ | $3.0 \pm 1.1 \pm 1.5$ | $2.8 \pm 0.8_{-0.6}^{+0.5}$ |

in Table 3. Our present result of 2.41 , based on the ${ }^{3} P_{0}$ approach, is in good agreement with the average experimental value of 2.35 . We can also predict the decay ratio $\frac{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{+} \pi^{-}\right)}{\Gamma\left(\mathrm{D}_{2}^{*}(2460) \rightarrow \mathrm{D}^{*+} \pi^{-}\right)}$with some other methods such as heavy-quark symmetry theory [58] and heavy meson effective theory [59]. In Ref. [59], heavy meson effective theory gave almost the same expression as that of heavyquark symmetry theory. Thus, our calculation is just a primary verification, which indicates that the ${ }^{3} P_{0}$ model can reproduce experimental data to some extent.

We know that $\mathrm{X}^{*}(3860)$ was favored to be a $0^{++}\left(2^{3} P_{0}\right)$ charmonium-like state by the Belle collaboration and that $\mathrm{X}(3915)$ was once been explained as this assignment. Lately, the latter was corrected to be the same state as another charmonium-like state, X(3930), which had been determined to have the $2^{++}\left(2^{3} P_{2}\right)$ assignment. In order to further confirm these conclusions, we study the strong decay behaviors of $\mathrm{X}^{*}(3860)$ by considering it as the $2^{3} P_{0}$ and $2^{3} P_{2}$ charmonia. We also do this for the $\mathrm{X}(3915)$ state. Furthermore, we analyze the decay behaviors of $\mathrm{X}(3930), \mathrm{X}(3872)$ and $\mathrm{X}(3940)$, which have been favored to be the $2^{++}\left(2^{3} P_{2}\right), 1^{++}\left(2^{3} P_{1}\right)$ and $0^{-+}\left(3^{1} S_{0}\right)$ states, respectively. As mentioned in Ref. [9], the mass difference $M_{\mathrm{X}(3930)}-M_{\mathrm{X}(3915)}=9.7 \pm 3.7$ MeV is smaller than the fine splitting of the $1 P$ state $M_{\chi_{c 2}}-M_{\chi_{c 0}}=141.45 \pm 0.32 \mathrm{MeV}$ [17]. This is important evidence to recognize $\mathrm{X}(3915)$ and $\mathrm{X}(3930)$ to be the same state. In order to determine its mass precisely, we calculate the decay width of $2^{3} P_{2}\left(\chi_{c 2}\right)$ state for different masses. All of these results are illustrated in Figs. 2-9.

## $3.1 \quad \mathrm{X}^{*}(3860)$

Whether we consider $\mathrm{X}^{*}(3860)$ as a $0^{++}$or $2^{++}$charmonium state, there is only one strong decay mode, $\mathrm{X}^{*}(3860) \rightarrow \mathrm{D} \overline{\mathrm{D}}$, where D refers to either $\mathrm{D}^{0}$ or $\mathrm{D}^{+}$. From Figs. 2 and 3, we can clearly see the difference between these two states. Taking $R=2.3 \sim 2.5 \mathrm{GeV}^{-1}$ as discussed above, the total width of the $0^{++}$state ranges from 110 to 180 MeV , which is compatible with the experimental data in Ref. [1]. The total width of the $2^{++}$state, which ranges from $0.4 \sim 1.9 \mathrm{MeV}$, is much smaller than the experimental data. That means, if $\mathrm{X}^{*}(3860)$ is assumed to be a $0^{++}$charmonium, its decay mode and total decay width are all consistent with the experimental data. The measured $\mathrm{X}^{*}(3860)$ mass is also close to potential model expectations for the $\chi_{c 0}(2 P)$. For example, the predicted mass in the Ebert-Faustov-Galkin model is 3854 $\mathrm{MeV} / c^{2}$ [88]. According to these analyses, the $\chi_{c 0}(2 P)$ assignment seems to be a good candidate for $\mathrm{X}^{*}(3860)$.

However, in our previous work [2], we also studied the mass and width of $\mathrm{X}^{*}(3860)$ as a $C \gamma_{5} \otimes \gamma_{5} C$ type scalar tetraquark state with the QCD sum rules. The results were also in agreement with the experiments. It was indicated in Ref. [2] that as a scalar tetraquark state, $\mathrm{X}^{*}(3860)$ can also decay into $\eta_{c} \pi^{-}$, with its total width $\Gamma\left(\mathrm{X}^{*}(3860) \rightarrow \eta_{c} \pi^{-}\right)=3.4 \mathrm{MeV}$. Whether $\mathrm{X}^{*}(3860)$ is a pure cē charmonium system or a scalar tetraquark state needs to be further confirmed experimentally in the future.


