

Scalar leptoquark and vector-like quark extended models as the explanation of the muon $g-2$ anomaly: bottom partner chiral enhancement case*

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Abstract: Leptoquark (LQ) models are well motivated solutions to the $(g-2)_\mu$ anomaly. In the minimal LQ models, only specific representations can lead to chiral enhancements. For the scalar LQs, R_2 and S_1 can lead to the top quark chiral enhancement. For the vector LQs, V_2 and U_1 can lead to the bottom quark chiral enhancement. When we consider the LQ and vector-like quark (VLQ) simultaneously, there can be more scenarios. In our previous study, we considered the scalar LQ and VLQ extended models with up-type quark chiral enhancement. Here, we study the scalar LQ and VLQ extended models with down-type quark chiral enhancement. We find two new models with B quark chiral enhancements, which originate from the bottom and bottom partner mixing. Then, we propose new LQ and VLQ search channels under the constraints of $(g-2)_\mu$.

Keywords: muon $g-2$, scalar leptoquark, vector-like quark

DOI: 10.1088/1674-1137/acce1d

I. INTRODUCTION

The $(g-2)_\mu$ anomaly is a longstanding puzzle in the standard model (SM) of elementary particle physics. It was first announced by the BNL E821 experiment [1]. Last year, the FNAL muon $g-2$ experiment revealed increased deviation from the SM prediction [2]. When combining the BNL and FNAL data, the averaged result is $a_\mu^{\text{Exp}} = 116592061(41) \times 10^{-11}$. Compared to the SM prediction $a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$ [3–23], the deviation is $\Delta a_\mu \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$, which shows a 4.2σ discrepancy. Many new physics models were proposed to explain the anomaly [24–29].

For the mediators with mass above the TeV level, the chiral enhancements are required, which can appear when left-handed and right-handed muons couple to a heavy fermion simultaneously. In the new lepton extended models [30–33], the chiral enhancements originate from the large lepton mass. The LQ models are an alternative choice [34–40], in which the chiral enhancements originate from the large quark mass. For the minimal LQ models, there are scalar LQs R_2/S_1 with top quark chiral enhancement and vector LQs V_2/U_1 with bottom quark chiral enhancement. The LQ can connect the lepton sec-

tor and quark sector. On the other hand, the VLQ naturally occurs in many new physics models and is free of quantum anomaly. It can mix with SM quarks and provide new source of CP violation. Hence, the LQ and VLQ extended models can lead to interesting flavour physics in both the lepton sector and quark sector. In our previous study [41], we investigated the scalar LQ and VLQ¹⁾ extended models with top and top partner chiral enhancements. In this study, we investigate the scalar LQ and VLQ extended models, which can produce the bottom partner chiral enhancements. This paper is complementary to our previous paper [41]. Moreover, the top partner and bottom partner lead to different collider signatures.

In Sec. II, we introduce the models and show the related interactions. Then, we derive the new physics contributions to $(g-2)_\mu$ and perform the numerical analysis in Sec. III. In Sec. IV, we discuss the possible collider phenomenology. Finally, we present a summary and conclusions in Sec. V.

II. MODEL SETUP

Typically, there are six types of scalar LQs [35],

Received 14 February 2023; Accepted 11 April 2023; Published online 12 April 2023

* Supported by an appointment to the Young Scientist Training Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government. This work was also supported by the Korean Local Governments-Gyeongangbuk-do Province and Pohang City

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1) The terminology VLQ should not be confused with the vector leptoquark in some bibliographies.



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which carry a conserved quantum number $F \equiv 3B + L$. Here, B and L are the baryon and lepton numbers. As for the VLQs, there are seven typical representations [42]. In Table 1, we list their representations and labels.

For the six types of scalar LQs and seven types of VLQs, there can be a total of 42 combinations, which are named "LQ+VLQ" for convenience. Only 17 of them can lead to the chiral enhancements. In Table 2, we list these models that feature the chiral enhancements. The contributions in the four models $R_2 + B_{L,R}/(B, Y)_{L,R}$ and $S_1 + B_{L,R}/(B, Y)_{L,R}$ are almost the same as those in the minimal R_2 and S_1 models. There are nine models $R_2 + T_{L,R}/(X, T)_{L,R}/(T, B)_{L,R}/(T, B, Y)_{L,R}$ and $S_1 + T_{L,R}/(X, T)_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$, which produce the top and top partner chiral enhancements. For the two models $R_2/S_3 + (X, T, B)_{L,R}$, there are top, top partner, bottom, and bottom partner chiral enhancements at the same time. The models including T quarks were investigated in our previous work [41]. Here, we will study the pure bottom partner chirally enhanced models $\tilde{R}_2/\tilde{S}_1 + (B, Y)_{L,R}$.

A. VLQ Yukawa interactions with Higgs

Let us start with the $(B, Y)_{L,R}$ related Higgs Yukawa

Table 1. Scalar LQ (left) and VLQ (right) representations.

$SU(3)_C \times SU(2)_L \times U(1)_Y$	label	F	$SU(3)_C \times SU(2)_L \times U(1)_Y$	label
representation			representation	
$(\bar{3}, 3, 1/3)$	S_3	-2	$(3, 1, 2/3)$	$T_{L,R}$
$(3, 2, 7/6)$	R_2	0	$(3, 1, -1/3)$	$B_{L,R}$
$(3, 2, 1/6)$	$\tilde{R}_2$	0	$(3, 2, 7/6)$	$(X, T)_{L,R}$
			$(3, 2, 1/6)$	$(T, B)_{L,R}$
$(\bar{3}, 1, 4/3)$	$\tilde{S}_1$	-2	$(3, 2, -5/6)$	$(B, Y)_{L,R}$
$(\bar{3}, 1, 1/3)$	S_1	-2	$(3, 3, 2/3)$	$(X, T, B)_{L,R}$
$(\bar{3}, 1, -2/3)$	$\tilde{S}_1$	-2	$(3, 3, -1/3)$	$(T, B, Y)_{L,R}$

Table 2. Chiral enhancements in the minimal LQ and LQ+VLQ models.

Model	Chiral enhancement
R_2	m_t/m_μ
S_1	m_t/m_μ
$R_2 + B_{L,R}/(B, Y)_{L,R}$	m_t/m_μ
$S_1 + B_{L,R}/(B, Y)_{L,R}$	m_t/m_μ
$R_2 + T_{L,R}/(X, T)_{L,R}/(T, B)_{L,R}/(T, B, Y)_{L,R}$	$m_t/m_\mu, m_T/m_\mu$
$S_1 + T_{L,R}/(X, T)_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$	$m_t/m_\mu, m_T/m_\mu$
$R_2 + (X, T, B)_{L,R}$	$m_t/m_\mu, m_T/m_\mu, m_b/m_\mu, m_B/m_\mu$
$S_3 + (X, T, B)_{L,R}$	$m_t/m_\mu, m_T/m_\mu, m_b/m_\mu, m_B/m_\mu$
$\tilde{R}_2 + (B, Y)_{L,R}$	$m_b/m_\mu, m_B/m_\mu$
$\tilde{S}_1 + (B, Y)_{L,R}$	$m_b/m_\mu, m_B/m_\mu$

interactions. In the gauge eigenstates, there are two interactions $\overline{Q}_L^i d_R^j \phi$ and $(\overline{B}_L, \overline{Y}_L) d_R^j \tilde{\phi}$ and the mass term $-M_B(\bar{B}B + \bar{Y}Y)$. Here, we define the SM Higgs doublet $\tilde{\phi} \equiv i\sigma^2 \phi^*$ with $\sigma^a (a = 1, 2, 3)$ to be the Pauli matrices. The Q_L^i and d_R^i ($i = 1, 2, 3$) represent the SM quark fields. We can parametrize ϕ as $[0, (v+h)/\sqrt{2}]^T$ in the unitary gauge. After the electroweak symmetry breaking (EWSB), there are mixings between d^i and B . For simplicity, we only consider mixing between the third generation and B quark. Thus, we can perform the following transformations to rotate b and B quarks into mass eigenstates:

$$\begin{bmatrix} b_L \\ B_L \end{bmatrix} \rightarrow \begin{bmatrix} c_L^b & s_L^b \\ -s_L^b & c_L^b \end{bmatrix} \begin{bmatrix} b_L \\ B_L \end{bmatrix}, \quad \begin{bmatrix} b_R \\ B_R \end{bmatrix} \rightarrow \begin{bmatrix} c_R^b & s_R^b \\ -s_R^b & c_R^b \end{bmatrix} \begin{bmatrix} b_R \\ B_R \end{bmatrix}. \quad (1)$$

