Chinese Physics C Vol. 45, No. 5 (2021) 053103

Properties of the $P_c(4312)$ pentaquark and its bottom partner

K. Azizi[1†] Y. Sarac[2‡] H. Sundu[3§]

[1]Department of Physics, University of Tehran, North Karegar Ave., Tehran 14395-547, Iran and Department of Physics,
Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey
[2]Electrical and Electronics Engineering Department, Atilim University, 06836 Ankara, Turkey
[3]Department of Physics, Kocaeli University, 41380 Izmit, Turkey

Abstract: We present an analysis of the newly observed pentaquark $P_c(4312)^+$ to shed light on its quantum numbers. To do that, the QCD sum rules approach is used. The measured mass of this particle is close to the $\Sigma_c^{++}\bar{D}^-$ threshold and has a small width, which supports the possibility of its being a molecular state. We consider an interpolating current in a molecular form and analyze both the positive and negative parity states with spin-$1/2$. We also consider the bottom counterpart of the state with similar molecular form. Our mass result for the charm pentaquark state supports that the quantum numbers of the observed state are consistent with $J^P=1/2^-$.

Keywords: pentaquarks, QCD sum rules, $P_c(4312)$ sate

DOI: 10.1088/1674-1137/abe8ce

I. INTRODUCTION

Hadrons with non-conventional structures have been a subject of interest for many years. Since quantum chromodynamics (QCD) does not rule out their existence, they have been investigated extensively both theoretically and experimentally. Though their roots extend from before the proposal of QCD, the first experimental evidence for such states came in 2003 with the observation of the $X(3872)$ [1]. This observation has placed the exotic hadrons and their identification at the focus of interest. Since then, many exotic state candidates have been observed [1-10] and listed in the Review of Particle Physics (PDG) [11]. On the other hand, their internal structures are still not certain, and there are many studies devoted to the identification of their substructure. It seems that we will possibly witness more such exotic states in the future. Therefore, it is important to understand the nature and substructure of such observed states, as well as to provide information for possible future observations by offering candidate states.

The pentaquarks are among these exotic states. Their existence was controversial before 2015. In 2015 the LHCb collaboration investigated the $\Lambda_b\to J/\psi pK$ process and reported the observation of two candidates for pentaquark states, namely $P_c(4380)^+$ and $P_c(4450)^+$ [12], in the $J/\psi p$ invariant mass distribution. After the observation of $P_c(4380)^+$ and $P_c(4450)^+$, different approaches were considered to clarify their inner structures and quantum numbers. They were interpreted as diquark-diquark-antiquark or diquark-triquark states [13-22] and meson baryon molecules [23-28]. The observed peaks were discussed in Refs. [29-32] considering the possibility of their being kinematical effects corresponding to the triangle singularity. There were also investigations on the properties of other candidate pentaquark states with different quark contents, such as strange hidden charm pentaquark states [33-40], hidden bottom pentaquark states [41-44], and single or triple charmed pentaquark states [45].

In the LHCb collaboration's recent analysis, conducted by revisiting the $\Lambda_b^0\to J/\psi pK^-$ process with more accumulated data, the peak corresponding to the $P_c(4450)^+$ state was observed to split into two peaks. Besides these two structures, labeled $P_c(4440)^+$ and $P_c(4457)^+$, a new pentaquark state, $P_c(4312)^+$, was also reported. The masses and widths for the observed resonances were reported as [46]:

Received 29 December 2021; Accepted 22 February 2021; Published online 10 April 2021

[†] E-mail: kazem.azizi@ut.ac.ir
[‡] E-mail: yasemin.sarac@atilim.edu.tr
[§] E-mail: hayriye.sundu@kocaeli.edu.tr

$$\begin{aligned}
m_{P_c(4312)^+} &= 4311.9 \pm 0.7^{+6.8}_{-0.6}\ \text{MeV} \\
\Gamma_{P_c(4312)^+} &= 9.8 \pm 2.7^{+3.7}_{-4.5}\ \text{MeV}, \\
m_{P_c(4440)^+} &= 4440.3 \pm 1.3^{+4.1}_{-4.7}\ \text{MeV} \\
\Gamma_{P_c(4440)^+} &= 20.6 \pm 4.9^{+8.7}_{-10.1}\ \text{MeV}, \\
m_{P_c(4457)^+} &= 4457.3 \pm 0.6^{+4.1}_{-1.7}\ \text{MeV} \\
\Gamma_{P_c(4457)^+} &= 6.4 \pm 2.0^{+5.7}_{-1.9}\ \text{MeV}. \qquad (1)
\end{aligned}$$

