# Reassessing aspects of LQG-modified dispersion relations of photons*

P. A. L. Mourão[1†] G. L. L. W. Levy[1‡] J. A. Helayël-Neto[1§]

[1]Centro Brasileiro de Pesquisas Físicas, Rio de Janeiro 22290-180, Brazil

**Abstract:** In this study, we investigate a scenario based on the effects of the Loop Quantum Gravity (LQG) on the electromagnetic sector of the Standard Model of Fundamental Interactions and Particle Physics (SM). Starting from a post-Maxwellian version of electromagnetism that includes LQG effects, we derive and discuss the influence of LQG parameters on classical quantities such as the components of the stress-tensor. Furthermore, we inspect the propagation of electromagnetic waves and examine the optical properties of the QED vacuum in this scenario. Among these, we contemplate the combined effect of the LQG parameters and a homogeneous background magnetic field on the propagation of electromagnetic waves by considering issues such as group velocities and refractive indices of the QED vacuum, in detail. Finally, with the aid of the previously analyzed LQG-extended photonic dispersion relations, we re-discuss the kinematics of the Compton effect and conclude that an interesting nonlinear profile emerges in the wavelengths of the incoming and deflected photons.

**Keywords:** electromagnetism, loop quantum gravity, modified dispersion relations, vacuum optical properties, Compton effect

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## I. INTRODUCTION

Our physical understanding points to the fact that vacuum exhibits strong fluctuations at the Planck scale by virtue of quantum effects. This phenomenon forms the basis for proposing the existence of a dynamic medium — referred to as *spin foam* [1−4] — that can affect the propagation and interactions of electromagnetic and gravitational waves, photons, neutrinos, electrons, and other highly energetic particles. This scenario is characterized by Lorentz-symmetry violation. One of the key-points of Loop Quantum Gravity (LQG) is the discreteness, or granularity, of spacetime. The latter is assumed to be four-dimensional. Rather than aiming at a complete unification of all the interactions, including gravity, the primary goal of LQG is to achieve a consistent description of the quantized gravitational field, as clearly discussed in previous studies. [1, 2, 5, 6].

Nowadays, there is a broad diversity of phenomena in which quantum gravity effects can be observable, among which there is an increasing interest in the category of the so-called tabletop experiments for quantum gravity. In this study, we focus on the phenomenon of the energy-dependent time of arrival of cosmic photons and neutrinos from distant sources [7, 8]. LQG is a relevant candidate to explain how the speed of light in vacuum may display an energy-dependent behavior. This occurs at the Planck energy scale because the basic principle of General Relativity (GR), namely, Lorentz invariance, is violated by the virtue of the strong vacuum fluctuations. This implies the formulation of field equations in the electromagnetic sector that incorporate the effects of LQG [7]. This yields different approaches and the development of various formulations [8−10]. Let us recall that Gamma-Ray Bursts (GRBs) [11−21] and Active Galactic Nuclei (AGNs) are examples of extremely energetic phenomena and appear as abundant sources of highly energetic particles such as cosmic photons and neutrinos. These ultra-energetic particles may reveal Lorentz-invariance violation (LIV) as an observable effect. Furthermore, LQG appears as a viable candidate to provide a theoretical basis to explain this sort of phenomenon. GRBs constitute a remarkable category of astrophysical structures that allow the detection of quantum-gravitational effects. The travel time dif-

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† E-mail: pedrolimamourao@gmail.com
‡ E-mail: guslevy9@hotmail.com
§ E-mail: josehelayel@gmail.com

ferences, which accumulate after photons propagate from long distances, are key elements to unravel these effects. Given that it is difficult to distinguish two massless particles with different energies that travel from distant regions where GRBs occur, it becomes necessary to observe short and intense bursts traveling large distances to guarantee a good experimental time resolution.

In this study, we focus exclusively on photons. Hence, it is necessary to adopt a model that describes the electromagnetic sector while incorporating LQG effects [7]. Consequently, we consider an electromagnetic model that incorporates physics beyond the Standard Model (SM), characterized by the presence of higher derivative terms and a nonlinear magnetic field contribution to the Ampère-Maxwell equation. This is of particular relevance and introduces some complexity in the study of classical aspects associated with the extended Maxwell equations. The method adopted in our study involves the linearization of the field equations before calculating the relevant classical physical quantities. This approach allows us to circumvent this complexity, and we attain expressions for components of the energy-momentum tensor and radiated energy with the additional LQG parameters. Each of the rotation angles that parametrize the effects of LQG contributes independently and significantly to the derived equations. Based on this modified electromagnetic scenario, the energy density, Poynting vector, and stress tensor exhibit significant deviations in comparison with their usual forms, highlighting how energy, momentum densities, and fluxes in spacetime differ substantially at the Planck scale.

However, the focus of our present study is on the vacuum dispersion relation emerging from the LQG-corrected set of Maxwell equations. We are concerned with a high-energy model such that the photons exhibit astrophysical-scale contributions [22, 23]. This leads to the possibility of analyzing deviations that can be compared with standard dispersion relations, potentially indicating imprints of quantum-gravitational effects. Other fundamental quantities that can serve as probes for analyzing Quantum Gravity (QG) include photons exhibiting a vacuum speed, which differs from the value predicted by General Relativity, or refractive indices deviating from the case of usual (Maxwellian) light. All the modifications of the equations lead to alterations in known events and optical properties of light in vacuum, such as birefringence, which is the dependence of the refractive index on the polarization, as well as dichroism and group-velocity dispersion, among others. These facts re-inforce the possibility of detecting QG effects that emerge in high-energy events.