Fig. 2. (color online) The strong decay of $\mathrm{X}^{*}(3860)$ as the $0^{++}\left(2^{3} P_{0}\right)$ state as a function of scale parameter $R_{\mathrm{X} *(3860)}$.


Fig. 3. (color online) The strong decay of $\mathrm{X}^{*}(3860)$ as the $2^{++}\left(2^{3} P_{2}\right)$ state as a function of scale parameter $R_{\mathrm{X} *(3860)}$.

## 3.2 $X(3915)$ and $X(3930)$

Considering $\mathrm{X}(3915)$ as the $0^{++}$and $2^{++}$charmonium states, we observe different decay behaviors in Figs. 4 and 5 . For the $0^{++}$state, its total strong decay width ranges from 159 to 220 MeV , which dominantly decays into $\mathrm{D} \overline{\mathrm{D}}$. Both the width and the decay channel are inconsistent with the experimental data in Ref. [4] (see Table 1). This means that assuming $\mathrm{X}(3915)$ to be a $0^{++}$charmonium state is disfavored. If it is treated as
a $2^{++}$charmonium state, its decay behavior is very similar to that of $\mathrm{X}(3930)$, which can be seen from Figs. 5 and 6 . They both decay into $\mathrm{D} \overline{\mathrm{D}}$ and $\mathrm{D} \overline{\mathrm{D}}^{*}$ with a total width ranging from 1.0 to 3.0 MeV . These values of the width fall in the range of the experimental data. Thus, it seems reasonable to assign both $\mathrm{X}(3915)$ and $\mathrm{X}(3930)$ to be the the same state, $2^{++}\left(\chi_{c 2}\right)$. If this conclusion is true, the mass of the $\chi_{c 2}$ charmonium state is wrong. Its value has been predicted by by the nonrelativistic potential model (NR) and the Godfrey-Isgur relativized potential model (GI) [69]. It is about 50 MeV higher than the experimental data for $\mathrm{X}(3930)$. We plot the dependence of the partial and total decay widths on the mass of $\chi_{c 2}\left(2^{++}\right)$in Fig. 7. We can see that the partial width for the decay mode $\chi_{c 2} \rightarrow \mathrm{D} \overline{\mathrm{D}}$ decreases with the increase of the mass. For the $\chi_{c 2} \rightarrow \mathrm{D} \overline{\mathrm{D}}^{*}$ decay mode, the situation is completely opposite. With more experimental data from LHCb , BarBar, Belle etc in the future, these results can help us to determine its mass accurately.


Fig. 4. (color online) The strong decay of $\mathrm{X}(3915)$ as the $0^{++}\left(2^{3} P_{0}\right)$ state as a function of scale parameter $R_{\mathrm{X}(3915)}$.


Fig. 5. (color online) The strong decay of $\mathrm{X}(3915)$ as the $2^{++}\left(2^{3} P_{2}\right)$ state as a function of scale parameter $R_{\mathrm{X}(3915)}$.

The decay width of $\chi_{c 2} \rightarrow \mathrm{D} \overline{\mathrm{D}}$ is about $0.2 \sim$ 0.3 MeV , which should be observable in experiments
for both $\mathrm{X}(3930)$ and $\mathrm{X}(3915)$. However, it was reported by both the Belle and BaBar collaborations that $X(3930)$ and $\mathrm{X}(3915)$ were observed in two different decay channels, $\mathrm{X}(3930) \rightarrow \mathrm{D} \overline{\mathrm{D}}$ and $\mathrm{X}(3915) \rightarrow \mathrm{J} / \psi \omega$. A reanalysis presented in Ref. [14] showed that if the helicity-2 dominance assumption is abandoned and a sizable helicity-0 component allowed, the decay process $\mathrm{X}(3915) \rightarrow \mathrm{D} \overline{\mathrm{D}}$ may be reproduced in the experimental data. However, the large helicity-0 contribution means that $\mathrm{X}(3930) / \mathrm{X}(3915)$ might not be a pure ce charmonium state. Recently, it was also suggested in Ref. [71] that $\mathrm{X}(3930) / \mathrm{X}(3915)$ is dominantly molecular with a probability of bare $c \bar{c}$ states lower than $45 \%$.