This improvement has re-focused attention on these states. Complicated interactions between the participating quarks makes it difficult to differentiate their internal structures. Although there are various possible interpretations of their substructure, their closeness to the meson baryon threshold and small widths favor a molecular interpretation. Considering these, the previously reported state $P_c(4450)^+$ was investigated, taking into account the $\chi_{c1}p$ molecule [25], $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$ molecule [23, 24, 26, 27, 47], or admixture of $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Lambda_c$ [28] or $\bar{D}^*\Sigma_c$ and $\Lambda_c(2595)\bar{D}$ molecule [48, 49]. However, the recently released result, with analyses of more accumulated data, indicates that the peak corresponding to this state contains two separate peaks. Therefore, the suggested spin of 5/2 may differ, and these states may have molecular forms with smaller spins. Inspired by this, following the new announcement of their observation by LHCb [46], the newly observed states, $P_c(4440)^+$, $P_c(4457)^+$ and $P_c(4312)^+$, have been investigated via different methods. Based on their narrow widths and the proximity of their masses to the $\bar{D}^*\Sigma_c$, $D\Sigma_c$, and $\bar{D}\Sigma_c$ thresholds, they have been considered in various recent studies using the molecular interpretation. Among the studies that consider the possibility of molecular substructure is the one-boson-exchange model [50, 51]. In Ref. [50] the $P_c(4312)^+$, $P_c(440)^+$, and $P_c(4457)^+$ were interpreted as loosely bound states: $\Sigma_c\bar{D}$ with $J^P=1/2^-$ for $P_c(4312)^+$, and $\Sigma_c\bar{D}^*$ for $P_c(4440)^+$ and $P_c(4457)^+$ with $J^P=1/2^-$ and $3/2^-$, respectively. The molecular interpretation was also considered in the heavy hadron chiral perturbation theory [52]. With the quark delocalization current screening model [53], pentaquark systems with quark content $uudd\bar{s}$ were considered in the molecular interpretation. The QCD sum rules method was also used with the molecular pentaquark structure in Refs. [54-56]. Reference [54] interpreted the $P_c(4312)^+$ as a $\Sigma_c^{++}\bar{D}^-$ bound state with $J^P=1/2^-$, and the $P_c(4440)^+$ and $P_c(4457)^+$ as $\Sigma_c^+\bar{D}^0$ with $J^P=1/2^-$, $\Sigma_c^{*++}\bar{D}^-$ and $\Sigma_c^+\bar{D}^{*0}$ with $J^P=3/2^-$, or $\Sigma_c^{*+}\bar{D}^{*0}$ with $J^P=5/2^-$. The $P_c(4312)^+$ state was considered in Ref. [55] as $\Sigma_c\bar{D}$, and its mass range was found to be $4.07 \sim 4.97$ GeV. In Ref. [56], considering the pentaquark state as molecular led to the assignment of $\bar{D}\Sigma_c$ with $J^P=1/2^-$ for the $P_c(4312)^+$, and $\bar{D}^*\Sigma_c$ with $J^P=3/2^-$ or $\bar{D}^*\Sigma_c^*$ with $J^P=5/2^-$ for $P_c(4440/4457)^+$. Using the quasipotential Bethe-Salpeter equation approach [57], the two structures, $P_c(4440)^+$ and $P_c(4457)^+$, were assigned as $\Sigma_c\bar{D}^*$ state with $J^P=1/2^-$ and $3/2^-$, and the $P_c(4312)^+$ was assigned as the $\Sigma_c\bar{D}$ bound state with $J^P=1/2^-$. The spin-parity quantum numbers were predicted as $J^P=1/2^-$, $J^P=3/2^-$ and $J^P=1/2^-$ for $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$, respectively, in the quark delocalization color screening model [58]. Another molecular picture gave their quantum numbers as $J^P=1/2^-$, $J^P=1/2^-$ and $J^P=3/2^-$ for $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$, respectively [59]. They were also considered within the hadrocharmonium scenario [60], in which the $P_c(4440)^+$ and $P_c(4457)^+$ were interpreted as almost degenerate states with respective spin parities $J^P=1/2^-$ and $J^P=3/2^-$, while the $P_c(4312)^+$ was interpreted as $J^P=1/2^+$. In their identification, the diquark-diquark-antiquark framework was applied with different approaches [61-64]. In Ref. [61] the spins were stated as $J^P=1/2^-$ for $P_c(4312)^+$, $J^P=1/2^-$, $J^P=3/2^-$ or $J^P=5/2^-$ for $P_c(4440)^+$ and $J^P=1/2^-$ or $J^P=3/2^-$ for the $P_c(4457)^+$ state through the predicted masses using the QCD sum rules method. Reference [62] considered the spin-parity quantum numbers of the $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ states as $J^P=1/2^-$, $J^P=1/2^-$ and $J^P=3/2^-$, respectively. The suggested spin-parity quantum numbers in Refs. [63, 64] are $J^P=3/2^-$, $J^P=3/2^+$ and $J^P=5/2^+$ for $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$, respectively. These states were also investigated through their production and decay mechanisms, see for instance Refs. [65-70].

On the other hand, one cannot set aside the possible existence of b-partner states with quark substructures in which the charmed quark is replaced by the bottom one. Historically, observations of states with charmed quarks have generally been followed by the identification of their bottom counterparts. Therefore, the expectation of a similar possibility for pentaquarks is quite natural. With this expectation, experimental searches have also been conducted for these. For instance, the LHCb collaboration has searched for pentaquark states with a single bottom quark [71]. This also indicates that similar investigations may take place for hidden bottom pentaquarks in the future. This prospect has also attracted theoretical searches, and various features of pentaquarks with bottom quarks have been investigated [41-44, 72-74] to shed light on experimental searches. Pursuing this aim, the possible hidden bottom pentaquark states with $J^P=1/2^\pm$, $3/2^\pm$, and $5/2^\pm$ have been investigated within a chiral quark model [75]. In Ref. [76] a unitary coupled-channel model was used to study $B\Lambda_b - B\Sigma_b$ interactions, and the pole positions were predicted for hidden bottom pentaquark states with different spin-parities. In Refs. [77, 78] the masses and decay widths for hidden charm and hidden bottom pentaquarks with different spin parities were predicted within the quark delocalization color screening model. The resonance mass range of the $\Sigma_b B$

state was predicted as $11077.2 \sim 11079.8$ MeV [77] and it was obtained as 11070 MeV in Ref. [78].