Here, $s_{L,R}^b$ and $c_{L,R}^b$ are abbreviations of $\sin\theta_{L,R}^b$ and $\cos\theta_{L,R}^b$, respectively. In fact, θ_L^b can be correlated with θ_R^b through the relation $\tan\theta_L^b = m_b \tan\theta_R^b / m_B$ [42]. Here, m_b and m_B represent the physical b and B quark masses, respectively. Additionally, the mass of the Y quark is $m_Y = M_B = \sqrt{m_B^2(c_R^b)^2 + m_b^2(s_R^b)^2}$. Then, we can choose m_B and θ_R^b as the new input parameters. After the transformations in Eq. (1), we obtain the following mass eigenstate Higgs Yukawa interactions:

$$\begin{aligned} \mathcal{L}_H^{\text{Yukawa}} \supset & -\frac{m_b}{v}(c_R^b)^2 h \bar{b}b - \frac{m_B}{v}(s_R^b)^2 h \bar{B}B \\ & - \frac{m_b}{v}s_R^b c_R^b h(\bar{b}_L B_R + \bar{B}_L b_L) - \frac{m_B}{v}s_R^b c_R^b h(\bar{B}_L b_R + \bar{b}_R B_L). \end{aligned} \quad (2)$$

Note that the Y quark does not interact with Higgs at the tree level.

B. VLQ gauge interactions

Now, let us label the $SU(2)_L$ and $U_Y(1)$ gauge fields as W_μ^a and B_μ . Then, the electroweak covariant derivative D_μ is defined as $\partial_\mu - igW_\mu^a \sigma^a/2 - ig'Y_q B_\mu$ for a doublet and $\partial_\mu - ig'Y_q B_\mu$ for a singlet, in which Y_q is the $U_Y(1)$ charge of the quark field acted by D_μ . Thus, the related gauge interactions can be written as $\overline{Q}_L^i iD_\mu \gamma^\mu Q_L^i + \overline{d}_R^i iD_\mu \gamma^\mu d_R^i + (\overline{B}, \overline{Y}) iD_\mu \gamma^\mu (B, Y)^T$. After the EWSB, the W gauge interactions can be written as

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} W_\mu^+ (\bar{t}_L \gamma^\mu b_L + \bar{b}_L \gamma^\mu t_L) + \text{h.c.} \quad (3)$$

The Z gauge interactions can be written as

$$\mathcal{L} \supset \frac{g}{c_W} \left[\left(-\frac{1}{2} + \frac{1}{3} s_W^2 \right) \bar{b}_L \gamma^\mu b_L + \frac{1}{3} s_W^2 \bar{b}_R \gamma^\mu b_R + \left(\frac{1}{2} + \frac{1}{3} s_W^2 \right) \bar{B} \gamma^\mu B + \left(-\frac{1}{2} + \frac{4}{3} s_W^2 \right) \bar{Y} \gamma^\mu Y \right] Z_\mu. \quad (4)$$

After the rotations in Eq. (1), we have the mass eigenstate W gauge interactions:

$$\mathcal{L}_{\text{BY}}^{\text{gauge}} \supset \frac{g}{\sqrt{2}} W_\mu^+ [c_L^b \bar{t}_L \gamma^\mu b_L + s_L^b \bar{t}_L \gamma^\mu B_L + c_L^b \bar{B}_L \gamma^\mu Y_L - s_L^b \bar{b}_L \gamma^\mu Y_L + c_R^b \bar{B}_R \gamma^\mu Y_R - s_R^b \bar{b}_R \gamma^\mu Y_R] + \text{h.c.} \quad (5)$$

We also have the mass eigenstate Z gauge interactions:

$$\mathcal{L}_{\text{BY}}^{\text{gauge}} \supset \frac{g}{c_W} Z_\mu \left[\frac{(c_L^b)^2 - (s_L^b)^2}{2} (\bar{B}_L \gamma^\mu B_L - \bar{b}_L \gamma^\mu b_L) - s_L^b c_L^b (\bar{b}_L \gamma^\mu B_L + \bar{B}_L \gamma^\mu b_L) + \frac{(s_R^b)^2}{2} \bar{b}_R \gamma^\mu b_R + \frac{(c_R^b)^2}{2} \bar{B}_R \gamma^\mu B_R - \frac{s_R^b c_R^b}{2} (\bar{b}_R \gamma^\mu B_R + \bar{B}_R \gamma^\mu b_R) + \frac{s_W^2}{3} (\bar{b} \gamma^\mu b + \bar{B} \gamma^\mu B) + \left(-\frac{1}{2} + \frac{4s_W^2}{3} \right) \bar{Y} \gamma^\mu Y \right]. \quad (6)$$

C. VLQ Yukawa interactions with LQ

Let us denote the SM lepton fields as L_L^i and e_R^i . The $\tilde{R}_2$ can be parametrized as $[\tilde{R}_2^{2/3}, \tilde{R}_2^{-1/3}]^T$, where the superscript labels the electric charge. Then, the $\tilde{R}_2$ and $\tilde{S}_1$ can induce the following $F=0$ and $F=2$ type gauge eigenstate LQ Yukawa interactions:

$$\mathcal{L}_{\tilde{R}_2+(B,Y)_{L,R}} \supset x_i \bar{e}_R^i (\tilde{R}_2)^\dagger \begin{pmatrix} B_L \\ Y_L \end{pmatrix} + y_{ij} \bar{L}_L^i \epsilon (\tilde{R}_2)^* d_R^j + \text{h.c.}, \quad (7)$$

and

$$\mathcal{L}_{\tilde{S}_1+(B,Y)_{L,R}} \supset x_{ij} \bar{e}_R^i (d_R^j)^c (\tilde{S}_1)^* + y_i \bar{L}_L^i \epsilon \begin{pmatrix} B_L \\ Y_L \end{pmatrix}^c (\tilde{S}_1)^* + \text{h.c.} \quad (8)$$

After the EWSB, they can be parametrized as

$$\mathcal{L}_{\tilde{R}_2+(B,Y)_{L,R}} \supset y_L^{\tilde{R}_2 \mu B} \bar{\mu} \omega_- B (\tilde{R}_2^{2/3})^* + y_R^{\tilde{R}_2 \mu b} \bar{\mu} \omega_+ b (\tilde{R}_2^{2/3})^* + y_L^{\tilde{R}_2 \mu B} \bar{\mu} \omega_- Y (\tilde{R}_2^{-1/3})^* - y_R^{\tilde{R}_2 \mu b} \bar{\nu}_L \omega_+ b (\tilde{R}_2^{-1/3})^* + \text{h.c.}, \quad (9)$$

and

$$\mathcal{L}_{\tilde{S}_1+(B,Y)_{L,R}} \supset y_L^{\tilde{S}_1 \mu b} \bar{\mu} \omega_- b^C (\tilde{S}_1)^* + y_R^{\tilde{S}_1 \mu B} \bar{\mu} \omega_+ B^C (\tilde{S}_1)^* - y_R^{\tilde{S}_1 \mu B} \bar{\nu}_L \omega_+ Y^C (\tilde{S}_1)^* + \text{h.c.} \quad (10)$$