With the new class of modified photon dispersion relations that we examine in this study, we investigate how they modify the expression for the wavelength shift of the photon in a Compton scattering. Originally, this effect results from extensive investigations of the scattering of X-rays by matter [24]. This process played a crucial role in the synthesis of Quantum Mechanics and Electrodynamics, which culminated in the complete quantitative explanation proposed by Dirac through his formulation of Relativistic Quantum Mechanics [25]. Although the Compton effect highlights quantum interactions at the particle level, LQG extends these principles to spacetime, striving to describe the behavior of gravity at the most fundamental scales. Understanding such connections deepens our comprehension of the Universe, bridging quantum phenomena with the fabric of spacetime. The results presented later explore the consequences of this unification, potentially revealing deeper structures of spacetime that reconcile the quantum behavior of particles with the gravitational dynamics of the universe.

The paper is organized as follows. Section II briefly introduces the incorporation of the electromagnetic field within an LQG framework, establishing the general formulation of the model employed in this study. Section III examines the classical aspects of the system under consideration. In certain cases, we adopt a linearized magnetic field approximation, as one of the modified Maxwell equations contains a nonlinear magnetic field term. Section IV revisits the Compton effect using LQG-modified photon dispersion relations. In this analysis, the nonlinear magnetic field term is switched off, allowing us to isolate LQG-induced corrections to the photon wavelength shift. Finally, Section V presents our concluding remarks and general discussion.

## II. A QUICK GLANCE AT THE MODEL

To provide a clear context for our discussion, we begin by providing a summary of the model under consideration. The Hamiltonian formulation of LQG effects on electromagnetic theory was explored in a previous study [7] and can be expressed as

$$
\begin{aligned}
H_{\rm LQG} = \frac{1}{Q^2}\int \mathrm{d}^3x \Bigg\{ &\left[1+\theta_7\left(\frac{l_P}{\mathcal{L}}\right)^{2+2\Upsilon}\right]\frac{1}{2}(\underline{\vec{B}}^2+\underline{\vec{E}}^2) \\
&+\theta_3 l_P^2(\underline{B}^a\nabla^2\underline{B}_a+\underline{E}^a\nabla^2\underline{E}_a)+\theta_2 l_P^2\underline{E}^a\partial_a\partial_b\underline{E}^b \\
&+\theta_8 l_P[\underline{\vec{B}}\cdot(\nabla\times\underline{\vec{B}})+\underline{\vec{E}}\cdot(\nabla\times\underline{\vec{E}})] \\
&+\theta_4\mathcal{L}^2 l_P^2\left(\frac{\mathcal{L}}{l_p}\right)^{2\Upsilon}(\underline{\vec{B}}^2)^2+\dots\Bigg\}. \qquad (1)
\end{aligned}
$$

We specify some of the quantities present in Eq. (1). $Q^2$ denotes the electromagnetic coupling constant, and $l_p \approx 1.6\times10^{-35}\,\mathrm{m}$ denotes Planck length. The characteristic length $\mathcal{L}$ is constrained by the relation $l_P \ll \mathcal{L} \le \lambda$, where $\lambda$ denotes the de Broglie wavelength. The characteristic length, $\mathcal{L}$, has a maximum value at $\mathcal{L}=k^{-1}$. This implies that our analysis is conducted within an effective theoretical framework. Finally, $\Upsilon$ depends on the helicity of the particle under consideration [26−28] and $\theta_i$'s are dimensionless parameters of order one or they are extremely small, close to zero [7, 29]; $a,b$ are spatial tensor indices. To ensure clear notation, we eliminate all underlines in the electromagnetic parameters and, from Eq. (1), rewrite the vectors in bold. The field equations can be expressed as follows:

$$\nabla\cdot\mathbf{E}=0, \tag{2}$$

$$A_\gamma(\nabla\times\mathbf{B})-\frac{\partial\mathbf{E}}{\partial t}+2l_P^2\theta_3\nabla^2(\nabla\times\mathbf{B})-2\theta_8 l_P\nabla^2\mathbf{B}+4\theta_4\mathcal{L}^2\left(\frac{\mathcal{L}}{l_P}\right)^{2\Upsilon_\gamma}l_P^2\nabla\times(B^2\cdot\mathbf{B})=0, \tag{3}$$

$$\nabla\cdot\mathbf{B}=0, \tag{4}$$

$$A_\gamma(\nabla\times\mathbf{E})+\frac{\partial\mathbf{B}}{\partial t}+2l_P^2\theta_3\nabla^2(\nabla\times\mathbf{E})-2\theta_8 l_P\nabla^2\mathbf{E}=0, \tag{5}$$

with

$$A_\gamma=1+\theta_7\left(\frac{\ell_P}{\mathcal{L}}\right)^{2+2\Upsilon}. \tag{6}$$

## III. ELECTROMAGNETIC RADIATION

We discuss the issue of obtaining certain electromagnetic quantities with the LQG parameters cast above included. We believe that it is convenient to simplify the notation according to the conventions listed below:

$$\bar{\theta}_3=2l_p^2\theta_3,\ \bar{\theta}_8=2l_p\theta_8,\ \bar{\theta}_4=4\theta_4\mathcal{L}^2\left(\frac{\mathcal{L}}{l_P}\right)^{2\Upsilon_\gamma}l_P^2. \tag{7}$$

