Strong constraints on Lorentz violation using new γ-ray observations around PeV*

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Abstract: The tiny modification of dispersion relation induced by Lorentz violation (LV) is an essential topic in quantum gravity (QG) theories, which can be magnified into significant effects when dealing with astrophysical observations at high energies and long propagation distances. LV would lead to photon decay at high energies; therefore, observations of high-energy photons could constrain LV or even QG theories. The Large High Altitude Air Shower Observatory (LHAASO) is the most sensitive gamma-array instrument currently operating above 100 TeV. Recently, LHAASO reported the detection of 12 sources above 100 TeV with maximum photon energy exceeding 1 PeV. According to these observations, the most stringent restriction is achieved in this study, i.e., limiting the LV energy scale to 1.7×10^{33} eV, which is over 139,000 times that of the Planck energy, and achieving an improvement of approximately 1.9 orders of magnitude over previous limits.

Keywords: Lorentz Violation, PeV photon, LHAASO

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I. INTRODUCTION

Three of the four fundamental interactions are described within the quantum field theory framework as elementary particles in the Standard Model (SM). The current understanding of the fourth interaction, gravity, is based on general relativity (GR), which is well-confirmed in the observation. However, GR cannot be reconciled with SM in extreme astrophysical objects, such as black holes [1]. A correct and consistent unification of the SM and GR is one of the holy grails of modern physics that has stimulated several theoretical ideas towards quantum gravity (QG), which describes gravity in the regimes where quantum effects cannot be disregarded.

Several approaches to QG provide fascinating perspectives on the structure of space-time, including string theory [2], space-time foam [3-5], non-commutative geometry [6, 7], and loop QG [8, 9]. A few of them predict Lorentz violation (LV) at high energies, where the Lorentz symmetry of space-time breaks down. Generally, these QG effects are assumed to be explicit, while they approximates to Planck energy, $E_{\rm QG}\sim E_{\rm Pl}\equiv\sqrt{\hbar c^5/G}\simeq 1.22\times10^{28}$ eV [10]. An enormous energy gap exists between $E_{\rm Pl}$ and the highest energy particles known, the trans-GZK cosmic rays in 10^{20} eV [11], which precludes any Earth-based experiment and direct observation of LV in $E_{\rm Pl}$.

Fortunately, LV, which occurs in $E_{\rm Pl}$, possibly introduces a few tiny "LV relics" at relatively lower energies [12]. In addition, these "relic" effects can demonstrate themselves and modify the energy-momentum relationship with additional terms suppressed by $E_{\rm Pl}$.

The modified dispersion relations (MDR) induced by LV is an essential topic in QG theories, mainly because it can be magnified into significant effects with high energy and long-baseline propagation. These effects provide a number of available scenarios to validate these theories using the "windows on QG" [12, 13]. Photons, which originated from remote energetic astrophysical objects, are substantially promising in probing the "windows on QG" in various aspects:

- Anomalous threshold reactions induced by LV terms (photon decay and vacuum Cherenkov radiation);

- Cumulative effects with long-baseline propagation (photon flight of time lags);

- LV induced decays not characterized by a threshold (photon splitting).

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In the 1990s, Amelino-Camelia suggested using gamma-ray bursts (GRBs) to verify LV effects and LV limit, $E_{\rm LV}$ [3, 4]. Based on the GRB observations, Fermi-LAT restricted $E_{\rm LV}^{(1)}$ to 9.3×10^{28} eV in 2013 [14]. Furthermore, in 2020, the High-Altitude Water Cherenkov (HAWC) Observatory determined the evidence of 100 TeV photon emissions from at least four astrophysical sources, which set a more stringent $E_{\rm LV}^{(1)}$ to 2.2×10^{31} eV [15].

Owing to its excellent energy resolution and background rejection power, the Large High Altitude Air Shower Observatory (LHAASO) is the most sensitive detector above few tens of TeV, which can help to detect LV signatures at ultra-high energies (UHEs). Even with only five-month data and half configuration, the achieved sensitivity of KM2A is significantly better than all previous observations at energies above 100 TeV [16]. Based on nearly one-year observations, LHAASO collaboration recently revealed significant number of PeVatrons in our Galaxy, reporting 12 gamma-ray sources over 100 TeV, with a maximum observational energy of 1.42±0.13 PeV [17].

Inspired by HAWC's previous work [15], we adopt spectra from three sources and the UHE single photon event reported by LHAASO collaboration to explore the anomalous behavior of photons induced by LV, which yields even more stringent limits to the LV energy scale $E_{\rm LV}$ than before.