In the above, we define the chiral operators $\omega_\pm$ as $(1 \pm \gamma^5)/2$. After the rotations in Eq. (1), we have the mass eigenstate interactions:

$$\mathcal{L}_{\tilde{R}_2+(B,Y)_{L,R}} \supset \bar{\mu} (-y_L^{\tilde{R}_2 \mu B} s_L^b \omega_- + y_R^{\tilde{R}_2 \mu b} c_R^b \omega_+) b (\tilde{R}_2^{2/3})^* + \bar{\mu} (y_L^{\tilde{R}_2 \mu B} c_L^b \omega_- + y_R^{\tilde{R}_2 \mu b} s_R^b \omega_+) B (\tilde{R}_2^{2/3})^* + y_L^{\tilde{R}_2 \mu B} \bar{\mu} \omega_- Y (\tilde{R}_2^{-1/3})^* - y_R^{\tilde{R}_2 \mu b} \bar{\nu}_L \omega_+ (c_R^b b + s_R^b B) (\tilde{R}_2^{-1/3})^* + \text{h.c.}, \quad (11)$$

and

$$\mathcal{L}_{\tilde{S}_1+(B,Y)_{L,R}} \supset \bar{\mu} (y_L^{\tilde{S}_1 \mu b} c_R^b \omega_- - y_R^{\tilde{S}_1 \mu B} s_L^b \omega_+) b^C (\tilde{S}_1)^* + \bar{\mu} (y_L^{\tilde{S}_1 \mu b} s_R^b \omega_- + y_R^{\tilde{S}_1 \mu B} c_L^b \omega_+) B^C (\tilde{S}_1)^* - y_R^{\tilde{S}_1 \mu B} \bar{\nu}_L \omega_+ Y^C (\tilde{S}_1)^* + \text{h.c.} \quad (12)$$

III. CONTRIBUTIONS TO $(g-2)_\mu$

A. Analytical results of contributions

For the LQ μq interaction, there are quark-photon and LQ-photon vertex mediated contributions to $(g-2)_\mu$, which can be described by the functions $f^q(x)$ and $f^S(x)$. Then, we use the functions $f_{LL}^{q,S}(x)$ and $f_{LR}^{q,S}(x)$ to label the parts without and with chiral enhancements. Starting from the $f_{LL}^{q,S}(x)$ and $f_{LR}^{q,S}(x)$ given in our previous paper [41], let us define the following integrals:

$$\begin{aligned} f_{LL}^{\tilde{R}_2 \mu Y}(x) &\equiv 4f_{LL}^q(x) - f_{LL}^S(x) = \frac{3+2x-7x^2+2x^3+2x(4-x)\log x}{4(1-x)^4}, \\ f_{LL}^{\tilde{R}_2 \mu b}(x) &\equiv f_{LL}^q(x) + 2f_{LL}^S(x) = \frac{x[5-4x-x^2+(2+4x)\log x]}{4(1-x)^4}, \\ f_{LR}^{\tilde{R}_2 \mu b}(x) &\equiv f_{LR}^q(x) + 2f_{LR}^S(x) = -\frac{5-4x-x^2+(2+4x)\log x}{4(1-x)^3}, \\ f_{LL}^{\tilde{S}_1 \mu b}(x) &\equiv -f_{LL}^q(x) + 4f_{LL}^S(x) \\ &= -\frac{2-7x+2x^2+3x^3+2x(1-4x)\log x}{4(1-x)^4}, \\ f_{LR}^{\tilde{S}_1 \mu b}(x) &\equiv -f_{LR}^q(x) + 4f_{LR}^S(x) = -\frac{1+4x-5x^2-(2-8x)\log x}{4(1-x)^3}. \end{aligned} \quad (13)$$

For the $\tilde{R}_2+(B,Y)_{L,R}$ model, there are b , B , and Y quark contributions to the $(g-2)_\mu$. The complete expression is calculated as

$$\begin{aligned}
\Delta a_{\mu}^{\tilde{R}_2+BY} = & \frac{m_{\mu}^2}{8\pi^2} \left\{ \frac{|y_L^{\tilde{R}_2\mu B}|^2}{m_{\tilde{R}_2}^2} f_{LL}^{\tilde{R}_2\mu Y} \left(\frac{m_Y^2}{m_{\tilde{R}_2}^2} \right) \right. \\
& + \frac{|y_L^{\tilde{R}_2\mu B}|^2 (s_L^b)^2 + |y_R^{\tilde{R}_2\mu b}|^2 (c_R^b)^2}{m_{\tilde{R}_2}^2} f_{LL}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right) \\
& - \frac{2m_b}{m_{\mu}} \frac{s_L^b c_R^b}{m_{\tilde{R}_2}^2} \text{Re} [y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*] f_{LR}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right) \\
& + \frac{|y_L^{\tilde{R}_2\mu B}|^2 (c_L^b)^2 + |y_R^{\tilde{R}_2\mu b}|^2 (s_R^b)^2}{m_{\tilde{R}_2}^2} f_{LL}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right) \\
& \left. + \frac{2m_b}{m_{\mu}} \frac{c_L^b s_R^b}{m_{\tilde{R}_2}^2} \text{Re} [y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*] f_{LR}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right) \right\}. \quad (14)
\end{aligned}$$

At the tree level, we have $m_{\tilde{R}_2}^2 = m_{\tilde{R}_2}^2 \equiv m_{\tilde{R}_2}$. Compared with the bottom partner chirally enhanced contribution

(i.e., the $f_{LR}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right)$ related term), the non-chirally enhanced parts are suppressed by the factor $m_{\mu}/(m_B s_R^b) \sim 1/(10^4 s_R^b)$ and the bottom quark chirally enhanced part is suppressed by the factor $(m_b s_L^b)/(m_B s_R^b) \sim (m_b^2/m_B^2)$. For the interesting values of s_R^b at $O(0.01 \sim 0.1)$, $\Delta a_{\mu}^{\tilde{R}_2+BY}$ is dominated by the bottom partner chirally enhanced contribution. Then, the above expression can be approximated as

$$\Delta a_{\mu}^{\tilde{R}_2+BY} \approx \frac{m_{\mu} m_B}{4\pi^2 m_{\tilde{R}_2}^2} s_R^b \text{Re} [y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*] f_{LR}^{\tilde{R}_2\mu b} \left(\frac{m_b^2}{m_{\tilde{R}_2}^2} \right). \quad (15)$$

For the $\tilde{S}_1 + (B, Y)_{L,R}$ model, there are b and B quark contributions to $(g-2)_{\mu}$. The complete expression is calculated as

$$\begin{aligned}
\Delta a_{\mu}^{\tilde{S}_1+BY} = & \frac{m_{\mu}^2}{8\pi^2} \left\{ \frac{|y_R^{\tilde{S}_1\mu B}|^2 (s_L^b)^2 + |y_L^{\tilde{S}_1\mu b}|^2 (c_R^b)^2}{m_{\tilde{S}_1}^2} f_{LL}^{\tilde{S}_1\mu b} \left(\frac{m_b^2}{m_{\tilde{S}_1}^2} \right) - \frac{2m_b}{m_{\mu}} \frac{s_L^b c_R^b}{m_{\tilde{S}_1}^2} \text{Re} [y_R^{\tilde{S}_1\mu B} (y_L^{\tilde{S}_1\mu b})^*] f_{LR}^{\tilde{S}_1\mu b} \left(\frac{m_b^2}{m_{\tilde{S}_1}^2} \right) \right. \\
& + \frac{|y_R^{\tilde{S}_1\mu B}|^2 (c_L^b)^2 + |y_L^{\tilde{S}_1\mu b}|^2 (s_R^b)^2}{m_{\tilde{S}_1}^2} f_{LL}^{\tilde{S}_1\mu b} \left(\frac{m_b^2}{m_{\tilde{S}_1}^2} \right) + \frac{2m_b}{m_{\mu}} \frac{c_L^b s_R^b}{m_{\tilde{S}_1}^2} \text{Re} [y_R^{\tilde{S}_1\mu B} (y_L^{\tilde{S}_1\mu b})^*] f_{LR}^{\tilde{S}_1\mu b} \left(\frac{m_b^2}{m_{\tilde{S}_1}^2} \right) \left. \right\}. \quad (16)
\end{aligned}$$