We consider that the dimensional parameters $\bar{\theta}_i$ obey the conditions below:

$$\bar{\theta}_i\cdot\bar{\theta}_j=\begin{cases}0, & \text{if}\quad i\neq j\\ \bar{\theta}_i^2, & \text{if}\quad i=j,\end{cases} \tag{8}$$

This may introduce a simplification because each parameter, $\bar{\theta}_i$, can be associated with a different physical phenomenon. Similar to what we normally do in classsical electromagnetism, we can take the cross product of equations Eqs. (3) and (5) with the fields **E** and **B**, respectively. By proceeding as indicated, we can readily arrive at LQG-corrected expressions for the energy density and Poynting vector:

$$\nabla\cdot(\mathbf{S}-\mathbf{S}_1-\mathbf{S}_2+\mathbf{S}_3)+\frac{\partial}{\partial t}(u-u_1)=0. \tag{9}$$

Based on the continuity equation above, we can extract the expressions for the energy density and Poynting vector extended by the inclusion of the LQG parameters:

$$\mathbf{S}_{\text{Maxwell}}=A_\gamma(\mathbf{E}\times\mathbf{B}), \tag{10}$$

$$\mathbf{S}_1=\bar{\theta}_3[(E_i\partial_j\epsilon_{ikl}\partial_k B_l)+(\partial_k\partial_j E_i\epsilon_{ikl}B_l)-(B_i\partial_j\epsilon_{ikl}\partial_k E_l)-(\partial_k\partial_j B_i\epsilon_{ikl}E_l)], \tag{11}$$

$$\mathbf{S}_2=\bar{\theta}_8(E_i\partial_j B_i-B_i\partial_j E_i), \tag{12}$$

$$\mathbf{S}_3=\bar{\theta}_4(E_i\epsilon_{ijk}B^2B_k), \tag{13}$$

$$u_{\text{Maxwell}}=\frac{E^2}{2}+\frac{B^2}{2}, \tag{14}$$

$$u_1=\bar{\theta}_4\frac{B^4}{4}. \tag{15}$$

Eqs. (10) and (14) reduce to the usual Poynting vector and energy density of the electromagnetic field, respectively, whenever we remove the LQG parameters. The continuity equation of the Poynting vector is kept unchanged, but the radiated energy, as shown above, has new terms coming from LQG: each $\bar{\theta}_i$ independently contributes one term to the spatial form, as we can observe in terms of $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$. At this stage, it is important to note that the nonlinear magnetic field term is the sole contribution that modifies the expression for the electromagnetic energy density. Taking the time derivative of the Poynting vector given in Eq. (9), we derive a continuity equation that enables us to obtain the purely spatial components of the energy-momentum tensor, namely the normal and shear stresses:

$$\partial_t(\mathbf{S}-\mathbf{S}_1-\mathbf{S}_2+\mathbf{S}_3)=-\partial_k\left\{A_\gamma^2\left[\delta_{ik}\left(\frac{B^2}{2}+\frac{E^2}{2}\right)-(B_iB_k+E_iE_k)\right]+\bar{\theta}_3[\delta_{ik}[(\partial_o\partial_o B_m)B_m-(\partial_o\partial_o E_m)E_m\right.$$

$$-(E_j\partial_m(\partial_m E_j))+(E_m\partial_n(\partial_m E_n))-(B_j\partial_m(\partial_m B_j))+(B_m\partial_n(\partial_m B_n))+\bar{\theta}_3[\partial_o\partial_o(E_m\partial_n(\partial_m E_n)) +2(\partial_o\partial_o(\partial_m\partial_m B_l)B_l)-\partial_o\partial_o(B_j\partial_m(\partial_m B_j))+\partial_o\partial_o(B_m\partial_n(\partial_m B_n))+2(\partial_o\partial_o(\partial_m\partial_m E_l)E_l)]] +(\partial_o\partial_o B_i)\cdot B_k+(\partial_o\partial_o E_i)\cdot E_k]+\bar{\theta}_8\delta_{ik}[(\bar{\theta}_8\partial_o\partial_o B_j-\epsilon_{jmn}\partial_m B_n)\cdot B_j-(\bar{\theta}_8\partial_o\partial_o E_j -\epsilon_{jmn}\partial_m E_n)E_j]+\bar{\theta}_4[\delta_{ik}[B^4+\frac{\bar{\theta}_4}{2}(B^2B_m)^2]-2(B^2B_iB_k)+\bar{\theta}_4[(B^2B_i)(B^2B_k)]-3(E_kE_i)B^2]\Big\}, \quad (16)$$

or we can rewrite it in compact form, as shown below:

$$\partial_t(\mathbf{S}-\mathbf{S}_1-\mathbf{S}_2+\mathbf{S}_3)+\partial_k T_{ik}=0. \quad (17)$$

Our next step comprises deriving and analyzing the dispersion relation for an electromagnetic wave. To conduct this step, we expand the magnetic field around a constant and homogeneous external magnetic background:

$$\mathbf{B}=\boldsymbol{\zeta}+\mathbf{b}_p, \quad (18)$$

where $\boldsymbol{\zeta}$ denotes a constant and homogeneous vector and $\mathbf{b}_p$ denotes the magnetic field of the propagating wave. Hence, we can rewrite the nonlinear term of the magnetic field as shown below:

$$\vec{B}^2\cdot\mathbf{B}=(\boldsymbol{\zeta}+\mathbf{b}_p)^2\cdot(\boldsymbol{\zeta}+\mathbf{b}_p)\approx\zeta^2\cdot\mathbf{b}_p+2(\boldsymbol{\zeta}\cdot\mathbf{b}_p)\cdot\boldsymbol{\zeta}, \quad (19)$$

