Transition magnetic moments of $J^P = \frac{3}{2}^+$ decuplet to $J^P = \frac{1}{2}^+$ octet baryons in the chiral constituent quark model

Harleen Dahiya

Department of Physics, Dr. B.R. Ambedkar National Institute of Technology, Jalandhar, 144011, India

Abstract: In light of the developments of the chiral constituent quark model (χCQM) in studying low energy hadronic matrix elements of the ground-state baryons, we extend this model to investigate their transition properties. The magnetic moments of transitions from the $J^P = \frac{3}{2}^+$ decuplet to $J^P = \frac{1}{2}^+$ octet baryons are calculated with explicit valence quark spin, sea quark spin and sea quark orbital angular momentum contributions. Since the experimental data is available for only a few transitions, we compare our results with the results of other available models. The implications of other complicated effects such as chiral symmetry breaking and SU(3) symmetry breaking arising due to confinement of quarks are also discussed.

1   Introduction

Understanding the internal structure of hadrons within the nonperturbative regime of quantum chromodynamics (QCD) is one of the most challenging areas in theory as well as experiment. The electromagnetic properties, obtained from the measurements of electromagnetic Dirac and Pauli form factors, are further related to static low-energy observables like mass, charge radius, magnetic moment, etc. It constitutes one of the most promising areas and can provide valuable insights into the underlying dynamics and the nonperturbative aspects of QCD. At present, electromagnetic form factors have been precisely obtained for nucleons [1–6], while for other baryons, the experimental data are available only for magnetic moments.

The magnetic moment is one of the most important quantities in scrutinizing the structure and properties of light baryons. Continuous theoretical efforts are being made to investigate the magnetic moments, and the calculations have benefited a lot from the information being made available through the experiments. At present, the magnetic moments of the $J^P = \frac{1}{2}^+$ octet baryons (except Σ^0) have been accurately measured experimentally [6]. Information about the $J^P = \frac{3}{2}^+$ decuplet baryons, however, is limited to only Δ^+ and Ω^- because of the difficulty in measuring their properties experimentally on account of their short lifetimes.

Further, the low-lying baryon decuplet to octet electromagnetic transitions play a very important role in probing the internal spin structure as well as the deformation of the octet and decuplet baryons. The $\Delta(1232)$ resonance is the lowest-lying excited state of the nucleon in which the search for transition amplitudes from the spin-parity selection rules has been carried out. The $\Delta^+ \to p\gamma$ transition amplitude contains the magnetic dipole moment (G_{M1}), the electric quadrupole moment (G_{E2}), and the Coulomb quadrupole moment (G_{C2}). The information on the magnetic moment is obtained from the G_{M1} amplitude, while the G_{E2} and G_{C2} amplitudes give us information on the intrinsic quadrupole moment. In spite of considerable efforts put in over the past few decades to determine the magnetic moments of the octet as well as the decuplet baryons, the decuplet to octet transition magnetic moments are less well-known. The transition magnetic moments are difficult to understand since the decuplet baryons have very short lifetimes and also the magnetic moments receive contributions from various interrelated effects, for example, spin and orbital angular momentum contributions, relativistic and exchange current effects, spin-0 meson cloud contributions, effect of the confinement on quark masses, etc.

The magnetic moments of the $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons have been extensively calculated theoretically using numerous different approaches. The approaches include a SU(6) symmetric naive quark model (NQM) [7, 8], nonrelativistic quark model [9], relativistic quark model [10], QCD-based quark mod-

el [11, 12], chiral perturbation theory [13], QCD string approach [14], light cone QCD sum rule [15], QCD sum rule [16], hypercentral model [17], Skyrme model [18], soliton model [19, 20], large-N_c chiral perturbation theory [21], lattice QCD [22, 23], and chiral quark model with exchange currents [24]. These studies indicate the growing interest in this field. Studies of the transition magnetic moments are rather limited. A few attempts have been made in chiral perturbation theory (χPT) [25], light cone QCD sum rules and light cone QCD (LCQCDSR) [26], large-N_c chiral perturbation theory (large N_C PT) [27, 28], relativistic quark model (Rel-QM) [29], cloudy bag model (CBM) [30], Skyrme model (SM) [31] , QCD sum rules (QCDSR) [32, 33], lattice QCD [34], chiral quark model (χQM) [35], effective mass quark model (EMQM) [36], meson cloud model (MCM) [37], U-spin [38], etc.