Similarly, it can be approximated as

$$\Delta a_{\mu}^{\tilde{S}_1+BY} \approx \frac{m_{\mu} m_B}{4\pi^2 m_{\tilde{S}_1}^2} s_R^b \text{Re} [y_R^{\tilde{S}_1\mu B} (y_L^{\tilde{S}_1\mu b})^*] f_{LR}^{\tilde{S}_1\mu b} \left(\frac{m_b^2}{m_{\tilde{S}_1}^2} \right). \quad (17)$$

B. Numerical analysis

The input parameters are chosen as $m_{\mu} = 105.66$ MeV, $m_b = 4.2$ GeV, $m_t = 172.5$ GeV, $G_F = 1.1664 \times 10^{-5}$ GeV $^{-2}$, $m_W = 80.377$ GeV, $m_Z = 91.1876$ GeV, and $m_h = 125.25$ GeV [43]. The v , g , θ_W are defined by $G_F = 1/(\sqrt{2}v^2)$, $g = 2m_W/v$, $\cos\theta_W \equiv c_W = m_W/m_Z$. There are also new parameters m_B , m_{LQ} , θ_R^b , and the LQ Yukawa couplings $y_{L,R}^{LQ\mu q}$. The VLQ mass can be constrained from the direct search, which is required to be above 1.5 TeV [44–47]. The mixing angle is mainly bounded by the electro-weak precision observables (EWPOs). The VLQ contributions to T parameter are suppressed by the factor $(s_R^b)^4$ or $m_b^2 (s_R^b)^2 / m_B^2$ [48, 49], which leads to a less constrained θ_R^b . The weak isospin third component of B_R is positive; thus, the mixing with the bottom quark enhances the right-handed Zbb coupling. As a result, the A_{FB}^b deviation [50, 51] can be compensated, which leads to looser constraints on θ_R^b . Conservatively, we can choose the mixing angle s_R^b to be smaller than 0.1 [42]. The LQ mass can also be constrained from the direct search, which is required to be above 1.7 TeV assuming $\text{Br}(LQ \rightarrow b\mu) = 1$

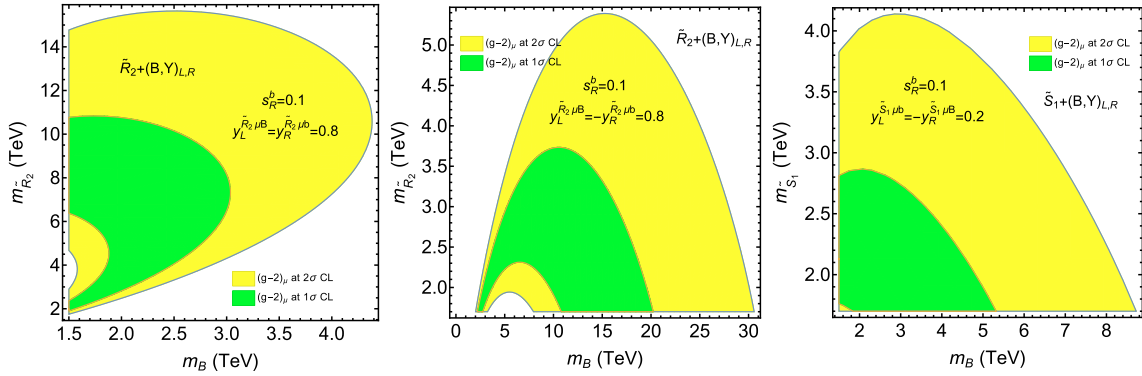
[52, 53].

We can choose benchmark points of m_B, m_{LQ}, s_R^b to constrain the LQ Yukawa couplings. Here, we consider two scenarios: $m_{LQ} > m_B$ and $m_{LQ} < m_B$. For the scenario of $m_{LQ} > m_B$, we adopt mass parameters of $m_B = 1.5$ TeV and $m_{LQ} = 2$ TeV. For the scenario of $m_{LQ} < m_B$, we adopt mass parameters of $m_B = 2.5$ TeV and $m_{LQ} = 2$ TeV. In Table 3, we give the approximate numerical expressions of Δa_{μ} in the $\tilde{R}_2/\tilde{S}_1 + (B, Y)_{L,R}$ models. We also show the allowed ranges for $s_R^b = 0.1$ and $s_R^b = 0.05$. Of course, these behaviours can be understood from Eqs. (15) and (17). In the $\tilde{R}_2 + (B, Y)_{L,R}$ model, $f_{LR}^{\tilde{R}_2\mu b}(x)$ vanishes when $m_B = m_{\tilde{R}_2}$, which causes the $|\text{Re}[y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*]|$ to be under the stress of perturbative unitarity. If there is a large hierarchy between m_B and $m_{\tilde{R}_2}$, the allowed $|\text{Re}[y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*]|$ can be smaller. Additionally, $\text{Re}[y_L^{\tilde{R}_2\mu B} (y_R^{\tilde{R}_2\mu b})^*]$ should be positive (negative) when $m_B < m_{\tilde{R}_2}$ ($m_B > m_{\tilde{R}_2}$). In the $\tilde{S}_1 + (B, Y)_{L,R}$ model, $f_{LR}^{\tilde{S}_1\mu b}(x)$ is always negative, which requires $\text{Re}[y_R^{\tilde{S}_1\mu B} (y_L^{\tilde{S}_1\mu b})^*] < 0$.

We can also choose benchmark points of s_R^b and LQ Yukawa couplings to constrain the m_B and m_{LQ} . Fig. 1 presents the $(g-2)_{\mu}$ allowed regions in the plane of $m_B - m_{LQ}$. As shown, $m_B < m_{\tilde{R}_2}$ and $m_B > m_{\tilde{R}_2}$ are favored in the left and middle plots, respectively. This can be understood from the asymptotic behaviours $f_{LR}^{\tilde{R}_2\mu b}(x) \sim -\log(x)/2 > 0$ for $x \rightarrow 0$ and $f_{LR}^{\tilde{R}_2\mu b}(x) \sim -1/(4x) < 0$ for $x \rightarrow \infty$. To produce a positive Δa_{μ} , $m_B < m_{\tilde{R}_2}$ and

Table 3. In the third column, we show the leading order numerical expressions of the Δa_μ . In the fifth and sixth columns, we show the ranges allowed by $(g-2)_\mu$ at 1σ and 2σ confidence levels (CLs).