From the splitting provided by Eq. (19), we can rewrite Eq. (3) as

$$A_\gamma(\nabla\times\mathbf{b}_p)-\frac{\partial\mathbf{E}}{\partial t}+\bar{\theta}_3\nabla^2(\nabla\times\mathbf{b}_p)-\bar{\theta}_8\nabla^2\mathbf{b}_p +\bar{\theta}_4\nabla\times[\zeta^2\cdot\mathbf{b}_p+2(\boldsymbol{\zeta}\cdot\mathbf{b}_p)\cdot\boldsymbol{\zeta}]=0. \quad (20)$$

The wave equations for the eletric and magnetic fields of the propagating signal are cast as follows:

$$A_\gamma^2(\nabla^2\mathbf{E})-\frac{\partial^2\mathbf{E}}{\partial t^2}=-2\bar{\theta}_3\nabla^2(\nabla^2\mathbf{E})-2\bar{\theta}_8\nabla\times(\nabla^2\mathbf{E}) -\bar{\theta}_3^2\nabla^2(\nabla^2(\nabla^2\mathbf{E}))+\bar{\theta}_8^2\nabla^2(\nabla^2\mathbf{E}) +\bar{\theta}_4\{\zeta^2(\nabla^2\mathbf{E})-2[\boldsymbol{\zeta}\cdot(\nabla\times\mathbf{E})]\cdot(\nabla\times\boldsymbol{\zeta})\}; \quad (21)$$

For the magnetic field, we obtain

$$A_\gamma^2(\nabla^2\mathbf{b}_p)-\frac{\partial^2\mathbf{b}_p}{\partial t^2}=-2\bar{\theta}_3\nabla^2(\nabla^2\mathbf{b}_p)-2\bar{\theta}_8\nabla\times(\nabla^2\mathbf{b}_p) -\bar{\theta}_3^2\nabla^2(\nabla^2(\nabla^2\mathbf{b}_p))+\bar{\theta}_8^2\nabla^2(\nabla^2\mathbf{b}_p) +\bar{\theta}_4\{\zeta^2(\nabla^2\mathbf{b}_p)-2[\boldsymbol{\zeta}\cdot(\nabla\times\mathbf{b}_p)]\cdot(\nabla\times\boldsymbol{\zeta})\}. \quad (22)$$

A constant field $\boldsymbol{\zeta}$ is associated with the nonlinear term. We now derive the dispersion relation by considering a plane wave solution:

$$\mathbf{E}=\mathbf{e}_0\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)},\ \mathbf{b}_p=\mathbf{b}_0\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)},\ k=|\mathbf{k}|. \quad (23)$$

We arrive at

$$\mathbf{e}_0\cdot\mathbf{k}=0,\ \mathbf{b}_0\cdot\mathbf{k}=0 \quad (24)$$

$$(\mathbf{k}\times\mathbf{e}_0)(A_\gamma-\bar{\theta}_3\mathbf{k}^2)-\mathrm{i}\bar{\theta}_8\mathbf{k}^2\cdot\mathbf{e}_0-w\mathbf{b}_0=0, \quad (25)$$

$$(\mathbf{k}\times\mathbf{b}_0)(A_\gamma-\bar{\theta}_3\mathbf{k}^2)-\mathrm{i}\bar{\theta}_8\mathbf{k}^2\cdot\mathbf{b}_0+w\mathbf{e}_0+\bar{\theta}_4[\zeta^2(\mathbf{k}\times\mathbf{b}_0) +2\bar{\theta}_4(\mathbf{k}\times\boldsymbol{\zeta})\cdot(\boldsymbol{\zeta}\cdot\mathbf{b}_0)]=0. \quad (26)$$

By making use of the previous equations, we obtain

$$M_{ij}\mathrm{e}_{0j}=0, \quad (27)$$

where $M_{ij}$ denotes a matrix of the form

$$M_{ij}=[\mathbf{k}^2(A_\gamma-\bar{\theta}_3\mathbf{k}^2)^2-(\mathrm{i}\bar{\theta}_8\mathbf{k}^2)^2+w^2-\bar{\theta}_4(\boldsymbol{\zeta}\cdot\mathbf{k})^2(A_\gamma -\bar{\theta}_3\mathbf{k}^2)]\delta_{ij}+[\mathrm{i}\bar{\theta}_4\bar{\theta}_8(\boldsymbol{\zeta}\cdot\mathbf{k})^2-2\mathrm{i}\bar{\theta}_8\mathbf{k}^2(A_\gamma-\bar{\theta}_3\mathbf{k}^2)]\cdot \epsilon_{ijk}k_k-2\bar{\theta}_4(\mathbf{k}\times\boldsymbol{\zeta})_i\cdot(\mathbf{k}\times\boldsymbol{\zeta})_j(A_\gamma-\bar{\theta}_3\mathbf{k}^2) -2i\bar{\theta}_4\bar{\theta}_8(\mathbf{k}\times\boldsymbol{\zeta})_i\cdot\zeta_j\mathbf{k}^2. \quad (28)$$

It should be noted that Eq. (28) has the same form as the matrix equation:

$$M_{ij}=\alpha\delta_{ij}+\beta u_i\cdot u_j+c\epsilon_{ijk}v_k+\gamma u_i\cdot s_j, \quad (29)$$

whose determinant is provided by $\det\mathrm{M}=\alpha^3+c^2(\mathbf{u}\cdot\mathbf{v})\cdot$