One of the important models which finds application in the nonperturbative regime of QCD is the chiral constituent quark model (χCQM) [39–41], where chiral symmetry breaking and its spontaneous breaking is implemented. The χCQM uses the effective interaction Lagrangian approach of the strong interactions, where the important phenomenon of quark-antiquark excitations is included. This results in the presence of the meson cloud at low energies where the effective degrees of freedom are the valence quarks and the internal Goldstone bosons (GBs), which are coupled to the valence quarks [41–45]. This perspective is in common with the modern effective field theory approaches. The χCQM has successfully been applied to calculate the spin and flavor distribution functions including the strangeness content of the nucleon [43, 44], weak vector and axial-vector form factors [46], nucleon structure functions and longitudinal spin asymmetries [47], electromagnetic and axial-vector form factors of the quarks and nucleon [48], charge radii, and quadrupole moment [49]. The magnetic moments of octet baryons, the transition within the octet baryons $\Sigma \to \Lambda$ and the Coleman-Glashow sum rule have already been calculated [50]. The work was further extended to the calculations of the magnetic moments of decuplet baryons [51], the magnetic moments of baryon resonances [52], magnetic moments of Λ resonances [53], and so on.

Considering the above developments of the χCQM in studying low energy hadronic matrix elements of the ground-state baryons, it becomes desirable to extend this model to investigate their transition properties. We will calculate the magnetic moments of transitions from the $J^P = \frac{3}{2}^+$ decuplet to $J^P = \frac{1}{2}^+$ octet baryons. Benefiting from the earlier studies of $J^P = \frac{3}{2}^+$ decuplet and $J^P = \frac{1}{2}^+$ octet baryons [50], the explicit contributions coming from the valence quarks, quark sea polarization,

and orbital angular momentum are calculated. The implications of other complicated effects such as chiral symmetry breaking and SU(3) symmetry breaking arising due to confinement of quarks are also discussed.

2   Transition magnetic moments

In this section, we calculate the transition magnetic moments for the radiative decays $B_i \to B_f + \gamma$, where B_i and B_f are the initial and final baryons. Since the $M1$ transition involves the quark magnetic moments, it can lead to the transition from the spin $\frac{3}{2}^+$ decuplet to the spin $\frac{1}{2}^+$ octet. We consider here the magnetic moments of the spin $\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions. In the present calculations we have considered only the $S_z = \frac{1}{2}$ spin projection for the $\frac{3}{2}^+$ decuplet, as the matrix elements for other spin projections will come out to be zero.

The transition magnetic moment can be calculated from the matrix element

$$\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) = \left\langle B_{\frac{1}{2}^+}, S_z = \frac{1}{2}\middle| \mu_z \middle| B_{\frac{3}{2}^+}, S_z = \frac{1}{2}\right\rangle, \quad (1)$$

where μ_z corresponds to the magnetic moment operator, and $\left|B_{\frac{1}{2}^+}\right\rangle$ and $\left|B_{\frac{3}{2}^+}\right\rangle$ correspond to the spin-flavor wavefunctions of the octet and decuplet baryons respectively, expressed as

$$\left|B_{\frac{3}{2}^+}\right\rangle \equiv \left|10, \frac{3}{2}^+\right\rangle = \chi^s \phi^s, \quad (2)$$

$$\left|B_{\frac{1}{2}^+}\right\rangle \equiv \left|8, \frac{1}{2}^+\right\rangle = \frac{1}{\sqrt{2}}(\chi' \phi' + \chi'' \phi''). \quad (3)$$

The spin wavefunctions (χ^s for the case of decuplet baryons and χ' and χ'' for the case of octet baryons) are expressed as

$$\chi^s = \uparrow\uparrow\uparrow,$$
$$\chi' = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow),$$
$$\chi'' = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow). \quad (4)$$