Section II provides a brief introduction on the phenomenological description of LV, which is sufficient to elucidate our methods for restricting $E_{\rm LV}$ and independent of the particular form of Lorentz violating theory. Section III presents the $E_{\rm LV}$ derived from LHAASO's observations. The comparison between different methods are discussed in Section IV. Finally, we present our conclusion in Section V.

II. LORENTZ VIOLATION

A complete physical theory must include dynamics, and there have been several attempts to provide a comprehensive framework to compute interactions. Both SM and relativity can be considered as effective field theory (EFT). Hence, it is intuitive for EFT frameworks to embed LV effects using the LV operator [18, 19].

Although the dynamics of LV are poorly understood to date, we do not focus on the kinematics of LV. In most QG models, LV can be induced via an MDR that may describe photon behavior from high-energies or distant astrophysical objects. For a phenomenological uniform description in natural units ($c=\hbar=1$), we adopt the following MDR for photons:

$$E_\gamma^2 = p_\gamma^2\left[1+\xi_n\left(\frac{p_\gamma}{E_{\rm Pl}}\right)^n\right], \tag{1}$$

where $n=1$ (linear modification) and $n=2$ (quadratic modification) are relevant to current gamma-ray astronomy observations. Corresponding to the n-th order LV correction, ξ_n is a tiny dimensionless LV coefficient suppressed by $E_{\rm Pl}^{-n}$. Notably, $\xi_n>0$ represents superluminal photon propagation, while $\xi_n<0$ corresponds to subluminal propagation.

Subluminal photon propagation can be derived from radiative corrections triggered by any charged particle with non-zero LV operators with mass dimension of four [20]. Hence, we solely consider superluminal photon propagation to avoid lengthy discussion on radiative corrections. Before focusing on two interesting scenarios for superluminal propagation, a mathematical conversion $\alpha_n=\xi_n/E_{\rm Pl}^n$ is conducted by inducing α_n to interpret the n-th order $E_{\rm LV}^{(n)}$ more intuitively:

$$E_{\rm LV}^{(n)} = \alpha_n^{-1/n}. \tag{2}$$

A. Photon decay

The basic QED vertex in which single photon decays into electron and positron ($\gamma\to e^+e^-$) is kinematically forbidden by canonical energy-momentum conservation. Certainly, the matrix element and decay rate $\Gamma_{\gamma\to e^+e^-}$ can be calculated with the MDR. Once the photon decay process is allowed, the decay rate behaves like the energy E_γ above threshold of α_n with final momenta are non-parallel. Then, any photon that propagates over macroscopic distances must be below the energy threshold or function as a hard cutoff in gamma-ray spectra without any high-energy photons observed on Earth [21, 22]. Therefore, it will establish the α_n threshold for any order n when m_e and E_γ represent the electron mass and gamma-ray energy, respectively.

$$\alpha_n \leqslant \frac{4m_e^2}{E_\gamma^n(E_\gamma^2-4m_e^2)}. \tag{3}$$

Then, based on Eq. (2) and (3), Martínez-Huerta derived α_0, $E_{\rm LV}^{(1)}$, and $E_{\rm LV}^{(2)}$ with the LV generic approach [22]:

$$\alpha_0 \leqslant \frac{4m_e^2}{E_\gamma^2-4m_e^2}, \tag{4}$$

$$E_{\rm LV}^{(1)} \geqslant 9.57\times10^{23}\ {\rm eV}\left(\frac{E_\gamma}{\rm TeV}\right)^3, \tag{5}$$

$$E_{\rm LV}^{(2)} \geqslant 9.78\times 10^{17}\ {\rm eV}\left(\frac{E_\gamma}{\rm TeV}\right)^2. \tag{6}$$

B. Photon splitting

The process in which a photon splits into multiple photons ($\gamma \to N\gamma$) is facilitated by the energy-momentum conservation; however, it does not occur in standard QED because the matrix element and the phase space volume disappear. After introducing MDR, the matrix element and the phase space volume are non-zero, causing this non-threshold process to occur with a finite rate in the superluminal photon propagation case. However, this process can only provide limited contributions because the split into more final-state photons is suppressed by more powers of the fine-structure constant. Therefore, the widest channel with quadratic modification is the three-photon channel ($\gamma \to 3\gamma$) [23, 24]. Then, we have photon splitting decay rate $\Gamma_{\gamma\to 3\gamma}$ that is measured in the unit of energy [25-27].