As can be seen, for the pentaquark states, identifying their structures and quantum numbers are among the main issues. Despite many studies, there is no conclusive consensus on their substructure. Therefore, it is important to investigate their structure deeply through different approaches. These studies are also helpful to check and better understand the results of different works. In this work, we analyze the recently observed pentaquark state $P_c(4312)^+$ by investigating its mass via the QCD sum rules approach [79-81]. Additionally, we consider the bottom counterpart of this state. The QCD sum rules method has been widely used in the literature to gain information about the properties of hadrons, giving successful predictions quite consistent with the experimental ones. To this end, we apply interpolating currents for these states in the molecular form, and we consider both the negative and positive parity cases with spin-1/2 quantum numbers. The predictions obtained for the masses are compared with the experimental observations to conjecture the possible spin-parity quantum numbers of the $P_c(4312)^+$ state. We also obtain the current coupling constants, which can be adopted as inputs in searching the decay properties of the states considered. After the observation of $P_c(4380)^+$ and $P_c(4450)^+$ [12], in Ref. [28] these states were considered in a similar manner. Using molecular interpolating currents for the $P_c(4380)^+$ and $P_c(4450)^+$ states with spin-parity quantum numbers $J^P = 3/2^{\pm}$ and $J^P = 5/2^{\pm}$, respectively, they were investigated via the QCD sum rules method. The assigned possible spin-parity quantum numbers for the $P_c(4450)^+$ state, $J^P = 5/2^+$, with mass $m = 4.44 \pm 0.15$ GeV, is still consistent with the $P_c(4440)^+$ state after the report of the separation of $P_c(4450)^+$ into two peaks.

The article has the following structure. Section II contains the details of the QCD sum rules calculation for the aforementioned spectroscopic parameters. Section III gives the numerical analyses of the QCD sum rules results obtained analytically in the previous section. The last section includes a summary and a comparison of the obtained results with experimental and other theoretical predictions.

II. QCD SUM RULES CALCULATIONS

To obtain the mass and current coupling constant of the state of interest via the QCD sum rules method, the calculation starts with the following two-point correlation function:

$$\Pi(q) = \mathrm{i} \int \mathrm{d}x e^{\mathrm{i}qx} \langle 0|\mathcal{T}\{J(x)\bar{J}(0)\}|0\rangle, \tag{2}$$

where $\mathcal{T}$ denotes the time ordering operator. In the correlation function, $J(x)$ represents the interpolating field for the state, and it is formed considering the valence quark content and quantum numbers of the state using quark fields. The related interpolating current in this analysis has the following form:

$$J = [\epsilon^{abc}(u_a^T C\gamma_\mu u_b)\gamma^\mu\gamma_5 c_c][\bar{c}_d\gamma_5 d_d], \tag{3}$$

with u, d, and c being the respective quark fields, a, b and c subindices the color indices, and C the charge conjugation matrix. For the analyses of the bottom counterpart of the P_c pentaquark state, one can use the same interpolating current J given for the P_c pentaquark state in Eq. (3) by replacing the c quark fields with b quark fields.

To calculate the correlation function, there are two approaches. In the first approach, it is calculated in terms of hadronic degrees of freedom. The result from this approach emerges in terms of hadronic parameters such as mass and current coupling constant. Therefore, this approach is called the hadronic side or physical side of the calculation. The second approach in the calculation is the computation of the correlator in terms of the QCD degrees of freedom, which gives the result in terms of QCD degrees of freedom, such as quark-gluon condensates, quark masses, and QCD coupling constant. Therefore, this approach is called the QCD or theoretical side. The QCD sum rules giving the physical quantities are obtained by matching both sides of the calculations via a dispersion relation. To improve this match, suppressing the contributions of higher states and the continuum, a Borel transformation is applied to both sides. The results from both sides contain various Lorentz structures and the matching is performed considering the coefficient of the same Lorentz structure for both sides.

To calculate the physical side, one treats the interpolating current as the operator annihilating or creating the hadron, and inserts a complete set of the hadronic states in between the interpolating currents. The interpolating current couples both the negative and positive parity states. This application results in the following form of the correlator after the computation of the integral over four-x,

$$\Pi^{\mathrm{Had}}(q) = \frac{\langle 0|J|P_{c(b)}(q,s):\frac{1}{2}^{+}\rangle\langle P_{c(b)}(q,s):\frac{1}{2}^{+}|\bar{J}|0\rangle}{m_+^2 - q^2} + \frac{\langle 0|J|P_{c(b)}(q,s):\frac{1}{2}^{-}\rangle\langle P_{c(b)}(q,s):\frac{1}{2}^{-}|\bar{J}|0\rangle}{m_-^2 - q^2} + \cdots, \tag{4}$$

where $\cdots$ represents the contribution of higher states and the continuum. The matrix elements in the last results are represented in terms of the current coupling constants, λ, and Dirac spinor, $u(q,s)$, with spin s as

$$\langle 0|J|P_{c(b)}(q,s):\frac{1}{2}^{+}\rangle=\lambda_{+}\gamma_5 u(q,s),$$
$$\langle 0|J|P_{c(b)}(q,s):\frac{1}{2}^{-}\rangle=\lambda_{-}u(q,s). \tag{5}$$

The $|P_{c(b)}(q,s):1/2^{+}\rangle$ represents the one-particle P_c (P_b) state with positive parity and mass m_{+}, $|P_{c(b)}(q,s):1/2^{-}\rangle$ represents the state with opposite parity and mass m_{-}, and λ_{+}, λ_{-} are their corresponding current coupling constants. When Eq. (5) is used in Eq. (4) and the summation over spins is applied using

$$\sum_s u(q,s)\bar{u}(q,s)=\not{q}+m, \tag{6}$$

the hadronic side becomes

$$\Pi^{\rm Had}(q)=\frac{\lambda_{+}^2(\not{q}-m_{+})}{m_{+}^2-q^2}+\frac{\lambda_{-}^2(\not{q}+m_{-})}{m_{-}^2-q^2}+\cdots. \tag{7}$$