The flavor wavefunctions ϕ^s for the decuplet baryons of the types $B_{\frac{3}{2}^+}(Q_1Q_1Q_1)$, $B_{\frac{3}{2}^+}(Q_1Q_1Q_2)$ and $B_{\frac{3}{2}^+}(Q_1Q_2Q_3)$ are respectively expressed as

$$\phi_{B^*}^s = Q_1Q_1Q_1,$$
$$\phi_{B^*}^s = \frac{1}{\sqrt{3}}(Q_1Q_1Q_2 + Q_1Q_2Q_1 + Q_2Q_1Q_1),$$
$$\phi_{B^*}^s = \frac{1}{\sqrt{6}}(Q_1Q_2Q_3 + Q_1Q_3Q_2 + Q_2Q_1Q_3$$
$$+ Q_2Q_3Q_1 + Q_3Q_1Q_2 + Q_3Q_2Q_1), \quad (5)$$

whereas the flavor wavefunctions ϕ' and ϕ'' for the octet

baryons of the type $B_{\frac{1}{2}^+}(Q_1Q_1Q_2)$ are

$$\phi_B' = \frac{1}{\sqrt{2}}(Q_1Q_2Q_1 - Q_2Q_1Q_1),$$

$$\phi_B'' = \frac{1}{\sqrt{6}}(2Q_1Q_1Q_2 - Q_1Q_2Q_1 - Q_2Q_1Q_1), \quad (6)$$

where Q_1, Q_2, and Q_3 correspond to any of the u, d, and s quarks. For the case of $\Lambda(uds)$ and $\Sigma^0(uds)$, the wavefunctions are given as

$$\phi_\Lambda' = \frac{1}{2\sqrt{3}}(usd + sdu - sud - dsu - 2uds - 2dus),$$

$$\phi_\Lambda'' = \frac{1}{2}(sud + usd - sdu - dsu),$$

$$\phi_{\Sigma^0}' = \frac{1}{2}(sud + sdu - usd - dsu),$$

$$\phi_{\Sigma^0}'' = \frac{1}{2\sqrt{3}}(sdu + sud + usd + dsu - 2uds - 2dus). \quad (7)$$

The details of the spatial wave functions (ψ^s, ψ', ψ'') can be found in Ref. [54].

The magnetic moment of a given baryon in the χCQM receives contributions from the valence quark spin, sea quark spin and sea quark orbital angular momentum. The total magnetic moment is expressed as

$$\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{Total}} = \mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{V}}$$
$$+ \mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{S}}$$
$$+ \mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{O}}, \quad (8)$$

where $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{V}}$ and $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{S}}$ are the magnetic moment contributions of the valence quarks and the sea quarks respectively coming from their spin polarizations, while $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{O}}$ is the magnetic moment contribution due to the rotational motion of the two bodies constituting the sea quarks and Goldstone boson (GB) and is referred to as the orbital angular momentum contribution of the quark sea [41].

In terms of quark magnetic moments and spin polarizations, the valence spin $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{V}}$, sea spin $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{S}}$, and sea orbital $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{O}}$ contributions can be defined as

$$\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{V}} = \sum_{q=u,d,s} \Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{V}} \mu_q, \quad (9)$$

$$\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{S}} = \sum_{q=u,d,s} \Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{S}} \mu_q, \quad (10)$$

$$\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_{\text{O}} = \sum_{q=u,d,s} \Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{V}} \mu(q_+ \to), \quad (11)$$

where $\mu_q = \frac{e_q}{2M_q}$ $(q = u, d, s)$ is the quark magnetic moment in the units of μ_N (nuclear magneton), $\Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{V}}$ and $\Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{S}}$ are the valence and sea quark spin polarizations respectively, $\mu(q_+ \to)$ is the orbital moment for any chiral fluctuation, and e_q and M_q are the electric charge and the mass, respectively, for the quark q.

The spin structure of a decuplet to octet transition matrix element is defined as

$$\left\langle B_{\frac{1}{2}^+}, S_z = \frac{1}{2} \left| \mathcal{N}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) \right| B_{\frac{3}{2}^+}, S_z = \frac{1}{2} \right\rangle, \quad (12)$$

where the number operator $\mathcal{N}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)$ measures the number of quarks with spin up ($\uparrow$) or down ($\downarrow$) in the transition $\frac{3}{2}^+ \to \frac{1}{2}^+$, for example,

$$\mathcal{N}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)$$
$$= \sum_{q=u,d,s} \left(N_{q^\uparrow}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) + N_{q^\downarrow}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) \right). \quad (13)$$

This can be used to calculate the quark spin polarizations (for $q = u, d, s$) for a given transition,

$$\Delta q\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) = N_{q^\uparrow}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right) - N_{q^\downarrow}\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right). \quad (14)$$

The valence quark spin polarizations $\Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{V}}$ for a given baryon transition can be calculated using the SU(6) spin-flavor wave functions defined in Eqs. (2) and (3). Using these, the magnetic moment contributions coming from the valence quarks can be calculated from Eq. (9) and are summarized in Table 1 for all the decuplet to octet transitions.