$(\gamma\mathbf{s}\cdot\mathbf{v}+\beta(\mathbf{u}\cdot\mathbf{v}))+\alpha^2\beta\mathbf{u}^2+\alpha c(c\mathbf{v}^2+\gamma\mathbf{s}\cdot(\mathbf{v}\times(\mathbf{v}\times\mathbf{s})))$. Now, we can satisfy the condition $\det \mathrm{M}=0$ to find the modified dispersion relation:

$$w_\pm^2 = k^2[A_\gamma-\bar\theta_3(\mathbf{k})^2]\left\{[A_\gamma-\bar\theta_3(\mathbf{k})^2]+\bar\theta_4\zeta^2\right\}-(\bar\theta_8\cdot k^2)^2 -\psi\pm\left(4k^4\left\{k^2\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\bar\theta_4\frac{\zeta^2}{2}\right)^2 +\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\frac{\zeta^2}{2}\right)\bar\theta_4[\boldsymbol{\zeta}\cdot(\mathbf{k}\times(\mathbf{k}\times\boldsymbol{\zeta}))]\right\}\bar\theta_8^2+\psi^2\right)^{1/2}. \tag{30}$$

The term $\psi=-\bar\theta_4[A_\gamma-\bar\theta_3(\mathbf{k})^2]\cdot|\mathbf{k}|^2\cdot|\zeta|^2\sin^2\varphi$ is a scalar, but it manifests the anisotropy of the model, in that photons present propagation speeds that are no longer constant at the Planck scale. This is precisely due to the dependence that the $\psi$ parameter has on $\sin\varphi$, the angle between vectors $\mathbf{k}\cdot\boldsymbol{\zeta}$. The $\pm$ sign in the dispersion relation is an indication of the expected phenomenon of vacuum birefringence. If $\boldsymbol{\zeta}$ or the linear terms $\bar\theta_3=\bar\theta_8=\bar\theta_7=0$, we obtain the same results provided in a previous study [7]. Furthermore, the group velocity can be directly derived from the dispersion relation. Differentiating the latter expression yields the expression for the group velocity:

$$v_\pm=\frac{\mathrm{d}w}{\mathrm{d}k}=\frac{1}{w_\pm}\cdot\Bigg[\mathbf{k}\cdot[A_\gamma-\bar\theta_3(\mathbf{k})^2]\left\{[A_\gamma-\bar\theta_3(\mathbf{k})^2]+\bar\theta_4\zeta^2\right\}-2\bar\theta_3\mathbf{k}^3[A_\gamma-\bar\theta_3(\mathbf{k})^2]+2i\bar\theta_8\mathbf{k}^3-\bar\theta_4\mathbf{k}\cdot|\zeta|^2\sin^2\varphi \pm\frac{1}{4}\left(4k^4\left\{k^2\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\bar\theta_4\frac{\zeta^2}{2}\right)^2+\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\frac{\zeta^2}{2}\right)\cdot\bar\theta_4\cdot[\boldsymbol{\zeta}\cdot(\mathbf{k}\times(\mathbf{k}\times\boldsymbol{\zeta}))]\right\}\bar\theta_8^2+\psi^2\right)^{-1}\cdot \left(12\mathbf{k}^5\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\bar\theta_4\frac{\zeta^2}{2}\right)^2\bar\theta_8^2+2\mathbf{k}^3\cdot\left(\bar\theta_4[A_\gamma-\bar\theta_3(\mathbf{k})^2]\cdot|\zeta|^2\sin^2\varphi\right)^2-8\bar\theta_3\mathbf{k}^7\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\bar\theta_4\frac{\zeta^2}{2}\right)^2\bar\theta_8^2\right)\Bigg]. \tag{31}$$

Working to get The refraction index ($n=|k|/w$) leads to the expression

$$n_\pm=|k|\cdot\left(k^2[A_\gamma-\bar\theta_3(\mathbf{k})^2]\left\{[A_\gamma-\bar\theta_3(\mathbf{k})^2]+\bar\theta_4\zeta^2\right\}-(\bar\theta_8\cdot k^2)^2-\psi\pm\left(4k^4\left\{k^2\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\bar\theta_4\frac{\zeta^2}{2}\right)^2 +\left([A_\gamma-\bar\theta_3(\mathbf{k})^2]-\frac{\zeta^2}{2}\right)\bar\theta_4[\boldsymbol{\zeta}\cdot(\mathbf{k}\times(\mathbf{k}\times\boldsymbol{\zeta}))]\right\}\bar\theta_8^2+\psi^2\right)^{1/2}\right)^{-1/2}. \tag{32}$$

Based on this equation, it can be shown that dichroism and birefringence effects can take place under specific conditions. However, this is not discussed in this study.