This result turns into

$$\tilde{\Pi}^{\rm Had}(q)=\lambda_{+}^2{\rm e}^{-\frac{m_{+}^2}{M^2}}(\not{q}-m_{+})+\lambda_{-}^2{\rm e}^{-\frac{m_{-}^2}{M^2}}(\not{q}+m_{-})+\cdots, \tag{8}$$

after the Borel transformation. The $\tilde{\Pi}^{\rm Had}(q)$ represents the Borel-transformed form of the correlation function, with M^2 being the Borel parameter. The result emerges with two Lorentz structures, namely $\not{q}$ and I. In the analyses, the coefficients of these structures for both hadronic and QCD sides are considered, and matching the coefficients of the same structures from both sides leads to the QCD sum rules for the considered physical quantities. Therefore the next step is the calculation of the coefficients of these structures from the QCD side.

The QCD side is computed using the interpolating current explicitly inside the correlation function, Eq. (2). The quark fields are contracted via Wick's theorem and this application renders the result into the one given in terms of heavy and light quark propagators as

$$\begin{aligned}\Pi^{\rm QCD}(q)=&{\rm i}\int{\rm d}^4x{\rm e}^{{\rm i}qx}\epsilon_{abc}\epsilon_{a'b'c'}\Big[-{\rm Tr}[S_u(x)^{aa'}\gamma_{\mu'}CS_u^{T\,bb'}(x)C\gamma_{\mu}]\\&+{\rm Tr}[S_u(x)^{ba'}\gamma_{\mu'}CS_u^{T\,ab'}(x)C\gamma_{\mu}]\Big]\\&\times{\rm Tr}[\gamma_5 S_d(x)^{dd'}\gamma_5 S_{c(b)}^{d'd}(-x)]\gamma_{\mu}\gamma_5 S_{c(b)}^{cc'}(x)\gamma_5\gamma_{\mu'}.\end{aligned} \tag{9}$$

The propagators represented as $S_{u(d)}$ and $S_{c(b)}$ are light and heavy quark propagators with the following explicit forms:

$$\begin{aligned}S_{q,ab}(x)=&{\rm i}\delta_{ab}\frac{\not{x}}{2\pi^2x^4}-\delta_{ab}\frac{m_q}{4\pi^2x^2}-\delta_{ab}\frac{\langle\bar{q}q\rangle}{12}\\&+{\rm i}\delta_{ab}\frac{\not{x}m_q\langle\bar{q}q\rangle}{48}-\delta_{ab}\frac{x^2}{192}\langle\bar{q}g_{\rm s}\sigma Gq\rangle\\&+{\rm i}\delta_{ab}\frac{x^2\not{x}m_q}{1152}\langle\bar{q}g_s\sigma Gq\rangle\\&-{\rm i}\frac{g_{\rm s}G_{ab}^{\alpha\beta}}{32\pi^2x^2}\left[\not{x}\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not{x}\right]-{\rm i}\delta_{ab}\frac{x^2\not{x}g_s^2\langle\bar{q}q\rangle^2}{7776},\end{aligned} \tag{10}$$

and

$$\begin{aligned}S_{Q,ab}(x)=&\frac{\rm i}{(2\pi)^4}\int{\rm d}^4k{\rm e}^{-{\rm i}k\cdot x}\Bigg\{\frac{\delta_{ab}}{\not{k}-m_Q}\\&-\frac{g_sG_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma_{\alpha\beta}}{(k^2-m_Q^2)^2}\\&+\frac{\pi^2}{3}\langle\frac{\alpha_sGG}{\pi}\rangle\delta_{ij}m_Q\frac{k^2+m_Q\not{k}}{(k^2-m_Q^2)^4}+\cdots\Bigg\},\end{aligned} \tag{11}$$

where q is used for u and d quarks and Q represents c or b quarks, $G_{ab}^{\alpha\beta}=G_A^{\alpha\beta}t_{ab}^A$ and $GG=G_A^{\alpha\beta}G_A^{\alpha\beta}$ with a, $b=1,2,3$. Here $A=1,2,\cdots,8$ and $t^A=\dfrac{\lambda^A}{2}$ where λ^A is the Gell-Mann matrices. The propagators are used in Eq. (9) explicitly and then Fourier and Borel transformations are applied. These steps result in

$$\tilde{\Pi}_i^{\rm QCD}(s_0,M^2)=\int_{(2m_Q+2m_u+m_d)^2}^{s_0}{\rm d}s{\rm e}^{-\frac{s}{M^2}}\rho_i(s)+\Gamma_i(M^2), \tag{12}$$

where $\rho_i(s)$ is the spectral density obtained from the imaginary part of the result as $\rho(s)=\frac{1}{\pi}{\rm Im}\Pi_i^{\rm QCD}$ and sub-index i represents either of the Lorentz structures, $\not{q}$ and I. Since $\rho_i(s)$ and $\Gamma_i(M^2)$ are lengthy functions, we will not present their explicit forms here to avoid overwhelming mathematical representations, and concentrate on the results obtained from their analyses, which will be presented in the next section. In Eq. (12), s_0 is the threshold parameter that arises after the application of the continuum subtraction using the quark-hadron duality assumption, and M^2 is the Borel parameter. Matching the coefficients of the same Lorentz structures from the hadronic and QCD sides leads us to the following relations:

$$\lambda_{+}^2{\rm e}^{-\frac{m_{+}^2}{M^2}}+\lambda_{-}^2{\rm e}^{-\frac{m_{-}^2}{M^2}}=\tilde{\Pi}_{\not{q}}^{\rm QCD}(s_0,M^2), \tag{13}$$

and

$$-\lambda_{+}^2m_{+}{\rm e}^{-\frac{m_{+}^2}{M^2}}+\lambda_{-}^2m_{-}{\rm e}^{-\frac{m_{-}^2}{M^2}}=\tilde{\Pi}_{I}^{\rm QCD}(s_0,M^2). \tag{14}$$

In the calculations, we take into account both the positive and negative parity states. To obtain the related sum rules, Eqs. (13) and (14) and their derivatives with respect to $(-\frac{1}{M^2})$ are considered, and these four coupled equations are solved for the four unknown quantities, namely, m_+, m_-, λ_+ and λ_-. These last results are used for analyzing either the P_c or P_b pentaquarks.