Table 1.   Valence transition magnetic moments for $\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions.

$\frac{3}{2}^+ \to \frac{1}{2}^+$ transition	valence transition magnetic moment
$\mu(\Delta \to p)_{\text{V}}$	$\frac{2\sqrt{2}}{3}\mu_u - \frac{2\sqrt{2}}{3}\mu_d$
$\mu(\Sigma^{*+} \to \Sigma^+)_{\text{V}}$	$\frac{2\sqrt{2}}{3}\mu_u - \frac{2\sqrt{2}}{3}\mu_s$
$\mu(\Sigma^{*0} \to \Sigma^0)_{\text{V}}$	$\frac{\sqrt{2}}{3}\mu_u + \frac{\sqrt{2}}{3}\mu_d - \frac{2\sqrt{2}}{3}\mu_s$
$\mu(\Sigma^{*0} \to \Lambda)_{\text{V}}$	$\sqrt{\frac{2}{3}}\mu_u - \sqrt{\frac{2}{3}}\mu_d$
$\mu(\Sigma^{*-} \to \Sigma^-)_{\text{V}}$	$\frac{2\sqrt{2}}{3}\mu_d - \frac{2\sqrt{2}}{3}\mu_s$
$\mu(\Xi^{*0} \to \Xi^0)_{\text{V}}$	$\frac{2\sqrt{2}}{3}\mu_u - \frac{2\sqrt{2}}{3}\mu_s$
$\mu(\Xi^{*-} \to \Xi^-)_{\text{V}}$	$\frac{2\sqrt{2}}{3}\mu_d - \frac{2\sqrt{2}}{3}\mu_s$

For the calculation of the sea quark spin polarizations $\Delta q\left(\frac{3}{2}^+ \to \frac{1}{2}^+\right)_{\text{S}}$ for a given baryon transition, we will

use the basic idea of the chiral constituent quark model (χCQM) [39] where the set of internal GBs couples directly to the valence quarks in the interior of hadron and we have

$$q^{\uparrow\downarrow} \to P(q,GB)q^{\uparrow\downarrow} + P(q^{\uparrow\downarrow},GB). \quad (15)$$

Here the transition probability of the emission of a GB from any of the q quarks $P(q,GB)$ and the transition probability of the $q^{\uparrow\downarrow}$ quark $P(q^{\uparrow\downarrow},GB)$ can be calculated

from the effective Lagrangian describing the interaction between quarks and GBs, expressed as

$$\mathcal{L}_{\text{int}} = c_8 \bar{\psi}\left(\Phi + P_{\eta'}\frac{\eta'}{\sqrt{3}}I\right)\psi = c_8\bar{\psi}(\Phi')\psi, \quad (16)$$

where c_8 is the coupling constant for the octet GB. The GB field Φ' can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix} P_\pi\frac{\pi^0}{\sqrt{2}} + P_\eta\frac{\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} & P_\pi\pi^+ & P_K K^+ \\ P_\pi\pi^- & -P_\pi\frac{\pi^0}{\sqrt{2}} + P_\eta\frac{\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} & P_K K^o \\ P_K K^- & P_K \bar{K}^0 & -P_\eta\frac{2\eta}{\sqrt{6}} + P_{\eta'}\frac{\eta'}{\sqrt{3}} \end{pmatrix}. \quad (17)$$

The fluctuation process describing the effective Lagrangian is

$$q^{\uparrow\downarrow} \to GB + q'^{\downarrow\uparrow} \to (q\bar{q}') + q'^{\downarrow\uparrow}, \quad (18)$$

where $q\bar{q}' + q'$ constitute the sea quarks. In Eq. (17), the chiral fluctuations $u(d) \to d(u) + \pi^{+(-)}$, $u(d) \to s + K^{+(0)}$, $u(d,s) \to u(d,s) + \eta$, and $u(d,s) \to u(d,s) + \eta'$ are given in terms of the transition probabilities P_π, P_K, P_η and $P_{\eta'}$ respectively [41, 42, 44].