Fig. 6. (color online) The strong decay of $\mathrm{X}(3927)$ as the $2^{++}\left(2^{3} P_{2}\right)$ state as a function of scale parameter $R_{\mathrm{X}(3927)}$.


Fig. 7. (color online) Dependence of the strong decay width on the mass of $\chi_{c 2}\left(2^{3} P_{2}\right)$ (scale parameter $R_{\chi_{c 2}}=2.4 \mathrm{GeV}^{-1}$ ).

The present prediction for the $\mathrm{X}(3930)$ width is about $1 \sim 3 \mathrm{MeV}$, still smaller than the lower limit from experiment of $21.3 \pm 6.8 \pm 3.6 \mathrm{MeV}$. This can be ascribed to the coupled-channel effect of the $\mathrm{X}(3930)$. It has been indicated that the lower the proportion of bare $c \bar{c}$ component in $\mathrm{X}(3930)$, the smaller the decay width of this mixed state [71]. In this work, we performed our calculations based on a supposition of $\mathrm{X}(3930)$ being a bare
$c \bar{c}$ system, $\chi_{c 2}$. Thus, it can be understood why our predicted decay width is smaller than the experimental data. In view of these analyses, we identify $\mathrm{X}(3930) / \mathrm{X}(3915)$ to be a mixed state with quantum numbers of $2^{++}$.

### 3.3 X(3872)

Since the $\mathrm{X}(3872)$ was observed, abundant experimental information has been accumulated, which can be seen in Table 1. The Belle experiment indicated $\mathrm{B}\left(\mathrm{X}(3872) \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{0} \pi^{0} \mathrm{~K}^{+}\right)=9.4_{-4.3}^{+3.6} \mathrm{~B}(\mathrm{X}(3872) \rightarrow$ $\mathrm{J} / \psi \pi^{+} \pi^{-} \mathrm{K}^{+}$) [22]. Based on these experimental data, we can draw the conclusion that $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ is a dominant decay. Although the underlying structure of $\mathrm{X}(3872)$ is very controversial, there is no doubt that its quantum number is $1^{++}$. As a charmonium state, $\chi_{c 1}\left(1^{++}\right)$, we show the dependence of the strong decay width on the scale parameter $R$ in Fig. 8. Taking $R=2.3-2.5 \mathrm{GeV}^{-1}$, the width of the inclusive decay channel $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ ranges from 0.2 to 1.0 MeV , which falls in the range of the experimental data in Table 1 and which is also consistent with the conclusion of $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ being its dominant decay mode. It seems reasonable to assign $\mathrm{X}(3872)$ to be a $\chi_{c 1}\left(1^{++}\right)$charmonium state based on these results.


Fig. 8. (color online) The strong decay of $\mathrm{X}(3872)$ as the $1^{++}\left(2^{3} P_{1}\right)$ state as a function of scale parameter $R_{\mathrm{X}(3872)}$.

However, the mass of the $\mathrm{X}(3872)$ resonance is 50 MeV lower than the predictions of the most lucky naive potential models for the mass of the $\chi_{c 1}(2 P)$ resonance, $m_{X}-m_{\chi_{c 1}(2 P)}=-\Delta \approx-50 \mathrm{MeV}[47]$. Besides, it was also indicated by experiment that $\mathrm{X}(3872)$ has almost the same decay width for the decays $\mathrm{X}(3872) \rightarrow$ $\mathrm{J} / \psi \pi^{+} \pi^{-} \pi^{0}$ and $\mathrm{X}(3872) \rightarrow \mathrm{J} / \psi \pi^{+} \pi^{-}[23]$, which implies a strong isospin-violating effect. Theoretically, this isospin-violating process $\mathrm{X}(3872) \rightarrow \mathrm{J} / \psi \rho \rightarrow \mathrm{J} / \psi \pi^{+} \pi^{-}$ can be explained by the $\mathrm{X}(3872) \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{* 0}+$ h.c.. rescattering effect. However, the numerical results in Ref. [89] showed that $\mathrm{B}\left(\mathrm{X}(3872) \rightarrow \mathrm{D}^{0} \overline{\mathrm{D}}^{* 0}+\right.$ h.c. $\left.\rightarrow \mathrm{J} / \psi \rho\right)$ is too tiny. The large isospin-violating effect can hardly be attributed to this re-scattering effect. This means
$\mathrm{X}(3872)$ cannot be considered as a pure cē charmonium system. If it is seen as a molecular state, its tiny binding energy $\left(m_{\mathrm{D}^{0}}+m_{\overline{\mathrm{D}}^{* 0}}\right)-m_{\mathrm{X}}=0.142 \pm 0.220 \mathrm{MeV}$ suggests a small prompt production rate for such a loosely bound state in $\mathrm{p} \overline{\mathrm{p}}$ collisions. Thus, this supposition also seems not to be reasonable. In view of these arguments, we speculate that $\mathrm{X}(3872)$ is also a mixture of $c \bar{c}$ charmonium and $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ molecular states. The hadronic decays into $\mathrm{DD} \pi$, $\mathrm{DD} \gamma$ as well as $\mathrm{J} / \psi \rho$ and $\mathrm{J} / \psi \omega$ proceed dominantly through the $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ component. The results, consistent with the experimental data, suggest that the main component of $\mathrm{X}(3872)$ is most likely to be a c $\overline{\mathrm{c}}$ system. This conclusion is consistent also with that of Ref. [45], which showed that $\mathrm{X}(3872)$ is approximately $97 \%$ charmonium state with $3 \% \overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ molecule.