Here, we should note that a similar current is used in Refs. [54, 82] to extract the masses of P_c states. In the present study, unlike these works, we simultaneously include both the negative and positive parity states that couple to the selected interpolating current, as we mentioned previously. We extract the spectroscopic properties of both parities by simultaneous solving the sum rules obtained above. The obtained mass results are then compared to the experimental value to fix the quantum numbers of the observed state, P_c(4312). In the present study, we also extract the values of the current couplings for both parities, which may be used as main inputs in investigation of the electromagnetic, weak or strong decays of these states. We also obtain the parameters of b-partner states that may help experimental groups in searching for P_b states. In what follows we shall explain the numerical analyses of the results obtained from the QCD sum rules calculations.

III. NUMERICAL ANALYSES

In this section the results obtained in Section II are applied to obtain the numerical values of the spectral properties of the candidate P_c(4312) and its opposite parity state. We also present the analyses for the bottom counterparts of these states. The analytic results contain some input parameters and auxiliary parameters. Some of the input parameters used in the calculations are gathered in Table 1.

Table 1. Some input parameters used in the analyses.

Parameters	Values
m_c	1.27 ± 0.02 GeV [11]
m_b	$4.18^{+0.03}_{-0.02}$ GeV [11]
m_u	$2.16^{+0.49}_{-0.26}$ MeV [11]
m_d	$4.67^{+0.48}_{-0.17}$ MeV [11]
$\langle\bar{q}q\rangle(1\text{ GeV})$	$(-0.24\pm0.01)^3$ GeV^3 [83]
m_0^2	(0.8 ± 0.1) GeV^2 [83]
$\langle\frac{\alpha_s}{\pi}G^2\rangle$	(0.012 ± 0.004) GeV^4 [84]

In addition to these input parameters, we need two auxiliary parameters in the analyses. These are the Borel parameter, M^2, and the threshold parameter, s_0. These two parameters are determined by considering the analyses of the sum rules, imposing some standard criteria. These criteria include the mild dependence of the results on these auxiliary parameters, the dominance of the contributions of the states in question to the higher states and continuum, and convergence of the operator product expansion (OPE). The working intervals of the auxiliary parameters are those for which these criteria are satisfied. To deduce M^2, the contribution of higher ordered terms on the QCD side should be sufficiently small, and the contribution of the lower states should dominate that of the higher states. To determine the lower limit of M^2, we take into account the region where the contribution of the higher dimensional term in the OPE is less than ~ 1%. For the upper limit of M^2, we consider the ratio of the pole term to the total as PC, that is:

$$PC(M^2)=\frac{\tilde{\Pi}_i^{\rm QCD}(s_0,M^2)}{\tilde{\Pi}_i^{\rm QCD}(\infty,M^2)}, \tag{15}$$

and in our analysis, we require this ratio to be larger than ~ 27%. Sticking to these criteria, we fix the interval for the Borel parameters as

$$5.5\ {\rm GeV}^2\leqslant M^2\leqslant 6.5\ {\rm GeV}^2, \tag{16}$$

for P_c states and

$$11.5\ {\rm GeV}^2\leqslant M^2\leqslant 13.5\ {\rm GeV}^2, \tag{17}$$

for P_b states. For the determination of the threshold parameter, we consider the relatively weak dependence of the results on this parameter. Accordingly, its working intervals for the analyses are set as

$$20.5\ {\rm GeV}^2\leqslant s_0\leqslant 22.5\ {\rm GeV}^2 \tag{18}$$

for P_c states and

$$132.0\ {\rm GeV}^2\leqslant s_0\leqslant 136.0\ {\rm GeV}^2 \tag{19}$$

for P_b states.

Having determined the auxiliary parameters, the masses and the current coupling constants can be determined using their intervals and the input parameters given in Table 1. The obtained results are as follows:

$$\begin{aligned} m_- &=4322\pm342\ {\rm MeV},\\ \lambda_- &=(0.24\pm0.09)\times10^{-3}\ {\rm GeV}^5, \end{aligned} \tag{20}$$

$$\begin{aligned} m_+ &=4776\pm380\ {\rm MeV},\\ \lambda_+ &=(0.38\pm0.12)\times10^{-3}\ {\rm GeV}^5, \end{aligned} \tag{21}$$

for the pentaquark states containing charm quarks, and

$$\begin{aligned} m_- &=11087\pm73\ {\rm MeV},\\ \lambda_- &=(0.11\pm0.03)\times10^{-3}\ {\rm GeV}^5, \end{aligned} \tag{22}$$

$$\begin{aligned} m_+ &= 11105 \pm 78\ \text{MeV}, \\ \lambda_+ &= (0.18 \pm 0.05) \times 10^{-3}\ \text{GeV}^5, \end{aligned} \tag{23}$$

for the pentaquark states containing bottom quarks.

The results contain the errors arising from the uncertainty carried by the determinations of the auxiliary parameters and other input parameters used in the analyses. The order of magnitude of errors in the b-channel is considerably smaller than the charmed case, and the results in the b-channel are more stable with respect to the changes in auxiliary parameters. Our results for the parameters of both the parities in both the c and b channels may be checked via different approaches.