From Eq. (15) the transition probability of the emission of a GB from any of the q quark is expressed in terms of the transition probabilities P_π, P_K, P_η and $P_{\eta'}$ as

$$P(u,GB) = P(d,GB) = -\frac{1}{6}(9P_\pi + P_\eta + 2P_{\eta'} + 6P_K), \quad (19)$$

$$P(s,GB) = -\frac{1}{3}(2P_\eta + P_{\eta'} + 6P_K), \quad (20)$$

whereas the transition probability of the $q^{\uparrow\downarrow}$ quark can be expressed as

$$P(u^{\uparrow\downarrow},GB) = \frac{1}{6}(3P_\pi + P_\eta + 2P_{\eta'})u^{\downarrow\uparrow} + P_\pi d^{\downarrow\uparrow} + P_K s^{\downarrow\uparrow}, \quad (21)$$

$$P(d^{\uparrow\downarrow},GB) = P_\pi u^{\downarrow\uparrow} + \frac{1}{6}(3P_\pi + P_\eta + 2P_{\eta'})d^{\downarrow\uparrow} + P_K s^{\downarrow\uparrow}, \quad (22)$$

$$P(s^{\uparrow\downarrow},GB) = P_K u^{\downarrow\uparrow} + P_K d^{\downarrow\uparrow} + \frac{1}{3}(2P_\eta + P_{\eta'})s^{\downarrow\uparrow}. \quad (23)$$

Using the sea spin polarizations, the magnetic moment contributions coming from the sea quarks can be calculated from Eq. (10) and are summarized in Table 2 for all the decuplet to octet transitions.

The magnetic moment contribution of the angular momentum of a given sea quark can be expressed in terms of the orbital angular momenta of quarks and G-B $\langle L_q, L_{GB}\rangle$, which are further related to the masses of

quarks and GB (M_q, M_{GB}) as

$$\langle L_q\rangle = \frac{M_{GB}}{M_q + M_{GB}} \text{ and } \langle L_{GB}\rangle = \frac{M_q}{M_q + M_{GB}}. \quad (24)$$

The magnetic moment arising from all the possible transitions of a given valence quark to the GBs is obtained by multiplying the orbital moment of each process by the probability for such a process to take place. The general orbital moment for any quark q is given as

$$\mu(q^\uparrow \to q'^\downarrow) = \frac{e_{q'}}{2M_q}\langle L_q\rangle + \frac{e_q - e_{q'}}{2M_{GB}}\langle L_{GB}\rangle. \quad (25)$$

The orbital moments of u, d and s quarks after including the transition probabilities P_π, P_K, P_η and $P_{\eta'}$ as well as the masses of GBs M_π, M_K and M_η can be expressed as

$$[\mu(u^\uparrow \to)] = \left[\frac{3P_\pi M_u^2}{2M_\pi(M_u + M_\pi)} - \frac{P_K(M_K^2 - 3M_u^2)}{2M_K(M_u + M_K)} + \frac{P_\eta M_\eta}{6(M_u + M_\eta)} + \frac{P_{\eta'} M_{\eta'}}{3(M_u + M_{\eta'})}\right]\mu_u, \quad (26)$$

$$[\mu(d^\uparrow \to)] = -\left[\frac{3P_\pi(M_\pi^2 - 2M_d^2)}{2M_\pi(M_d + M_\pi)} - \frac{P_K M_K}{(M_d + M_K)} - \frac{P_\eta M_\eta}{6(M_d + M_\eta)} - \frac{P_{\eta'} M_{\eta'}}{3(M_d + M_{\eta'})}\right]\mu_d, \quad (27)$$

$$[\mu(s^\uparrow \to)] = -\left[\frac{P_K(M_K^2 - 3M_s^2)}{M_K(M_s + M_K)} - \frac{2P_\eta M_\eta}{3(M_s + M_\eta)} - \frac{P_{\eta'} M_{\eta'}}{3(M_s + M_{\eta'})}\right]\mu_s. \quad (28)$$

The orbital contribution to the magnetic moment of the decuplet to octet transition $\mu\left(B_{\frac{3}{2}^+} \to B_{\frac{1}{2}^+}\right)_O$ for the

Table 2.   Sea quark transition magnetic moments for $\frac{3}{2}^+\to\frac{1}{2}^+$ transitions.