Model	$(m_B, m_{LQ})/\text{TeV}$	$\Delta a_\mu \times 10^7$	s_R^b	$\text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu B})^*]$ or $\text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu B})^*]$	
				1σ	2σ
$\tilde{R}_2 + (B, Y)_{L,R}$	(1.5, 2)	$0.35 s_R^b \text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu B})^*]$	0.1	(0.55, 0.88)	(0.38, 1.05)
			0.05	(1.1, 1.77)	(0.76, 2.11)
	(2.5, 2)	$-0.224 s_R^b \text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu B})^*]$	0.1	(-1.38, -0.86)	(-1.65, -0.59)
			0.05	(-2.76, -1.71)	(-3.29, -1.19)
$\tilde{S}_1 + (B, Y)_{L,R}$	(1.5, 2)	$-6.88 s_R^b \text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu B})^*]$	0.1	(-0.045, -0.028)	(-0.054, -0.019)
			0.05	(-0.09, -0.056)	(-0.11, -0.039)
	(2.5, 2)	$-6.37 s_R^b \text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu B})^*]$	0.1	(-0.049, -0.03)	(-0.058, -0.021)
			0.05	(-0.097, -0.06)	(-0.12, -0.042)

**Fig. 1.** (color online) $(g-2)_\mu$ allowed regions at 1σ (green) and 2σ (yellow) CLs with $s_R^b = 0.1$. The parameters are chosen as $y_L^{\tilde{R}_2 \mu B} = y_R^{\tilde{R}_2 \mu B} = 0.8$ in the $\tilde{R}_2 + (B, Y)$ model (left), $y_L^{\tilde{R}_2 \mu B} = -y_R^{\tilde{R}_2 \mu B} = 0.8$ in the $\tilde{R}_2 + (B, Y)$ model (middle), and $y_L^{\tilde{S}_1 \mu B} = -y_R^{\tilde{S}_1 \mu B} = 0.2$ in the $\tilde{S}_1 + (B, Y)$ model (right).

$m_B > m_{\tilde{R}_2}$ are favored for $\text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu B})^*] > 0$ and $\text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu B})^*] < 0$, respectively. The $f_{LR}^{\tilde{S}_1 \mu B}(x)$ has asymptotic behaviours $f_{LR}^{\tilde{S}_1 \mu B}(x) \sim \log(x)/2 < 0$ for $x \rightarrow 0$ and $f_{LR}^{\tilde{S}_1 \mu B}(x) \sim -5/(4x) < 0$ for $x \rightarrow \infty$. To produce a positive Δa_μ , $\text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu B})^*] < 0$ is favored. Furthermore, the allowed regions in the plane of $m_B - m_{LQ}$ are sensitive to the choice of $y_{L,R}^{\text{LQ}\mu q}$. Generally, a larger $|y_{L,R}^{\text{LQ}\mu q}|$ corresponds to a larger m_B and m_{LQ} .

IV. LQ AND VLQ PHENOMENOLOGY AT HADRON COLLIDERS

In Table 4, we list the main LQ and VLQ decay channels¹⁾. The decay formulae of LQ and VLQ are given in Appendices A and B, respectively. For the scenario of $m_{LQ} > m_B$, there are new LQ decay channels. When searching for the LQ $\tilde{R}_2^{2/3}$, we propose the $\mu j_b Z$ and $\mu j_b h$ signatures. When searching for the LQ $\tilde{R}_2^{-1/3}$, we propose the $\mu j_b W$ signatures. When searching for the LQ $\tilde{S}_1$, we

propose the $\mu j_b Z$, $\mu j_b h$, $\mu j_b W$ signatures. For the scenario of $m_{LQ} < m_B$, there are new VLQ decay channels. When searching for the VLQ B , we propose the $\mu^+ \mu^- j_b$ signatures. When searching for the VLQ Y , we propose the $\mu \mu j_b$ signatures. It seems that such decay channels have not been searched for by the experimental collaborations.

To estimate the effects of new decay channels, we will compare the ratios of new partial decay widths with the traditional ones. Because of gauge symmetry, the different partial decay widths can be correlated. Then, we choose the following four ratios:

$$\begin{aligned} \frac{\Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ B)}{\Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ b)} &\sim \frac{|y_L^{\tilde{R}_2 \mu B}|^2}{|y_R^{\tilde{R}_2 \mu B}|^2}, & \frac{\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{B})}{\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{b})} &\sim \frac{|y_R^{\tilde{S}_1 \mu B}|^2}{|y_L^{\tilde{S}_1 \mu B}|^2}, \\ \frac{\Gamma(B \rightarrow \mu^- \tilde{R}_2^{2/3})}{\Gamma(B \rightarrow b h)} &\sim \frac{v^2 |y_L^{\tilde{R}_2 \mu B}|^2}{m_B^2 (s_R^b)^2}, & \frac{\Gamma(B \rightarrow \mu^+ (\tilde{S}_1)^*)}{\Gamma(B \rightarrow b h)} &\sim \frac{v^2 |y_R^{\tilde{S}_1 \mu B}|^2}{m_B^2 (s_R^b)^2}. \end{aligned} \quad (18)$$

1) The $\tilde{R}_2^{-1/3} \rightarrow \nu_L B$ and $B \rightarrow \nu_L \tilde{R}_2^{-1/3}$ decay channels are suppressed by the factor $(s_R^b)^2$. The $B \rightarrow t W^-$ decay channel is suppressed by the factor m_b^2/m_B^2 .

In Fig. 2, we show the contour plots of above four ratios under the consideration of $(g-2)_\mu$ constraints. In these plots, we include the full contributions. We find that the new LQ decay channels can become important for larger $|y_L^{\tilde{R}_2 \mu B}|$ in the $\tilde{R}_2 + (B, Y)_{L,R}$ model and $|y_R^{\tilde{S}_1 \mu B}|$ in the $\tilde{S}_1 + (B, Y)_{L,R}$ model. As for the VLQ decay, the importance of new decay channels depends significantly on s_R^b . For $s_R^b = 0.1$ and $m_B = 2.5$ TeV, the new VLQ decay chan-

nels are less significant. For smaller s_R^b , the new VLQ decay channels can play an important role.

For the LQ and VLQ production at hadron colliders, there are pair and single production channels, which are very sensitive to the LQ and VLQ masses. We can adopt the **FeynRules** [54] to generate the model files and compute the cross sections with **MadGraph5_aMC@NLO** [55]. For the 2 TeV scale LQ pair production [56–58], the

Table 4. In the third column, we show the main LQ decay channels. In the fourth column, we show the main VLQ decay channels. In the fifth column, we show the new LQ or VLQ signatures.

Model	Scenario	LQ decay	VLQ decay	new signatures
$\tilde{R}_2 + (B, Y)_{L,R}$	$m_{LQ} > m_B$	$\tilde{R}_2^{2/3} \rightarrow \mu^+ b, \mu^+ B$	$B \rightarrow bZ, bh$	$\tilde{R}_2^{2/3} \rightarrow \mu j_b Z, \mu j_b h$
		$\tilde{R}_2^{-1/3} \rightarrow \mu^+ Y, \nu_L b$	$Y \rightarrow bW^-$	$\tilde{R}_2^{-1/3} \rightarrow \mu j_b W$
	$m_{LQ} < m_B$	$\tilde{R}_2^{2/3} \rightarrow \mu^+ b$	$B \rightarrow bZ, bh, \mu^- \tilde{R}_2^{2/3}$	$B \rightarrow \mu^+ \mu^- j_b$
		$\tilde{R}_2^{-1/3} \rightarrow \nu_L b$	$Y \rightarrow bW^-, \mu^- \tilde{R}_2^{-1/3}$	$Y \rightarrow \mu \tilde{H}_T j_b$
$\tilde{S}_1 + (B, Y)_{L,R}$	$m_{LQ} > m_B$	$\tilde{S}_1 \rightarrow \mu^+ \bar{b}, \mu^+ \bar{B}, \nu_L \bar{Y}$	$B \rightarrow bZ, bh$	$\tilde{S}_1 \rightarrow \mu j_b Z, \mu j_b h, \tilde{H}_T j_b W$
			$Y \rightarrow bW^-$	
	$m_{LQ} < m_B$	$\tilde{S}_1 \rightarrow \mu^+ \bar{b}$	$B \rightarrow bZ, bh, \mu^+ (\tilde{S}_1)^*$	$B \rightarrow \mu^+ \mu^- j_b$
			$Y \rightarrow bW^-, \nu_L (\tilde{S}_1)^*$	$Y \rightarrow \mu \tilde{H}_T j_b$