IV. DISCUSSION AND CONCLUSIONS

The recent report of the LHCb collaboration for the pentaquark states included a new pentaquark state $P_c(4312)^+$ and a split in the peak of the previously observed $P_c(4450)^+$, which are named $P_c(4440)^+$ and $P_c(4457)^+$. There are different interpretations of their substructure and quantum numbers, which need to be elucidated with further investigations. Among the different probable options, one possible structure favors their having a molecular form. With this motivation, this work is devoted to the analysis of the $P_c(4312)^+$ through the QCD sum rules method and predictions for its spectroscopic parameters. To this end, an interpolating current was chosen in the $\Sigma_c^{++}\bar{D}^-$ molecular form with negative parity, and the opposite parity state was also studied.

The ambiguity in their quantum numbers and substructures put these pentaquark states at the focus of interest. Therefore these states are investigated via alternative models. Table 2 presents predictions for the masses and possible J^P quantum numbers obtained by some of these models, as well as the experimental observations. This table also contains our result from this study for $P_c(4312)^+$, and our results for its possible bottom counterpart.

The central value of our mass prediction, $m_- = 4322 \pm 342$, obtained for the negative parity spin-1/2 state, is consistent with the experimental result for $P_c(4312)^+$ and the predictions of different theoretical studies for this particle, which suggests the possible spin-parity of this particle being $J^P = 1/2^-$. Our mass prediction for the opposite parity state with the same spin is $m_+ = 4776 \pm 380$ MeV. However, this mass value is higher than the reported masses of the $P_c(4312)^+$ and the other observed pentaquark states.

As is seen from Table 2, we also considered the bottom counterpart of the $P_c(4312)$ state with a molecular form $\Sigma_b^+\bar{B}^0$, labeled P_b in the table. The mass predictions for the negative and positive parity cases are presented as $m_- = 11087 \pm 73$ MeV and $m_+ = 11105 \pm 78$ MeV, respectively. The prediction for the negative parity case is compared with the other predictions available in the literature, and it is seen to be consistent with their results within the errors.

In Ref. [28], we considered the $P_c(4380)^+$ and $P_c(4450)^+$ states in the molecular form with $J^P = 3/2^\pm$ and $J^P = 5/2^\pm$, respectively, and made mass calculations for both the positive and negative parity cases. The obtained results are shown in Table 3 for comparison with the results of the cases with $J^P = 1/2^\pm$. In the calculations made in Ref. [28], we considered a molecular current for $P_c(4380)^+$ and a mixed interpolating current in the molecular form, which is an admixture of $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Lambda_c$ for $P_c(4450)^+$. In light of the new experimental report indicating the split in the peak of $P_c(4450)^+$ state,

Table 2. Mass and possible quantum numbers predicted for the $P_c(4312)^+$ resonance and its bottom partner from different studies.

Resonance		J^P	This study (MeV)	[54] (GeV)	[58, 77] (MeV)	[64] (MeV)	[59] (MeV)	[61] (GeV)	Experiment (MeV)
$P_c(4312)$	m	$1/2^-$	4322 ± 342	$4.33^{+0.17}_{-0.33}$	$4306.7 \sim 4311.3$		4306.4	4.31 ± 0.11 4.34 ± 0.14	
		$1/2^+$	4776 ± 380						$4311.9 \pm 0.7^{+6.8}_{-0.6}$
		$3/2^-$				4240			
Resonance		J^P	**This study (MeV)**	[77] (MeV)	[78] (MeV)	[75] (MeV)			
P_b	m	$1/2^-$	11087 ± 73	$11077.2 \sim 11079.8$	11070	$11072 - 11074$			
		$1/2^+$	11105 ± 78						

Table 3. Mass predictions for hidden-charm pentaquark states with different quantum numbers, from this study and Ref. [28].

State	This study		[28]			
	P_c		P_c		P_c	
J^P	$1/2^-$	$1/2^+$	$3/2^-$	$3/2^+$	$5/2^-$	$5/2^+$
Mass(MeV)	4322 ± 342	4776 ± 380	4300 ± 100	4240 ± 160	4200 ± 150	4440 ± 150

our final remark for the state $P_c(4450)^+$ in our previous work in Ref. [28] still supports one of these new states, the state $P_c(4440)$, to possibly have spin-parity quantum numbers $J^P = 5/2^+$. As is seen from Table 3, compared with the experimental value, the mass of the state with $J^P = 5/2^+$ is consistent with the experimental result, $m_{P_c(4440)^+} = 4440.3 \pm 1.3^{+4.1}_{-4.7}$ MeV [46], and supports the molecular structure for this state.

The investigations of pentaquark states support different possibilities for their substructures, leaving their structures still ambiguous. Therefore, to determine their substructure, we need further theoretical and experimental investigations. The results obtained for the masses and current coupling constants of both the negative and positive parity hidden-charmed/hidden-bottom states in the present study may supply inputs for these further investigations.