$\frac{3}{2}^+\to\frac{1}{2}^+$ transition	Sea quark transition magnetic moment
$\mu(\Delta\to p)_{\mathrm{S}}$	$-\frac{2\sqrt{2}}{3}\left(P_\pi+P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u+\frac{2\sqrt{2}}{3}\left(P_\pi+P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_d$
$\mu(\Sigma^{*+}\to\Sigma^+)_{\mathrm{S}}$	$-\frac{2\sqrt{2}}{3}\left(2P_\pi+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u-\frac{2\sqrt{2}}{3}(P_\pi-P_K)\mu_d$
	$+\frac{2\sqrt{2}}{3}\left(P_K+\frac{4}{3}P_\eta+\frac{2}{3}P_{\eta'}\right)\mu_s$
$\mu(\Sigma^{*0}\to\Sigma^0)_{\mathrm{S}}$	$-\frac{\sqrt{2}}{3}\left(3P_\pi-P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u-\frac{\sqrt{2}}{3}\left(3P_\pi-P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_d$
	$+\frac{2\sqrt{2}}{3}\left(P_K+\frac{4}{3}P_\eta+\frac{2}{3}P_{\eta'}\right)\mu_s$
$\mu(\Sigma^{*0}\to\Lambda)_{\mathrm{S}}$	$-\sqrt{\frac{2}{3}}\left(P_\pi+P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u+\sqrt{\frac{2}{3}}\left(P_\pi+P_K+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_d$
$\mu(\Sigma^{*-}\to\Sigma^-)_{\mathrm{S}}$	$-\frac{2\sqrt{2}}{3}(P_\pi-P_K)\mu_u-\frac{2\sqrt{2}}{3}\left(P_\pi+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_d$
	$+\frac{2\sqrt{2}}{3}a\left(P_K+\frac{4}{3}P_\eta+\frac{2}{3}P_{\eta'}\right)\mu_s$
$\mu(\Xi^{*0}\to\Xi^0)_{\mathrm{S}}$	$-\frac{2\sqrt{2}}{3}\left(2P_\pi+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u-\frac{2\sqrt{2}}{3}(P_\pi-P_K)\mu_d$
	$+\frac{2\sqrt{2}}{3}\left(P_K+\frac{4}{3}P_\eta+\frac{2}{3}P_{\eta'}\right)\mu_s$
$\mu(\Xi^{*-}\to\Xi^-)_{\mathrm{S}}$	$-\frac{2\sqrt{2}}{3}\left(2P_\pi+\frac{P_\eta}{3}+\frac{2}{3}P_{\eta'}\right)\mu_u-\frac{2\sqrt{2}}{3}(P_\pi-P_K)\mu_d$
	$+\frac{2\sqrt{2}}{3}\left(P_K+\frac{4}{3}P_\eta+\frac{2}{3}P_{\eta'}\right)\mu_s$

baryon the type $B(Q_1Q_2Q_3)$ is given as

$$\Delta Q_1\left(\frac{3}{2}^+\to\frac{1}{2}^+\right)_{\mathrm{V}}\mu(Q_1^\uparrow\to)$$

$$+\ \Delta Q_2\left(\frac{3}{2}^+\to\frac{1}{2}^+\right)_{\mathrm{V}}\mu(Q_2^\uparrow\to)$$

$$+\ \Delta Q_3\left(\frac{3}{2}^+\to\frac{1}{2}^+\right)_{\mathrm{V}}\mu(Q_3^\uparrow\to) \qquad (29)$$

3   Results and discussion

The transition probabilities P_π, P_K, P_η and $P_{\eta'}$ as well as the masses of GBs M_π, M_K and M_η are the input parameters needed for the numerical calculations of the baryon transition magnetic moments $\mu\left(B_{\frac{3}{2}^+}\to B_{\frac{1}{2}^+}\right)$ in the χCQM. The hierarchy followed by the transition probabilities P_π, P_K, P_η and $P_{\eta'}$, which represent respectively the probabilities of fluctuations of a constituent quark into pions, K, η and η', is given as

$$P_{\eta'}<P_\eta<P_K<P_\pi. \qquad (30)$$

This order is because the probability of emission of a particular GB is dependent on its mass, implying that the probability of emitting a heavier meson from a lighter quark is much smaller than that of emitting a lighter meson. The transition probabilities are usually fixed by the experimentally known spin and flavor distribution functions measured from the DIS experiments [3, 4, 6, 44]. A

detailed analysis leads to the following probabilities:

$$P_{\eta'}=0.03, \quad P_\eta=0.04, \quad P_K=0.06, \quad P_\pi=0.12. \qquad (31)$$

On the other hand, the orbital angular momentum contributions are characterized by the masses of quarks and GBs (M_q and M_{GB}). The on-mass shell mass values can be used in accordance with several other similar calculations [41, 50].