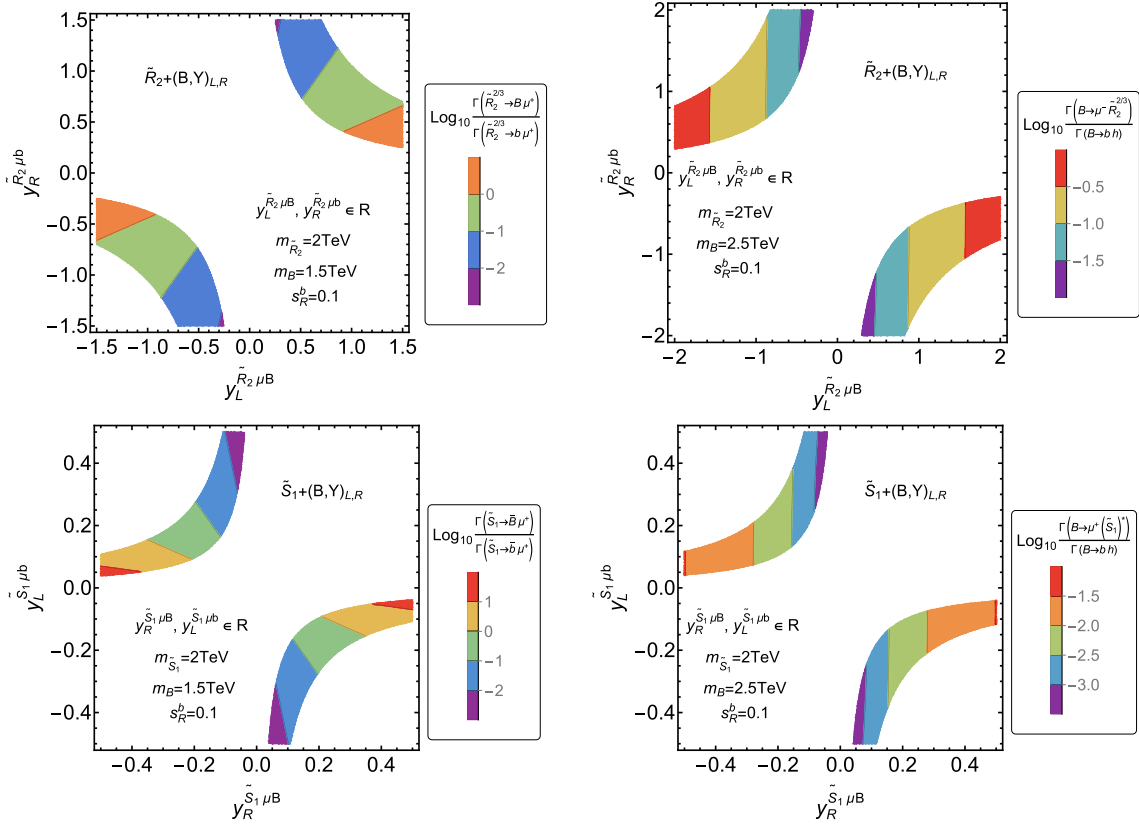


Fig. 2. (color online) Contour plots of $\log_{10} \frac{\Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ B)}{\Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ b)}$ (upper left), $\log_{10} \frac{\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{B})}{\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{b})}$ (lower left), $\log_{10} \frac{\Gamma(B \rightarrow \mu^- \tilde{R}_2^{2/3})}{\Gamma(B \rightarrow b h)}$ (upper right), and $\log_{10} \frac{\Gamma(B \rightarrow \mu^+ (\tilde{S}_1)^*)}{\Gamma(B \rightarrow b h)}$ (lower right), where the colored regions are allowed by the $(g-2)_\mu$ at 2σ CL. For the LQ decay, we choose $m_B = 1.5$ TeV and $m_{LQ} = 2$ TeV. For the VLQ decay, we choose $m_B = 2.5$ TeV and $m_{LQ} = 2$ TeV.

cross section can be ~ 0.01 fb at the 13 TeV LHC. For the 1.5 TeV and 2.5 TeV scale VLQ pair production [59–61], the cross section can be ~ 2 fb and ~ 0.01 fb at the 13 TeV LHC. For the single LQ and VLQ production channels, they depend on the electroweak couplings [42, 62, 63]. In the parameter space of large LQ Yukawa couplings, the single LQ production can be important, which may give some constraints at HL-LHC. To generate enough events, higher energy hadron colliders, for example, 27 and 100 TeV, can be necessary. In addition to the collider direct search, there can be indirect footprints, for example, B physics related decay modes $\Upsilon \rightarrow \mu^+ \mu^- \nu \bar{\nu} \gamma$. If we consider a more complex flavour structure (e.g., turn on the $LQus$ interaction), this can affect the $B \rightarrow K \mu^+ \mu^-$ channel. Here, we will not study this detailed phenomenology.

V. SUMMARY AND CONCLUSIONS

In this study, we investigate the scalar LQ and VLQ extended models to explain the $(g-2)_\mu$ anomaly. Then, we find two new models $\tilde{R}_2/\tilde{S}_1 + (B, Y)_{L,R}$, which can lead to the B quark chiral enhancements because of the bottom and bottom partner mixing. In the numerical analysis, we consider two scenarios: $m_{LQ} > m_B$ and $m_{LQ} < m_B$. After considering the experimental constraints, we choose relative light masses, which are adopted to be $(m_B, m_{LQ}) = (1.5 \text{ TeV}, 2 \text{ TeV})$ for the first scenario and $(m_B, m_{LQ}) = (2.5 \text{ TeV}, 2 \text{ TeV})$ for the second scenario. In the $\tilde{R}_2 + (B, Y)_{L,R}$ model, the $|\text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu b})^*]|$ is bounded

to be $O(1)$, because $f_{LR}^{\tilde{R}_2 \mu b}(x)$ vanishes accidentally as $m_B = m_{\tilde{R}_2}$. Meanwhile, we can expect smaller $|\text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu b})^*]|$ for largely splitted $m_{\tilde{R}_2}$ and m_B . In the $\tilde{S}_1 + (B, Y)_{L,R}$ model, the $\text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu b})^*]$ is bounded to the range $(-0.06, -0.02)$ at a 2σ CL if $s_R^b = 0.1$.

Under the constraints from $(g-2)_\mu$, we propose new LQ and VLQ search channels. In the scenario of $m_{LQ} > m_B$, there are new LQ decay channels: $\tilde{R}_2^{2/3} \rightarrow \mu^+ B$, $\tilde{R}_2^{-1/3} \rightarrow \mu^+ Y$, and $\tilde{S}_1 \rightarrow \mu^+ \tilde{B}, \nu_L \tilde{Y}$. For larger $y_L^{\tilde{R}_2 \mu B}$ and $y_R^{\tilde{S}_1 \mu B}$, it is important to take into account these decay channels. In the scenario of $m_{LQ} < m_B$, there are new VLQ decay channels: $B \rightarrow \mu^- \tilde{R}_2^{2/3}, \mu^+ (\tilde{S}_1)^*$ and $Y \rightarrow \mu^- \tilde{R}_2^{-1/3}, \nu_L (\tilde{S}_1)^*$. For $s_R^b = 0.1$, these channels are negligible compared with the traditional $B \rightarrow bZ, b\gamma$ and $Y \rightarrow bW^-$ channels. For smaller s_R^b , these new VLQ decay channels can also become important.