## IV. LQG CORRECTIONS TO THE KINEMATICS OF THE COMPTON EFFECT

In this Section, we address the corrections to the Compton effect stemming from the LQG-corrected photon dispersion relations. Before initiating the computation, it is important to emphasize that we do not employ the magnetic field splitting given in Eq. (7) in this section. Instead, we work with Eqs. (3) and (5), in which the nonlinear magnetic field term is neglected. We then consider plane wave solutions for the electric and magnetic fields.

$$\mathbf{E}=\mathbf{E}_0\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)},\mathbf{B}=\mathbf{B}_0\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-\omega t)},k=|\mathbf{k}|, \tag{33}$$

we obtain

$$(\mathbf{k}\times\mathbf{E}_0)\left[A_\gamma-2\bar\theta_3(\ell_p\mathbf{k})^2\right]-2\mathrm{i}\bar\theta_8\ell_p\mathbf{k}^2\mathbf{E}_0-w\mathbf{B}_0=0, \tag{34}$$

$$(\mathbf{k}\times\mathbf{B}_0)\left[A_\gamma-2\bar\theta_3(\ell_p\mathbf{k}^2)\right]-2\mathrm{i}\bar\theta_8\ell_p\mathbf{k}^2\cdot\mathbf{B}_0+w\mathbf{E}_0=0. \tag{35}$$

It should be noted that we do not consider the nonlinear contribution in the magnetic field in Eq. (5). Hence, Eqs. (25) and (26) differ from the Eqs. (33) and (34) that were previously obtained. From Eq. (27) and the condition $\det \mathrm{M} = 0$, it is possible to obtain the modified dispersion relation, which takes the form

$$\omega = c|\mathbf{k}|\left[A_\gamma - 2\theta_3(|\mathbf{k}|\ell_p)^2 \pm 2\theta_8(|\mathbf{k}|\ell_p)\right]. \tag{36}$$

The aforementioned equation is the same as the one coming from Eq. (30), whenever $\zeta = 0$. We adopt this dispersion relation in the derivation of the kinematics of the Compton effect (for a better understanding of the procedure as indicated in [30] and [31]). Based on energy and momentum conservation, the difference in wavelengths between the scattered and incident photons, considering LQG effects, is as follows:

$$\lambda' - \lambda = \lambda_c \cos(1-\theta) - \theta_7 \ell_p^{2+2\Upsilon}\lambda_c\left\{\left[\frac{\lambda'}{\lambda}\left(\frac{1}{\mathcal{L}}\right)^{2+2\Upsilon} + \frac{\lambda}{\lambda'}\left(\frac{1}{\mathcal{L}'}\right)^{2+2\Upsilon}\right] - \left[\left(\frac{1}{\mathcal{L}}\right)^{2+2\Upsilon} + \left(\frac{1}{\mathcal{L}'}\right)^{2+2\Upsilon}\right]\right.$$
$$\left. + \frac{1}{\lambda_c}\left[\lambda'\left(\frac{1}{\mathcal{L}}\right)^{2+2\Upsilon} + \lambda\left(\frac{1}{\mathcal{L}'}\right)^{2+2\Upsilon}\right]\right\} - \theta_3 \ell_p^2 8\pi^2 \lambda_c \left[\left(\frac{1}{\lambda^2} + \frac{1}{\lambda'^2}\right) - \left(\frac{\lambda'}{\lambda^3} + \frac{\lambda}{\lambda'^3}\right)\right.$$
$$\left. - \frac{1}{\lambda_c}\left(\frac{\lambda'}{\lambda^2} - \frac{\lambda}{\lambda'^2}\right)\right] \pm \theta_8 \ell_p 4\pi\lambda_c\left[\left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right) + \left(\frac{\lambda'}{\lambda^2} + \frac{\lambda}{\lambda'^2}\right) + \frac{1}{\lambda_c}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'}\right)\right]. \tag{37}$$

Now, let us simplify the equation above. First, let us disregard the third term on the right-hand side, as it is of second order in the $\ell_p$ scale. We are interested in maintaining only first-order terms in the LQG parameters. We adopt the moving scale, which is related to the characteristic scale $\mathcal{L}$ and the momentum by $\mathcal{L} = k^{-1}$. Our result for the shift in the photon wavelength is cast below:

$$\lambda' - \lambda = \lambda_e(1-\cos\theta) - \theta_7\lambda_e\left\{\frac{\lambda'}{\lambda}(\ell_p k)^{2+2\Upsilon} + \frac{\lambda}{\lambda'}(\ell_p k')^{2+2\Upsilon}\right.$$
$$- (\ell_p k)^{2+2\Upsilon} - (\ell_p k')^{2+2\Upsilon} + \frac{1}{\lambda_e}\left[\lambda'(\ell_p k)^{2+2\Upsilon}\right.$$
$$\left.\left. + \lambda(\ell_p k')^{2+2\Upsilon}\right]\right\} \pm 4\pi\theta_8\ell_p\lambda_e\left[\left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right)\right.$$
$$\left. + \left(\frac{\lambda'}{\lambda^2} + \frac{\lambda}{\lambda'^2}\right) + \frac{1}{\lambda_e}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'}\right)\right]. \tag{38}$$

Here, we define the Compton wavelength $\lambda_e = h/mc$, where $m$ denotes the electron mass. It should be noted that, at the right side of Eq. (38), we have the helicity term, where we choose the value $\Upsilon = -1/2$, in such a way that our framework can be qualitatively reproduced. Thus, considering that $k = 2\pi/\lambda$, the equation becomes

$$\lambda' - \lambda = \lambda_e(1-\cos\theta) - 2\pi\theta_7\ell_p\lambda_e\left[\left(\frac{\lambda'}{\lambda^2} + \frac{\lambda}{\lambda'^2}\right) - \left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right)\right.$$
$$\left. + \frac{1}{\lambda_e}\left(\frac{\lambda'}{\lambda} - \frac{\lambda}{\lambda'}\right)\right] \pm 4\pi\theta_8\ell_p\lambda_e\left[\left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right)\right.$$
$$\left. + \left(\frac{\lambda'}{\lambda^2} + \frac{\lambda}{\lambda'^2}\right) + \frac{1}{\lambda_e}\left(\frac{\lambda'}{\lambda} + \frac{\lambda}{\lambda'}\right)\right]. \tag{39}$$