$$\Gamma_{\gamma\to 3\gamma} = 5\times 10^{-14}\frac{E_\gamma^{19}}{m_e^8 E_{\rm LV}^{(2)\ 10}}. \tag{7}$$

In parallel with the optical depth concept, the survival probability $P(E_\gamma, L_{\rm obs}; E_{\rm LV})$ for photons after a traveling distance $L_{\rm obs}$ from astrophysical sources to Earth is governed by an exponential distribution.

$$P(E_\gamma, L_{\rm obs}; E_{\rm LV}) = {\rm e}^{-\Gamma_{\gamma\to 3\gamma}\times L_{\rm obs}}. \tag{8}$$

The free path of photon propagation λ will fall with the increase in E_γ, given $E_{\rm LV}^{(2)}$. The universe with the LV effect will become too opaque to observe gamma-rays traveling in the distance of $L_{\rm obs}$. Therefore, if we determine the evidence of spectra energy distribution (SED) cutoff from energetic astrophysical sources in $L_{\rm obs}$, we can infer that $\Gamma_{\gamma\to 3\gamma}\times L_{\rm obs} = 1$, where $L_{\rm obs}$ is the critical value of $L_{\rm LV}$. Then we can restrict $E_{{\rm LV}(3\gamma)}^{(2)}$:

$$E_{{\rm LV}(3\gamma)}^{(2)} \geqslant 3.33\times 10^{19}\ {\rm eV}\left(\frac{L_{\rm obs}}{\rm kpc}\right)^{0.1}\left(\frac{E_\gamma}{\rm TeV}\right)^{1.9}. \tag{9}$$

III. CONSTRAINTS ON LORENTZ VIOLATION

A. Constraints based on spectra cutoff

In this analysis, we used the information from three luminous sources reported by LHAASO (LHAASO J2226+6057, LHAASO J1908+0621, and LHAASO J1825-1326) in [17]. These spectra prefer log-parabolic fits. Although there is a steepening sign of spectrum with energy, there is no indication of an abrupt cutoff behavior. In addition, the energy resolution of the detector for gamma-ray events varies from different zenith angles. For showers with zenith angles less than 20°, the energy resolution is approximately 13% at 100 TeV[16]. So we performed χ^2-fitting to log-parabolic spectra for these sources convolved with a moderate estimation on energy resolution of 20%. The $\Delta\chi^2$ between the fitting with cutoff at $E_{\rm cut}$ and that without cutoff is calculated as

$$\Delta\chi^2 = \chi^2(E_{\rm cut}) - \chi^2(E_{\rm cut}\to\infty). \tag{10}$$

As the result of preference to non-cutoff behavior, the $\Delta\chi^2$ decreases with an increase in energy. Therefore, we proceed to set one-side lower limit on $E_{\rm cut}$, in which 95% CL corresponds to $\Delta\chi^2 = 2.71$. The achieved lower limits on $E_{\rm cut}$ for the three sources are presented in Table 1. The highest limit, 370.5 TeV, is from LHAASO J1908+0621.

In the photon decay case, the spectra of any astrophysical source would exhibit the cutoff behavior at the same energy, provided these sources are not limited by their acceleration mechanism; hence, we can accumulate the three sources into the combined statistic estimator to conduct the combined analysis. We also compared the best-fitting spectra before and after using the same cutoff presented in Fig. 1. The achieved 95% CL lower limit for $E_{\rm cut}$ is improved by approximately 30% to 483.3 TeV after the combined analysis. In the combined analysis, $\Delta\chi^2$ is a function of $E_{\rm cut}$, as presented in Fig. 2. In addition, the most stringent limits on $E_{\rm LV}$ are well above $E_{\rm Pl}$, which is up to 4.87×10^{31} eV, using a single source, and 1.08×10^{32} eV, using the combined analysis in linear modification. In the quadratic modification, $E_{\rm LV}^{(2)}$ values remain significantly below $E_{\rm Pl}$.