Note added: In a previous study [64], the authors examined the model with $\tilde{R}_2, S_3$, and $(B, Y)_{L,R}$. In this work, they did not consider the bottom and B quark mixing, and the chiral enhancements were produced through the $\tilde{R}_2$ and S_3 mixing. In [65], the authors explained the $(g-2)_\mu$ and B physics anomalies in the $S_1 + (B, Y)_{L,R}$ model.

APPENDIX A: LQ DECAY WIDTH FORMULAE

When the $\tilde{R}_2$ masses are degenerate, there are no gauge boson decay channels such as $\tilde{R}_2^{2/3} \rightarrow \tilde{R}_2^{-1/3} W^+$. For the $\tilde{R}_2^{2/3}$ to $\mu^+ b$ and $\mu^+ B$ decay channels, the widths are calculated as

$$\begin{aligned} \Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ b) &= \frac{m_{\tilde{R}_2}}{16\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_b^2}{m_{\tilde{R}_2}^2}\right)^2 - \frac{4m_\mu^2 m_b^2}{m_{\tilde{R}_2}^4}} \\ &\quad \times \left\{ \left(1 - \frac{m_\mu^2 + m_b^2}{m_{\tilde{R}_2}^2}\right) [|y_L^{\tilde{R}_2 \mu B}|^2 (s_L^b)^2 + |y_R^{\tilde{R}_2 \mu b}|^2 (c_R^b)^2] + \frac{4m_\mu m_b}{m_{\tilde{R}_2}^2} s_L^b c_R^b \text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu b})^*] \right\}, \\ \Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ B) &= \frac{m_{\tilde{R}_2}}{16\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_B^2}{m_{\tilde{R}_2}^2}\right)^2 - \frac{4m_\mu^2 m_B^2}{m_{\tilde{R}_2}^4}} \\ &\quad \times \left\{ \left(1 - \frac{m_\mu^2 + m_B^2}{m_{\tilde{R}_2}^2}\right) [|y_L^{\tilde{R}_2 \mu B}|^2 (c_L^b)^2 + |y_R^{\tilde{R}_2 \mu b}|^2 (s_R^b)^2] - \frac{4m_\mu m_B}{m_{\tilde{R}_2}^2} c_L^b s_R^b \text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu b})^*] \right\}. \end{aligned} \quad (A1)$$

For the $\tilde{R}_2^{-1/3}$ to $\mu^+ Y, \nu_L b, \nu_L B$ decay channels, the widths are calculated as

$$\Gamma(\tilde{R}_2^{-1/3} \rightarrow \mu^+ Y) = \frac{m_{\tilde{R}_2}}{16\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_Y^2}{m_{\tilde{R}_2}^2}\right)^2 - \frac{4m_\mu^2 m_Y^2}{m_{\tilde{R}_2}^4}} \left(1 - \frac{m_\mu^2 + m_Y^2}{m_{\tilde{R}_2}^2}\right) |y_L^{\tilde{R}_2 \mu B}|^2,$$

$$\begin{aligned}\Gamma(\tilde{R}_2^{-1/3} \rightarrow \nu_L b) &= \frac{m_{\tilde{R}_2}}{16\pi} \left(1 - \frac{m_b^2}{m_{\tilde{R}_2}^2}\right)^2 |y_R^{\tilde{R}_2 \mu b}|^2 (c_R^b)^2, \\ \Gamma(\tilde{R}_2^{-1/3} \rightarrow \nu_L B) &= \frac{m_{\tilde{R}_2}}{16\pi} \left(1 - \frac{m_B^2}{m_{\tilde{R}_2}^2}\right)^2 |y_R^{\tilde{R}_2 \mu b}|^2 (s_R^b)^2.\end{aligned}\quad (\text{A2})$$

Considering $m_\mu, m_b \ll m_B$ and $\theta_{L,R}^b \ll 1$, we have the following approximations:

$$\begin{aligned}\Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ B) &\approx \Gamma(\tilde{R}_2^{-1/3} \rightarrow \mu^+ Y) \approx \frac{m_{\tilde{R}_2}}{16\pi} \left(1 - \frac{m_B^2}{m_{\tilde{R}_2}^2}\right)^2 |y_L^{\tilde{R}_2 \mu B}|^2, \\ \Gamma(\tilde{R}_2^{2/3} \rightarrow \mu^+ b) &\approx \Gamma(\tilde{R}_2^{-1/3} \rightarrow \nu_L b) \approx \frac{m_{\tilde{R}_2}}{16\pi} |y_R^{\tilde{R}_2 \mu b}|^2.\end{aligned}\quad (\text{A3})$$

For the $\tilde{S}_1$ to $\mu^+ \bar{b}, \mu^+ \bar{B}, \nu_L \bar{Y}$ decay channels, the widths are calculated as

$$\begin{aligned}\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{b}) &= \frac{m_{\tilde{S}_1}}{16\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_b^2}{m_{\tilde{S}_1}^2}\right)^2 - \frac{4m_\mu^2 m_b^2}{m_{\tilde{S}_1}^4}} \\ &\quad \times \left\{ \left(1 - \frac{m_\mu^2 + m_b^2}{m_{\tilde{S}_1}^2}\right) [|y_R^{\tilde{S}_1 \mu B}|^2 (s_L^b)^2 + |y_L^{\tilde{S}_1 \mu b}|^2 (c_R^b)^2] + \frac{4m_\mu m_b}{m_{\tilde{S}_1}^2} s_L^b c_R^b \text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu b})^*] \right\}, \\ \Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{B}) &= \frac{m_{\tilde{S}_1}}{16\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_B^2}{m_{\tilde{S}_1}^2}\right)^2 - \frac{4m_\mu^2 m_B^2}{m_{\tilde{S}_1}^4}} \\ &\quad \times \left\{ \left(1 - \frac{m_\mu^2 + m_B^2}{m_{\tilde{S}_1}^2}\right) [|y_R^{\tilde{S}_1 \mu B}|^2 (c_L^b)^2 + |y_L^{\tilde{S}_1 \mu b}|^2 (s_R^b)^2] - \frac{4m_\mu m_B}{m_{\tilde{S}_1}^2} c_L^b s_R^b \text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu b})^*] \right\}, \\ \Gamma(\tilde{S}_1 \rightarrow \nu_L \bar{Y}) &= \frac{m_{\tilde{S}_1}}{16\pi} \left(1 - \frac{m_Y^2}{m_{\tilde{S}_1}^2}\right)^2 |y_R^{\tilde{S}_1 \mu B}|^2.\end{aligned}\quad (\text{A4})$$

Considering $m_\mu, m_b \ll m_B$ and $\theta_{L,R}^b \ll 1$, we have the following approximations:

$$\begin{aligned}\Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{B}) &\approx \Gamma(\tilde{S}_1 \rightarrow \nu_L \bar{Y}) \approx \frac{m_{\tilde{S}_1}}{16\pi} \left(1 - \frac{m_B^2}{m_{\tilde{S}_1}^2}\right)^2 |y_R^{\tilde{S}_1 \mu B}|^2, \\ \Gamma(\tilde{S}_1 \rightarrow \mu^+ \bar{b}) &\approx \frac{m_{\tilde{S}_1}}{16\pi} |y_L^{\tilde{S}_1 \mu b}|^2.\end{aligned}\quad (\text{A5})$$