References

[1] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003), arXiv:hep-ex/0309032

[2] Z. Q. Liu *et al.* (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013), arXiv:1304.0121[hep-ex]

[3] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. D **74**, 091103 (2006), arXiv:hep-ex/0610018

[4] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **100**, 102003 (2008), arXiv:0712.1143[hep-ex]

[5] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013), arXiv:1303.5949[hep-ex]

[6] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **112**(9), 092001 (2014), arXiv:1310.4101[hep-ex]

[7] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **111**(24), 242001 (2013), arXiv:1309.1896[hep-ex]

[8] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **114**(9), 092003 (2015), arXiv:1410.6538[hep-ex]

[9] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **93**(1), 011102 (2016), arXiv:1511.08564[hep-ex]

[10] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **118**(9), 092002 (2017), arXiv:1610.07044[hep-ex]

[11] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**(3), 030001 (2018)

[12] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **115**, 072001 (2015), arXiv:1507.03414[hep-ex]

[13] R. F. Lebed, Phys. Lett. B **749**, 454 (2015), arXiv:1507.05867[hep-ph]

[14] G. N. Li, X. G. He, and M. He, JHEP **12**, 128 (2015), arXiv:1507.08252[hep-ph]

[15] L. Maiani, A. D. Polosa, and V. Riquer, Phys. Lett. B **749**, 289 (2015), arXiv:1507.04980[hep-ph]

[16] V. V. Anisovich, M. A. Matveev, J. Nyiri, A. V. Sarantsev and A. N. Semenova, arXiv: 1507.07652[hep-ph].

[17] Z. G. Wang and T. Huang, Eur. Phys. J. C **76**(1), 43 (2016), arXiv:1508.04189[hep-ph]

[18] Z. G. Wang, Eur. Phys. J. C **76**(2), 70 (2016), arXiv:1508.01468[hep-ph]

[19] Z. G. Wang, Nucl. Phys. B **913**, 163-208 (2016), arXiv:1512.04763[hep-ph]

[20] R. Ghosh, A. Bhattacharya, and B. Chakrabarti, Phys. Part. Nucl. Lett. **14**(4), 550-552 (2017), arXiv:1508.00356[hep-ph]

[21] R. Zhu and C. F. Qiao, Phys. Lett. B **756**, 259 (2016), arXiv:1510.08693[hep-ph]

[22] G. J. Wang, R. Chen, L. Ma *et al.*, Phys. Rev. D **94**(9), 094018 (2016), arXiv:1605.01337[hep-ph]

[23] H. X. Chen, W. Chen, X. Liu *et al.*, Phys. Rev. Lett. **115**(17), 172001 (2015), arXiv:1507.03717[hep-ph]

[24] R. Chen, X. Liu, X. Q. Li, and S. L. Zhu, Phys. Rev. Lett. **115**(13), 132002 (2015), arXiv:1507.03704[hep-ph]

[25] U. G. Meißner and J. A. Oller, Phys. Lett. B **751**, 59 (2015), arXiv:1507.07478[hep-ph]

[26] J. He, Phys. Lett. B **753**, 547 (2016), arXiv:1507.05200[hep-ph]

[27] L. Roca, J. Nieves and E. Oset, Phys. Rev. D **92**(9), 094003 (2015), arXiv:1507.04249[hep-ph]

[28] K. Azizi, Y. Sarac and H. Sundu, Phys. Rev. D **95**(9), 094016 (2017), arXiv:1612.07479[hep-ph]

[29] F. K. Guo, U. G. Meißner, W. Wang, and Z. Yang, Phys. Rev. D **92**(7), 071502 (2015), arXiv:1507.04950[hep-ph]

[30] F. K. Guo, U. G. Meißner, J. Nieve,s and Z. Yang, Eur. Phys. J. A **52**(10), 318 (2016), arXiv:1605.05113[hep-ph]

[31] M. Mikhasenko, arXiv: 1507.06552[hep-ph]

[32] X. H. Liu, Q. Wang, and Q. Zhao, Phys. Lett. B **757**, 231-236 (2016), arXiv:1507.05359[hep-ph]

[33] X. Liu, H. Huang and J. Ping, Phys. Rev. C **102**(2), 025204 (2020), arXiv:2005.09646[hep-ph]

[34] H. X. Chen, L. S. Geng, W. H. Liang *et al.*, Phys. Rev. C **93**(6), 065203 (2016), arXiv:1510.01803[hep-ph]

[35] A. Feijoo, V. K. Magas, A. Ramos, and E. Oset, Eur. Phys. J. C **76**(8), 446 (2016), arXiv:1512.08152[hep-ph]

[36] J. X. Lu, E. Wang, J. J. Xie *et al.*, Phys. Rev. D **93**, 094009 (2016), arXiv:1601.00075[hep-ph]

[37] Y. Irie, M. Oka, and S. Yasui, Phys. Rev. D **97**(3), 034006 (2018), arXiv:1707.04544[hep-ph]

[38] R. Chen, J. He, and X. Liu, Chin. Phys. C **41**(10), 103105 (2017), arXiv:1609.03235[hep-ph]

[39] Q. Zhang, B. R. He, and J. L. Ping, arXiv: 2006.01042[hep-ph]

[40] A. Pimikov, H. J. Lee, and P. Zhang, Phys. Rev. D **101**(1), 014002 (2020)

[41] E. Y. Paryev, arXiv: 2007.01172[nucl-th]

[42] T. Gutsche and V. E. Lyubovitskij, Phys. Rev. D **100**(9), 094031 (2019), arXiv:1910.03984[hep-ph]

[43] K. Azizi, Y. Sarac, and H. Sundu, Phys. Rev. D **96**(9), 094030 (2017), arXiv:1707.01248[hep-ph]

[44] X. Cao, F. K. Guo, Y. T. Liang *et al.*, Phys. Rev. D **101**(7), 074010 (2020), arXiv:1912.12054[hep-ph]

[45] K. Azizi, Y. Sarac, and H. Sundu, Phys. Rev. D **98**(5), 054002 (2018), arXiv:1805.06734[hep-ph]

[46] R. Aaij et al. (LHCb), Phys. Rev. Lett. **122**, no.22, 222001 (2019)[arXiv: 1904.03947[hep-ex]].