In the photon splitting case, the expected $E_{\rm cut}$ is distance dependent, but $E_{\rm LV}$ would remain a universal en-

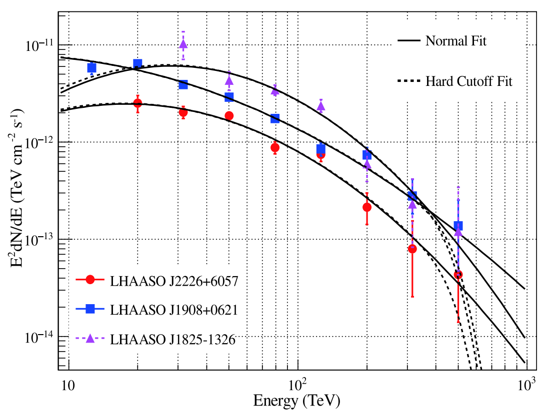

Fig. 1. (color online) Comparison of the normal best-fit spectra with those with a hard cutoff at $E_{\rm cut}$ = 483.3 TeV. The fitting lines have taken account of 20% energy resolution of LHAASO.

Table 1. LHAASO sources with the most conservative distance of the possible origin. We calculate the 95% CL lower limits for E_c and $E_{\rm LV}^{(n)}$ and upper limits for α_0.

Source	$E_{\rm cut}$/TeV	$L_{\rm obs}$/kpc	$\alpha_0(10^{-18})$	$E_{\rm LV}^{(1)}(10^{31}{\rm eV})$	$E_{\rm LV}^{(2)}(10^{22}{\rm eV})$	$E_{\rm LV(3\gamma)}^{(2)}(10^{24}{\rm eV})$
LHAASO J2226+6057	280.7	0.8[a]	13.26	2.11	7.71	1.46
LHAASO J1908+0621	370.5	2.37	7.61	4.87	13.43	2.76
LHAASO J1825-1326	169.9	1.55	36.18	0.47	2.82	0.60
Combined sources	483.3	–	4.47	10.80	22.84	4.37

[a]The distances of possible astrophysical objects associated with LHAASO J2226+6057 are provided in [28].

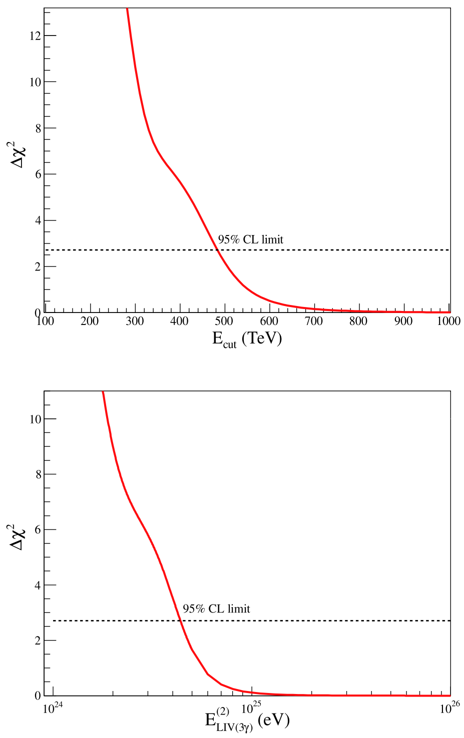

Fig. 2. (color online) In combined analysis, $\Delta\chi^2$ is a function of $E_{\rm cut}$ or $E_{\rm LIV(3\gamma)}^{(2)}$, and the dotted line indicates the 95% CL lower limit. Up: Photon decay scenario. Down: Photon splitting scenario.

ergy scale for different sources where the Lorentz symmetry breaks down. To conduct the combined analysis, we used the same $E_{\rm LV}$ rather than $E_{\rm cut}$ to introduce the three sources into the combined statistic estimator. In the combined analysis, $\Delta\chi^2$ is a function of $E_{\rm LV(3\gamma)}^{(2)}$, as illustrated in Fig. 2. The deduced lower limits on $E_{\rm LV}$ are presented in Table 1. We adopt the nearest distance among possible origins to set the most conservative $E_{\rm LV}$. In addition, the most stringent limits on $E_{\rm LV}$ are up to 2.76×10^{24} eV using a single source and 4.37×10^{24} eV using combined analysis in the quadratic modification. The results with quadratic modification are significantly blow $E_{\rm Pl}$. In fact, the conservative distance seems to underestimate the lower bound of $E_{\rm LV}$. Only when we confirm several galactic Pevatrons, can we improve $E_{\rm LV}$.