Fig. 9. (color online) The strong decay of $\mathrm{X}(3940)$ as the $0^{-+}\left(3^{1} S_{0}\right)$ state as a function of scale parameter $R_{\mathrm{X}(3940)}$.

### 3.4 X(3940)

Finally, we can see in Fig. 9 that, as a $3^{1} S_{0}$ charmonium state, $\mathrm{X}(3940)$ can decay into $\mathrm{D}^{* 0} \mathrm{D}^{0}$ and $\mathrm{D}^{*+} \mathrm{D}^{-}$ final states. This is consistent with experiment, where $\mathrm{X}(3940)$ was truly observed from the inclusive process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{J} / \psi \overline{\mathrm{D}}^{*} \mathrm{D}$. However, it can also be seen that the maximum of the total width can only reach up to 10 MeV if $R$ is changed from 2.0 to $3.0 \mathrm{GeV}^{-1}$. The predicted decay width in experiment is $\Gamma=37_{-15}^{+26} \pm 8 \mathrm{MeV}$, which is much larger than this value. In addition, the mass of the $3^{1} S_{0}$ charmonium state was predicted to be about 100 MeV higher than $\mathrm{X}(3940)$ by the potential models [69]. These indicate that the $3^{1} S_{0}$ charmonium state might not be a good candidate for $\mathrm{X}(3940)$.

## 4 Conclusion

In summary, by considering both $\mathrm{X}^{*}(3860)$ and $\mathrm{X}(3915)$ as $0^{++}$and $2^{++}$charmonium states, and $\mathrm{X}(3872), \mathrm{X}(3930), \mathrm{X}(3940)$ as $1^{++}, 2^{++}$and $0^{-+}$charmonia respectively, we study their two-body open charm strong decay behaviors with the ${ }^{3} P_{0}$ decay model. We
find that $\mathrm{X}^{*}(3860)$ can be explained to be a $\chi_{c 0}$ state or a scalar tetraquark state, which needs to be further confirmed experimentally by the predicted decay channel $\mathrm{X}^{*}(3860) \rightarrow \eta_{c} \pi^{-}$. As for $\mathrm{X}(3872)$, if it is seen as the mixture of a cc charmonium and a $\overline{\mathrm{D}}^{* 0} \mathrm{D}^{0}$ molecular state, the experimental data can be reproduced and the isospin-violating effect can also be explained. The total width of $\mathrm{X}(3940)$ is inconsistent with the experimental result if it is supposed to be a $3^{1} S_{0}$ charmonium state. Thus, the $3^{1} S_{0}$ charmonium state is not a good candidate for $\mathrm{X}(3940)$ in the present work. If we treat $\mathrm{X}(3915)$ as a $0^{++}$charmonium, its decay behaviors are contradictory to experiment. Therefore, taking the $0^{++}$charmonium state as an assignment for the $\mathrm{X}(3915)$ is unreasonable.

If we suppose it is a $2^{++}$charmonium state, its decay behaviors match well with those of $\mathrm{X}(3930)$. Furthermore, the experimental data can also be reproduced. Thus, we tentatively assign $\mathrm{X}(3930)$ and $\mathrm{X}(3915)$ to be the same assignment, $2^{++}$. By comparing our results with those in Ref. [14, 71], we also speculate that $\mathrm{X}(3930) / \mathrm{X}(3915)$ is possibly a mixed state.

Certainly, the above results and conclusions are model-dependent to some extent. Therefore, there is still room for debate about the inner structures of these charmonium-like states. These above conclusions are just preliminary discussions which need to be further confirmed by other methods and experiments.

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