The inputs discussed above have been used to calculate the explicit valence $\mu\left(B_{\frac{3}{2}^+}\to B_{\frac{1}{2}^+}\right)_{\mathrm{V}}$, sea $\mu\left(B_{\frac{3}{2}^+}\to B_{\frac{1}{2}^+}\right)_{\mathrm{S}}$ and orbital $\mu\left(B_{\frac{3}{2}^+}\to B_{\frac{1}{2}^+}\right)_{\mathrm{O}}$ contributions corresponding to the transition magnetic moments, and the results are presented in Table 3. The limited experimental data available for the $\frac{3}{2}^+\to\frac{1}{2}^+$ transitions is also presented in the table. It can immediately seen that the contributions coming from valence and orbital contributions have same signs whereas the sea contributions have opposite signs. All of these ultimately add up to give the total magnetic moment. It is also observed that in some cases the orbital part dominates over the sea part, making the total magnetic moments even higher than the valence part. This is the case for the $\Delta\to p$, $\Sigma^{*+}\to\Sigma^+$, $\Sigma^{*0}\to\Lambda$, $\Sigma^{*-}\to\Sigma^-$ $\Xi^{*-}\to\Xi^-$ and $\Xi^{*0}\to\Xi^0$ transitions. One can generalize this as follows: whenever the number of u, d or s quarks is higher than that of the other quarks, the orbital part dominates, whereas when there are equal numbers of u, d and s quarks,

Table 3.   Magnetic moments in units of μ_N for the $B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}$ transitions.

$B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}$ transition	Data [6]	$\mu\left(B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}\right)_V$	$\mu\left(B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}\right)_S$	$\mu\left(B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}\right)_O$	$\mu\left(B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}\right)$
$\Delta\to p$	−3.43	2.83	−0.60	1.65	3.87
$\Sigma^{*+}\to\Sigma^+$	4.45	2.49	−0.54	0.65	2.60
$\Sigma^{*0}\to\Sigma^0$	−−	1.08	−0.24	0.18	1.02
$\Sigma^{*0}\to\Lambda$	3.69	2.45	−0.52	1.42	3.35
$\Sigma^{*-}\to\Sigma^-$	<0.85	−0.33	0.05	−0.99	−1.27
$\Xi^{*0}\to\Xi^0$	<5.39	2.50	−0.31	0.65	2.83
$\Xi^{*-}\to\Xi^-$	<5.39	−0.33	0.31	−1.00	−1.02

Table 4.   Phenomenological results of some other theoretical approaches for $B_{\frac{3}{2}+}\to B_{\frac{1}{2}+}$ transition magnetic moments.

other models	$\Delta\to p$	$\Sigma^{*+}\to\Sigma^+$	$\Sigma^{*0}\to\Sigma^0$	$\Sigma^{*0}\to\Lambda$	$\Sigma^{*-}\to\Sigma^-$	$\Xi^{*0}\to\Xi^0$	$\Xi^{*-}\to\Xi^-$
NQM [7]	2.65	2.42	1.05	2.31	−0.32	2.18	−0.29
χPT [25]	−3.50	4.46	−2.34	3.62	−0.21	5.38	0.20
LCQCDSR [26]	2.50	2.10	0.89	−−	−0.47	−2.77	0.47
Large N_C PT [27]	3.51	2.96	1.34	2.96	−0.27	2.96	−−
Large N_C PT [28]	3.51	2.97	1.39	2.93	−0.19	2.96	−0.19
Rel-QM [29]	3.25	2.59	1.07	2.86	−0.46	2.71	−0.47
QCDSR [32]	−2.76	2.24	1.01	−2.46	−0.22	2.46	−0.27
QCDSR [33]	3.86	3.38	1.47	4.44	−0.57	−1.24	0.23
Lattice QCD [34]	2.46	2.61	1.07	−−	−0.47	−2.77	0.47
χQM [35]	−3.31	2.17	−−	−2.74	−0.59	2.23	−0.59
EMQM [36]	2.63	2.33	1.02	2.28	0.30	2.33	0.30
MCM [37]	3.32	3.54	1.61	3.39	−0.34	3.62	−0.42
U-spin [38]	−−	3.22	1.61	2.68	0	3.21	−−
PDG [6]	−3.43	4.45	−−	3.69	<0.85	<5.39	<5.39
this work: χCQM	3.87	2.60	1.02	3.35	−1.27	2.83	−1.02