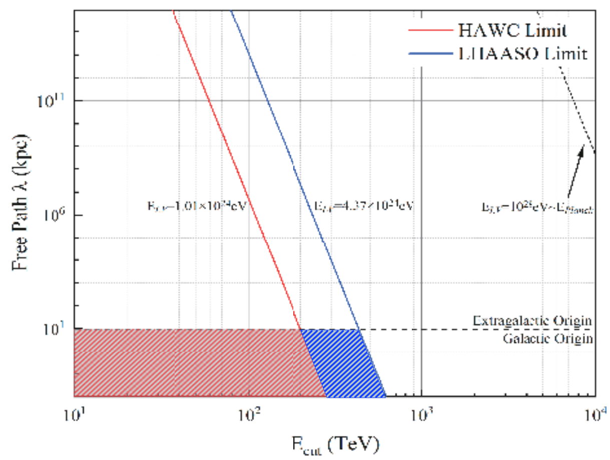

Fig. 3. (color online) In the photon splitting scenario, the relationship between the free path λ and spectra cutoff energy $E_{\rm cut}$ are plotted with solid lines, given different $E_{\rm LV}$s, which are set by HAWC, LHAASO, and $E_{\rm Pl}$.

However, it should be noted that the photon splitting induced by LV is a non-threshold process, as we mentioned in Sec. I and IIB. Hence, it is necessary to clarify that the photon splitting induced by LV derives "quasi-threshold" for the spectra cutoff behavior within galactic astrophysical sources on the kpc scale. To illustrate this phenomenon, we used Eq. (9) to calculate the relationship between the free path λ of photon propagation and spectra cutoff energy $E_{\rm cut}$ with the given $E_{\rm LV}$ values in Fig. 3. The free path λ shrinks exponentially with the increase in $E_{\rm cut}$ in the 19th power, such that the photon splitting could be considered a "quasi-threshold" scenario. Considering that TeV gamma-ray photons will interact with the intergalactic diffuse photon background, it is logical to choose galactic astrophysical sources to examine the photon splitting scenario. The characteristic size scale of the Milky Way is marked in a black dash line. In addition, the parameter space marked in the colored pattern is observed to set a limit to $E_{\rm LV}$ within the galactic distance in practice. Conversely, with an increase in $E_{\rm LV}$, the universe becomes more transparent for photons to travel. LHAASO achieved approximately 4 times the $E_{\rm LV}$ value of the previous result reported by the HAWC observatory based on spectra cutoff. Furthermore, the energy gap between $E_{\rm LV}$, set by LHAASO, and $E_{\rm Pl}$ remains to be tested in the future.

In addition, we quantified the effect on the fitting $E_{\rm cut}$ with different energy resolutions, because we have no access to the simulation information of LHAASO. In addition, $E_{\rm cut}$ increases with the increase in energy resolution. The combined analysis is affected the most; however, for the $E_{\rm cut}$ in 95% CL with 30% energy resolution, $E_{\rm cut}^{30\%}$ = 467.7 TeV. The uncertainty induced by the energy resolution is approximately 3%.

B. Constraint based on single event

While dealing with the phase space integration to calculate the photon decay rate, the condition that the electron or positron momentum should be real and positive provides a stringent limit to α_n [22]. According to Eq. (2), the photon decay rate $\Gamma_{\gamma\to e^+e^-}$ will grow with α_n in the fixed photon energy E_γ, final-state particles mass m_e, and the leading order of modification n, while α_n is above the threshold. If α_n is sufficiently large, $E_{\rm LV}$ will be approximately E_γ. In addition, the photo decay process will become substantialy efficient, such that the free path is limited to the millimeter scale. In other words, a tiny survival probability exists for the photon during the propagation to Earth, when $E_{\rm LV}$ is a few orders of magnitude higher than photon in this LV generic approach. If we determine the evidence of UHE photons, then $E_{\rm LV}$ will be pushed higher than E_γ.

In spectral analysis, the bin width is always larger than the energy resolution to ensure sufficient statistics, which is a trade off with energy information on a single event. In addition, the possibility of misidentifying a cosmic ray as a photo-like event is also not fully considered for spectral analysis. Owing to its excellent energy resolution and rejection power, LHAASO can distinguish the UHE photons among events.

Meanwhile, LHAASO determines the evidence of a PeV single gamma-ray event from LHAASO J2032+4102 in the Cygnus region. The non-rejection probability of a cosmic ray is estimated to be 0.028% in [17]. Owing to the decent energy resolution of LHAASO above 100 TeV, this event's energy uncertainty is limited within 1.42 ± 0.13 PeV. Hence, we can be confident that this UHE photon's lower energy bound is 1.21 PeV in 95% CL. The UHE single photon event can function as a counter example against low $E_{\rm LV}$ assumptions. Furthermore, the $E_{\rm LV}^{(n)}$ set by this event is provided in Table 2.