It is noteworthy that the term $\theta_8$ still corresponds to birefringence effects. As a result, the total rotation angle between two oppositely polarized photons with the same energy can be expressed as [32−34]

$$|\Delta\theta(E,z)| \simeq \frac{2\theta_8\ell_p E^2}{H_0}\int_0^z \frac{(1+z')\mathrm{d}z'}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}. \tag{40}$$

The best constraint on the rotation angle $\theta_8$ in the literature [34] can be transformed into a restriction $\theta_8 \leq 10^{-16}$. Thus, the $\theta_8\ell_p\lambda_e$ component is on the order of $10^{-63}$. Therefore, we also cancel out the term multiplied by these components. Consequently, the expression for the Compton effect with the LQG corrections becomes

$$\lambda' - \lambda = \lambda_e(1-\cos\theta) - 2\pi\theta_7\ell_p\lambda_e\left[\left(\frac{\lambda'}{\lambda^2} + \frac{\lambda}{\lambda'^2}\right)\right.$$
$$\left. - \left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right) + \frac{1}{\lambda_e}\left(\frac{\lambda'}{\lambda} - \frac{\lambda}{\lambda'}\right)\right]. \tag{41}$$

Taking the limit $\theta_7 \to 0$ of the only remaining angular parameter, we recover the conventional wavelength shift of the usual Compton effect. Another important characteristic of the LQG-corrected Compton effect is the absence of influences from the LQG parameters on the scattering angle, $\theta$, of the deflected photon. On qualitative grounds, we believe this is due to the inherent anisotropy present in the dispersion relation (36) we have utilized.

To extract an estimate of the LQG contribution to the kinematics of the Compton effect, it should be noted that, from Eq. (41), the shift in the wavelength of the photon, $\Delta\lambda = \lambda' - \lambda$, splits into two pieces. Specifically, $\Delta_c\lambda$ can be denoted by the usual Compton shift, whereas $\Delta_{\mathrm{LQG}}$ stands for the effect of LQG on $\Delta\lambda$ as follows:

$$\Delta\lambda = \Delta_c\lambda + \Delta_{\mathrm{LQG}}\lambda, \tag{42}$$

we may evaluate how they compete by deriving the ratio $\Delta_{\mathrm{LQG}}\lambda/\Delta_c\lambda$.

According to the inspection conducted by Li and Ma in a previous study [8], the factor $\theta_7\ell_p$, in natural units, turns out to be negative and can be estimated as

$$\theta_7 \ell_p \sim -2.8 \times 10^{-18}\ \mathrm{GeV}^{-1}; \tag{43}$$

in length units, and this corresponds to

$$\theta_7 \ell_p \sim -5.49 \times 10^{-32}\ \mathrm{cm}. \tag{44}$$

Now, by replacing $\lambda'$ as $\lambda' = \lambda + \Delta\lambda$ in Eq. (41) and by considering that $\Delta_{\mathrm{LQG}}\lambda \ll \lambda$ (terms in higher powers are neglected), we obtain

$$\frac{\Delta_{\mathrm{LQG}}\lambda}{\Delta_c \lambda} \simeq -4\pi \frac{\theta_7 \ell_p}{\lambda}. \tag{45}$$

We highlight that this result does not depend on the Compton wavelength of the particle that scatters the radiation. It should be noted that the LQG contribution is more sensitive to high frequencies/short wavelengths. For typical X-ray radiations,

$$\Delta_{\mathrm{LQG}}\lambda\Big|_X \sim 10^{-21} \Delta_c \lambda\Big|_X; \tag{46}$$

For radiation in the deep gamma spectrum ($\nu \sim 10^{25}$ Hz), the estimate is

$$\Delta_{\mathrm{LQG}}\lambda\Big|_\gamma \sim 10^{-14} \Delta_c \lambda\Big|_\gamma. \tag{47}$$

## V. FINAL COMMENTS AND GENERAL DISCUSSIONS

One of the open questions of contemporary Physics is the problem of gravity quantization. A number of approaches and theories aim to achieve a consistent and testable formulation. Considering the importance of the issue, in this study, through the incorporation of LQG effects into the electromagnetic sector, we seek to gain an understanding of how the electromagnetic radiation behaves and how the effects arising from LQG modify well-known electromagnetic phenomena when compared with the Maxwellian theory. To contemplate a concrete situation, we have chosen to reassess the kinematics of the Compton effect, considering the extension of traditional electromagnetism modified by LQG correction terms. This leads to a more complex analysis, as we are now dealing with Planck-scale collisions, such as scattering processes between highly-energetic photons and electrons, for example. Our specific purpose in this investigation is to compute how the photon wavelengths shift, and thus, an imprint of new physics can be identified.