APPENDIX B: VLQ DECAY WIDTH FORMULAE

If $m_B \sim \text{TeV}$ and $\theta_R^b \ll 0.1$, we have $m_B - m_Y \approx m_B (s_R^b)^2 / 2 \lesssim 5 \text{ GeV}$, which leads to the kinematic prohibition of some decay channels. For the $Y \rightarrow bW^-$ decay channel, the width is calculated as

$$\Gamma(Y \rightarrow bW^-) = \frac{g^2}{64\pi m_Y} \sqrt{\left(1 - \frac{m_b^2 + m_W^2}{m_Y^2}\right)^2 - \frac{4m_b^2 m_W^2}{m_Y^4}} \cdot \left\{ [(s_L^b)^2 + (s_R^b)^2] \frac{(m_Y^2 - m_b^2)^2 + m_W^2 (m_Y^2 + m_b^2) - 2m_W^4}{m_W^2} - 12m_Y m_b s_L^b s_R^b \right\}. \quad (\text{B1})$$

For the $B \rightarrow bZ, bh, tW^-$ decay channels, the widths are calculated as

$$\begin{aligned}
\Gamma(B \rightarrow bh) &= \frac{m_B}{32\pi} \sqrt{\left(1 - \frac{m_b^2 + m_h^2}{m_B^2}\right)^2 - \frac{4m_b^2 m_h^2}{m_B^4}} \left[\left(1 + \frac{m_b^2 - m_h^2}{m_B^2}\right) \frac{m_b^2 + m_B^2}{v^2} + 4 \frac{m_b^2}{v^2} \right] (s_R^b)^2 (c_R^b)^2, \\
\Gamma(B \rightarrow bZ) &= \frac{g^2}{32\pi c_W^2 m_B} \sqrt{\left(1 - \frac{m_b^2 + m_Z^2}{m_B^2}\right)^2 - \frac{4m_b^2 m_Z^2}{m_B^4}} \\
&\quad \times \left\{ \left[(s_L^b c_L^b)^2 + \frac{(s_R^b c_R^b)^2}{4} \right] \frac{(m_B^2 - m_b^2)^2 + m_Z^2(m_B^2 + m_b^2) - 2m_Z^4}{m_Z^2} - 6m_B m_b s_L^b s_R^b c_L^b c_R^b \right\}, \\
\Gamma(B \rightarrow tW^-) &= \frac{g^2 (s_L^b)^2}{64\pi} \sqrt{\left(1 - \frac{m_t^2 + m_W^2}{m_B^2}\right)^2 - \frac{4m_t^2 m_W^2}{m_B^4}} \frac{(m_B^2 - m_t^2)^2 + m_W^2(m_B^2 + m_t^2) - 2m_W^4}{m_W^2 m_B}. \tag{B2}
\end{aligned}$$

Considering $m_b, m_t, m_Z, m_W \ll m_B$ and $\theta_{L,R}^b \ll 1$, we have the following approximations:

$$\begin{aligned}
\Gamma(B \rightarrow bZ) &\approx \Gamma(B \rightarrow bh) \approx \frac{1}{2} \Gamma(Y \rightarrow bW^-) \approx \frac{m_B^3}{32\pi v^2} (s_R^b)^2, \\
\Gamma(B \rightarrow tW^-) &\approx \frac{m_b^2 m_B}{16\pi v^2} (s_R^b)^2. \tag{B3}
\end{aligned}$$

In the $\tilde{R}_2 + (B, Y)_{L,R}$ model, the VLQ can also decay into the $\tilde{R}_2$ final state. For the $Y \rightarrow \mu^- \tilde{R}_2^{-1/3}$ decay channel, the width is calculated as

$$\Gamma(Y \rightarrow \mu^- \tilde{R}_2^{-1/3}) = \frac{m_Y}{32\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_{\tilde{R}_2}^2}{m_Y^2}\right)^2 - \frac{4m_\mu^2 m_{\tilde{R}_2}^2}{m_Y^4}} \left(1 + \frac{m_\mu^2 - m_{\tilde{R}_2}^2}{m_Y^2}\right) |y_L^{\tilde{R}_2 \mu B}|^2. \tag{B4}$$

For the $B \rightarrow \mu^- \tilde{R}_2^{2/3}, \nu_L \tilde{R}_2^{-1/3}$ decay channels, the widths are calculated as

$$\begin{aligned}
\Gamma(B \rightarrow \mu^- \tilde{R}_2^{2/3}) &= \frac{m_B}{32\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_{\tilde{R}_2}^2}{m_B^2}\right)^2 - \frac{4m_\mu^2 m_{\tilde{R}_2}^2}{m_B^4}} \\
&\quad \times \left\{ \left(1 + \frac{m_\mu^2 - m_{\tilde{R}_2}^2}{m_B^2}\right) [|y_L^{\tilde{R}_2 \mu B}|^2 (c_L^b)^2 + |y_R^{\tilde{R}_2 \mu b}|^2 (s_R^b)^2] + \frac{4m_\mu m_{\tilde{R}_2}}{m_B^2} c_L^b s_R^b \text{Re}[y_L^{\tilde{R}_2 \mu B} (y_R^{\tilde{R}_2 \mu b})^*] \right\}, \\
\Gamma(B \rightarrow \nu_L \tilde{R}_2^{-1/3}) &= \frac{m_B}{32\pi} \left(1 - \frac{m_{\tilde{R}_2}^2}{m_B^2}\right)^2 |y_R^{\tilde{R}_2 \mu b}|^2 (s_R^b)^2. \tag{B5}
\end{aligned}$$

Considering $m_\mu \ll m_B$ and $\theta_{L,R}^b \ll 1$, we have the following approximations:

$$\Gamma(B \rightarrow \mu^- \tilde{R}_2^{2/3}) \approx \Gamma(Y \rightarrow \mu^- \tilde{R}_2^{-1/3}) \approx \frac{m_B}{32\pi} \left(1 - \frac{m_{\tilde{R}_2}^2}{m_B^2}\right)^2 |y_L^{\tilde{R}_2 \mu B}|^2. \tag{B6}$$

In the $\tilde{S}_1 + (B, Y)_{L,R}$ model, the VLQ can also decay into the $\tilde{S}_1$ final state. For the $Y \rightarrow \nu_L (\tilde{S}_1)^*$ decay channel, the width is calculated as

$$\Gamma(Y \rightarrow \nu_L (\tilde{S}_1)^*) = \frac{m_Y}{32\pi} \left(1 - \frac{m_{\tilde{S}_1}^2}{m_Y^2}\right)^2 |y_R^{\tilde{S}_1 \mu B}|^2. \tag{B7}$$

For the $B \rightarrow \mu^+ (\tilde{S}_1)^*$ decay channel, the width is calculated as

$$\Gamma(B \rightarrow \mu^+(\tilde{S}_1)^*) = \frac{m_B}{32\pi} \sqrt{\left(1 - \frac{m_\mu^2 + m_{\tilde{S}_1}^2}{m_B^2}\right)^2 - \frac{4m_\mu^2 m_{\tilde{S}_1}^2}{m_B^4}} \times \left\{ \left(1 + \frac{m_\mu^2 - m_{\tilde{S}_1}^2}{m_B^2}\right) [|y_R^{\tilde{S}_1 \mu B}|^2 (c_L^b)^2 + |y_L^{\tilde{S}_1 \mu b}|^2 (s_R^b)^2] + \frac{4m_\mu}{m_B} c_L^b s_R^b \text{Re}[y_R^{\tilde{S}_1 \mu B} (y_L^{\tilde{S}_1 \mu b})^*] \right\}. \quad (\text{B8})$$

Considering $m_\mu \ll m_B$ and $\theta_{L,R}^b \ll 1$, we have the following approximations:

$$\Gamma(B \rightarrow \mu^+(\tilde{S}_1)^*) \approx \Gamma(Y \rightarrow \nu_L(\tilde{S}_1)^*) \approx \frac{m_B}{32\pi} \left(1 - \frac{m_{\tilde{S}_1}^2}{m_B^2}\right)^2 |y_R^{\tilde{S}_1 \mu B}|^2. \quad (\text{B9})$$

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