IV. DISCUSSION

According to our calculations, the most stringent constraint on $E_{\rm LV}$ via the UHE single photon event reported by LHAASO is up to 1.7×10^{33} eV, which is over 139,000 times that of $E_{\rm Pl}$. The limitation methods on $E_{\rm LV}$ have undergone significant developments since Amelino-Camelia first suggested the adoption of astrophysical photons to verify QG in 1998 [4].

Regarding high-energy photons, it is a direct way to restrict $E_{\rm LV}$ by measuring the arrive-time delay with long-baseline propagation; however, it is affected by the uncertain emitting time and region from different luminous sources.

Several regression methods are adopted to determine the flight of time delay under the assumption that all GRBs have the same emitting mechanism and similar emitting-time postpone. Owing to this long-baseline propagation, we have to consider the absorption of extragalactic background light (EBL) or cosmic microwave background (CMB), making it harder to detect GRBs' photons with energies above TeVs via satellite-borne observation. Hence, the LV constraints with the cumulative effects method may be confined to the energy range near $E_{\rm Pl}$, i.e., $E_{\rm LV}$ set by Fermi-LAT is 7.6 times greater than $E_{\rm Pl}$ [14].

Certainly, the methods based on non-cumulative effect can circumvent this dilemma. It is more reasonable to set $E_{\rm LV}$ by the photon's collective behavior, for instance, the cutoff behavior of SED. With the accumulation of high-energy events, SED will reach a higher energy range, leading to a more stringent constraint on $E_{\rm LV}$. However, SED would probably exhibit steepening or cutoffs at some energy, owing to the acceleration and radiation mechanisms. In addition, the cutoff behavior of SED may be induced by photon propagation effects or the absorption with EBL or CMB. Both of them will introduce uncertainty to $E_{\rm LV}$ deduced by SED cutoff. Hence, the LV constrains set by the SED cutoff behavior is likely to be restricted within the energy range, which is three or four orders of magnitude above $E_{\rm Pl}$, i.e, HAWC and LHAASO set the $E_{\rm LV}^{(1)}$ to 2.22×10^{31} eV and 1.08×10^{32} eV which are 1800 and 8850 times of $E_{\rm Pl}$, respectively.

In this aspect, we can continue to set more stringent LV restrictions by an UHE single event. The only penalty we should pay is the uncertainty for failure to reject a cosmic-ray event. Although this method is heavily groun-

Table 2. UHE single photon event originating from LHAASO J2032+4102. The energy of this event reaches 1.42 ± 0.13 PeV and 95% CL lower bound of $E_{\gamma,{\rm low}}^{95\%}$ = 1.21 PeV. The $L_{\rm obs}$ is 1.40 ± 0.08 and 95% lower bound of $L_{\rm obs,low}^{95\%}$ = 1.27 kpc.

	E_γ/PeV	$L_{\rm obs}$/kpc	$\alpha_0(10^{-19})$	$E_{\rm LV}^{(1)}(10^{33}{\rm eV})$	$E_{\rm LV}^{(2)}(10^{24}{\rm eV})$	$E_{{\rm LV}(3\gamma)}^{(2)}(10^{25}{\rm eV})$
UHE event	1.21	1.27[a]	7.13	1.70	1.43	2.45

[a]The distances of possible astrophysical objects associated with LHAASO J2032+4102 are provided in [29].

ded on the background rejection power and energy resolution, LHAASO-KM2A is capable of rejecting the cosmic ray background by a factor of 10^{-4} above 100 TeV and functions at the energy resolution of 13% above 100 TeV, according to [17]. In short, LHAASO has competence in restricting the $E_{\rm LV}$ via a single-photon event in the photon decay scenario and improves $E_{\rm LV}$ to over five orders of magnitude greater than $E_{\rm Pl}$.

V. CONCLUSION

In this study, we adopted the information from three sources above 100 TeV and an UHE single photon event from the Cygnus region to set limits on $E_{\rm LV}$. We considered two scenarios of photon decay and photon splitting, among which the first-order photon decay process provides the most stringent Lorentz limit, 1.7×10^{33} eV. The newly given limit is about 1.9 orders of magnitude over previous limits [15].

It is surprising that the half-configured LHAASO-KM2A provides high quality data in a nearly one-year operation. LHAASO-KM2A will stand out to restrict $E_{\rm LV}$ by harnessing the advantages of the UHE photons from sources, owing to its unprecedented capabilities of background rejection, high energy, and angular resolutions. Hence, in the future, LHAASO-KM2A is likely to function as a unique probe for LV physics, based on UHE single photon events.

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