there is some variation from this behavior. For example, in the case of $\Delta\to p$ and $\Sigma^{*+}\to\Sigma^+$ the baryon quark content is uud and clearly the orbital contribution is dominant compared to the sea contribution. Similarly, in the cases of the $\Sigma^{*-}\to\Sigma^-$, $\Xi^{*0}\to\Xi^0$ and $\Xi^{*-}\to\Xi^-$ transitions, where the baryon quark contents are dds, uss and dss respectively, again the orbital contributions are large compared to the sea contributions. This can be easily explained from Eq. (29), where we see that the orbital contribution is governed by the valence quark spin polarization along with the orbital moments of the quarks. The orbital part dominates in the cases where the number of u, d or s quarks is higher than the other quarks, because the contributions of the individual quarks with the same magnitudes add up in the same direction. On the other hand, in the cases where the quark content is uds ($\Sigma^{*0}\to\Sigma^0$ and $\Sigma^{*0}\to\Lambda$ transitions) there is some variation from this behavior for different cases because the contributions of the individual quarks have different magnitudes. These observations clearly suggest that since the quark sea is created from quantum fluc-

tuations associated with bound state hadron dynamics and the process is completely determined by nonperturbative mechanisms, the constituent quarks and weakly interacting Goldstone bosons can provide the appropriate degree of freedom in the nonperturbative regime of QCD on which further corrections could be evaluated. A further precise measurement of these magnetic moments, therefore, would have great importance for the understanding of χCQM.

For the sake of comparison with other models, we present the results of the available phenomenological and theoretical models in Table 4. We present the results from chiral perturbation theory (χPT) [25], light cone QCD sum rules and light cone QCD (LCQCDSR) [26], large-N_c chiral perturbation theory (large N_C PT) [27, 28], relativistic quark model (Rel-QM)[29], QCD sum rules (QCDSR) [32, 33], lattice QCD [34], chiral quark model (χQM) [35], effective mass quark model (EMQM) [36], meson cloud model (MCM) [37], and U-spin [38]. The experimental values from the PDG [6] are also listed.

Since experimental data is available for the $\Delta \to p$, $\Sigma^{*+} \to \Sigma^+$, and $\Sigma^{*0} \to \Lambda$ transitions, we can compare these with the χCQM results as well as with the results of other available models. It is evident from the table that a good agreement corresponding to the case of $\Delta \to p$ is obtained. The magnetic moment of the $\Delta \to p + \gamma$ transition is a long-standing problem and most of the approaches in the literature underestimate it. The empirical estimate for the magnetic moment of the $\Delta \to p + \gamma$ transition can be made from the helicity amplitudes [6], $A_{\frac{1}{2}} = -0.135 \pm 0.005$ GeV$^{-\frac{1}{2}}$ and $A_{\frac{3}{2}} = -0.250 \pm 0.008$ GeV$^{-\frac{1}{2}}$ [6], as inputs in the decay rate. The extracted magnetic moment comes out to be $\mu_{\Delta \to p} = 3.46 \pm 0.03$ μ_N. Our predicted value of 3.87 μ_N is very close to the experimental results. The difference in sign with some of the other models may be due to the different model predictions and signs of the wavefunctions. In the case of $\Sigma^{*+} \to \Sigma^+$, even though our results are almost half of the experimental value, except for a very few models all other models predict a value close to our results. In the case of the $\Sigma^{*0} \to \Lambda$ transitions our results are more or less in good agreement with the results of other models as well as with the experimental data.

To summarize, the chiral constituent quark model (χCQM) is able to describe the transition properties of the low lying baryons. In a very interesting manner, the χCQM) is able to phenomenologically estimate the explicit contributions from the valence quarks, sea quarks and their orbital angular momentum to the total magnetic moments of the transitions from the $J^P = \frac{3}{2}^+$ decuplet to $J^P = \frac{1}{2}^+$ octet baryons. The results immediately suggest that the contributions coming from various sources with same and opposite signs ultimately add up to give the total magnetic moment. Whenever the number of u, d or s quarks is more than the other quarks, there is a dominance of the orbital part, whereas when the u, d and s quarks are in equal numbers, there is some variation from this behavior. These observations endorse that the sea quarks and the orbital angular momentum of the sea quarks perhaps provide the dominant dynamics of the constituents in the low-energy regime of QCD. The qualitative and quantitative description of the results confirms that constituent quarks and weakly interacting Goldstone bosons provide the appropriate degree of freedom in the nonperturbative regime of QCD. A further precise measurement of these magnetic moments, therefore, would have important implications for the χCQM.

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