[47] C. W. Xiao and U.-G. Meißner, Phys. Rev. D **92**(11), 114002 (2015), arXiv:1508.00924[hep-ph]

[48] L. Geng, J. Lu, and M. P. Valderrama, Phys. Rev. D **97**(9), 094036 (2018), arXiv:1704.06123[hep-ph]

[49] T. J. Burns, Eur. Phys. J. A **51**(11), 152 (2015), arXiv:1509.02460[hep-ph]

[50] R. Chen, Z. F. Sun, X. Liu *et al.*, Phys. Rev. D **100**(1),

011502 (2019), arXiv:1903.11013[hep-ph]

[51] F. L. Wang, R. Chen, Z. W. Liu *et al.*, Phys. Rev. C **101**(2), 025201 (2020), arXiv:1905.03636[hep-ph]

[52] L. Meng, B. Wang, G. J. Wang *et al.*, Phys. Rev. D **100**(1), 014031 (2019), arXiv:1905.04113[hep-ph]

[53] X. Liu, H. Huang, and J. Ping, Phys. Rev. C **100**(2), 025203 (2019), arXiv:1905.00583[hep-ph]

[54] H. X. Chen, W. Chen, and S. L. Zhu, Phys. Rev. D **100**(5), 051501 (2019), arXiv:1903.11001[hep-ph]

[55] J. R. Zhang, Eur. Phys. J. C **79**(12), 1001 (2019), arXiv:1904.10711[hep-ph]

[56] Z. G. Wang, Int. J. Mod. Phys. A **34**(19), 1950097 (2019), arXiv:1806.10384[hep-ph]

[57] J. He, Eur. Phys. J. C **79**(5), 393 (2019), arXiv:1903.11872[hep-ph]

[58] H. Huang, J. He, and J. Ping, arXiv: 1904.00221[hep-ph]

[59] C. W. Xiao, J. Nieves, and E. Oset, Phys. Rev. D **100**(1), 014021 (2019), arXiv:1904.01296[hep-ph]

[60] M. I. Eides, V. Y. Petrov, and M. V. Polyakov, Mod. Phys. Lett. A **35**(18), 2050151 (2020), arXiv:1904.11616[hep-ph]

[61] Z. G. Wang, Int. J. Mod. Phys. A **35**(01), 2050003 (2020), arXiv:1905.02892[hep-ph]

[62] A. N. Semenova, V. V. Anisovich, and A. V. Sarantsev, Eur. Phys. J. A **56**(5), 142 (2020), arXiv:1911.11994[hep-ph]

[63] A. Ali, I. Ahmed, M. J. Aslam *et al.*, JHEP **10**, 256 (2019), arXiv:1907.06507[hep-ph]

[64] A. Ali and A. Y. Parkhomenko, Phys. Lett. B **793**, 365-371 (2019), arXiv:1904.00446[hep-ph]

[65] K. Azizi, Y. Sarac, and H. Sundu, Phys. Lett. B **782**, 694-701 (2018), arXiv:1802.01384[hep-ph]

[66] Z. G. Wang and X. Wang, Chin. Phys. C **44**, 103102 (2020), arXiv:1907.04582[hep-ph]

[67] Z. G. Wang, H. J. Wang, and Q. Xin, arXiv: 2005.00535[hep-ph]

[68] Y. J. Xu, C. Y. Cui, Y. L. Liu and M. Q. Huang, arXiv: 1907.05097 [hep-ph]

[69] F. K. Guo, H. J. Jing, U. G. Meiner *et al.*, Phys. Rev. D **99**(9), 091501 (2019), arXiv:1903.11503[hep-ph]

[70] C. J. Xiao, Y. Huang, Y. B. Dong *et al.*, Phys. Rev. D **100**(1), 014022 (2019), arXiv:1904.00872[hep-ph]

[71] R. Aaij et al. (LHCb), Phys. Rev. D **97**(3), 032010 (2018) arXiv: 1712.08086 [hep-ex]

[72] Y. Shimizu, D. Suenaga, and M. Harada, Phys. Rev. D **93**(11), 114003 (2016), arXiv:1603.02376[hep-ph]

[73] Y. H. Lin, C. W. Shen, and B. S. Zou, Nucl. Phys. A **980**, 21-31 (2018), arXiv:1805.06843[hep-ph]

[74] F. Z. Peng, M. Z. Liu, Y. W. Pan, M. Sánchez Sánchez and M. Pavon Valderrama, arXiv: 1907.05322 [hep-ph]

[75] G. Yang, J. Ping, and J. Segovia, Phys. Rev. D **99**(1), 014035 (2019), arXiv:1809.06193[hep-ph]

[76] C. W. Shen, D. Rönchen, U. G. Meißner *et al.*, Chin. Phys. C **42**(2), 023106 (2018), arXiv:1710.03885[hep-ph]

[77] H. Huang and J. Ping, Phys. Rev. D **99**(1), 014010 (2019), arXiv:1811.04260[hep-ph]

[78] H. Huang, C. Deng, J. Ping, and F. Wang, Eur. Phys. J. C **76**(11), 624 (2016), arXiv:1510.04648[hep-ph]

[79] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979)

[80] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 448 (1979)

[81] B. L. Ioffe, Nucl. Phys. B **188**, 317 (1981) Erratum: [Nucl. Phys. B **191**, 591 (1981)]

[82] H. X. Chen, E. L. Cui, W. Chen *et al.*, Eur. Phys. J. C **76**(10), 572 (2016)

[83] V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP **56**, 493-501 (1982) [Zh. Eksp. Teor. Fiz. **83**, 876 (1982)]

[84] V. M. Belyaev and B. L. Ioffe, Sov. Phys. JETP **57**, 716 (1983) [Zh. Eksp. Teor. Fiz. **84**, 1236 (1983)]