The effects of LQG on the electromagnetic sector significantly modify the original Hamiltonian of the theory, leading to substantial changes in Maxwell's equations when considering the classical formalism. One of the most important expressions in classical electromagnetic theory includes the Poynting vector, electromagnetic energy density, and shear stresses. These quantities exhibit new correction terms due to the granular nature of spacetime at the Planck scale, as pictured by LQG. The Poynting vector has new spatial contributions arising from each of the rotation-angle parameters; however, it is interesting to point out that, for the electromagnetic energy density, the only new term comes from the nonlinear contribution in the magnetic field appearing in the extended Ampère-Maxwell equation. Based on this equation, the energy-momentum tensor can be immediately obtained. Hence, we observe how LQG effects modify the equations, as the resulting expression becomes significantly larger and more complex than the usual one. Again, each of the rotation angles contributes independently. It is important to highlight that LQG squared terms will appear, providing very small contributions, but these are retained for better correspondence with numerical/phenomenological results when these equations can be tested. It should be noted that, for special regions in the LQG-parameter space and strong external magnetic fields, we can check whether the pressure as calculated from the purely spatial components of the stress tensor of the energy-momentum tensor may become negative, characterizing a potential source of dark energy.

By considering monochromatic plane wave solutions, a matrix equation is obtained for the electric field of the propagating wave. The nonlinear term in the magnetic field term, upon linearization, introduces coupled quantities into the matrix expression. As previously known in the literature, the dispersion relation reveals photon polarization conditions due to the ± sign. The expression we have derived includes all LQG terms under a square root. If the result of these quantities is negative, two important optical effects emerge: dichroism and birefringence. Once the dispersion relation is established, the calculation of the vacuum refractive index becomes straightforward. The dispersive nature of vacuum is further examined through the evaluation of the group velocity, corresponding to the photon propagation speed in the presence of LQG effects in the electromagnetic sector. This analysis necessitates a numerical/phenomenological approach to assess how LQG corrections affect the photon velocity, particularly to investigate the possible emergence of supraluminal propagation modes.

Finally, we re-examined the kinematics of the Compton effect by including the corrections from LQG in the electromagnetic sector. Given that it was not critical to the Compton effect, we disregarded the nonlinear term in the magnetic field. This significantly simplified the dispersion relation. This allowed us to determine the difference in wavelengths, and thereby, certain conditions for the parameters in the expression had to be established. This is because the theoretical and experimental studies on the Compton effect required only three decimal places to achieve a satisfactory result—work that was carried out nearly 100 years ago. However, the challenge of performing numerical calculations with the Compton effect corrected by LQG arises because this theory involves much smaller length scales and much higher energy scales when compared with the conventional Compton effect. Thus, we need ultra-sensitive detectors to conduct these measurements and compare them with the theoretical results. The astrophysical environment itself might provide the necessary conditions, with its detectors and satellites, to achieve satisfactory results by comparing the corrected Compton effect theory from LQG with astrophysical measurements.

In future studies, we will report on ongoing investigations aimed at extending our previous study to the LQG-modified Yang-Mills theory. Considering the electroweak theory, based on an SU(2) x U(1) symmetry, the idea is to show how anomalous neutral couplings involving the photon and Z0-boson, which do not appear in the SM, can arise and be used to impose bounds on the LQG parameters. This part of the problem can be solved by using the data acquired via the ATLAS and CMS collaborations of the LHC on the search for electroweak anomalous tri- and four-gauge boson vertices. We intend to present the results of our research in a forthcoming study.

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## References


[1] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory*, First edition (Cambridge: Cambridge University Press, 2014), p. 3
[2] C. Rovelli, Living Rev. Rel. **1**, 1 (1998)
[3] T. Thiemann, *Modern Canonical Quantum General Relativity*, First edition (Cambridge: Cambridge University Press, 2007), p. 1
[4] R. Gambini and J. Pullin, *A First Course in Loop Quantum Gravity*, First edition (USA: Oxford University Press, 2011), p. 104
[5] A. Ashtekar and J. Lewandowski, Class. Quant. Grav. **21**(15), R53 (2004)
[6] A. Ashtekar and P. Singh, Class. Quant. Grav. **28**(21), 213001 (2011)
[7] J. Alfaro, H. A. Morales-Tecotl, and L. F. Urrutia, Phys. Rev. D **65**, 103509 (2002)
[8] H. Li and B. Q. Ma, Phys. Lett. B **836**, 137613 (2023)
[9] G. L. L. W. Levy and J. A. Helayël-Neto, Annals Phys. **473**, 169892 (2025)
[10] J. P. S. Melo, M. J. Neves, J. M. A. Paixão *et al*., Eur. Phys. J. C **84**(9), 938 (2024)
[11] L. Shao, Z. Xiao, and B. Q. Ma, Astropart. Phys. **33**, 312 (2010)
[12] S. Zhang and B. Q. Ma, Astropart. Phys. **61**, 108 (2015)
[13] H. Xu and B. Q. Ma, Astropart. Phys. **82**, 72 (2016)
[14] H. Xu and B. Q. Ma, Phys. Lett. B **760**, 602 (2016)
[15] G. Amelino-Camelia, G. D'Amico, G. Rosati *et al*., Nature Astron. **1**, 0139 (2017)
[16] G. Amelino-Camelia, G. D'Amico, F. Fiore *et al*., Symmetry **13**(4), 541 (2021)
[17] H. Xu and B. Q. Ma, JCAP **01**, 050 (2018)
[18] Y. Liu and B. Q. Ma, Eur. Phys. J. C **78**(10), 825 (2018)
[19] H. Li and B. Q. Ma, Sci. Bull. **65**, 262 (2020)
[20] J. Zhu and B. Q. Ma, Phys. Lett. B **820**, 136518 (2021)
[21] Y. Chen and B. Q. Ma, JHEAp **32**, 78 (2021)
[22] J. M. A. Paixão, L. P. R. Ospedal, M. J. Neves *et al*., JHEP **05**, 029 (2024)
[23] J. M. A. Paixão, L. P. R. Ospedal, M. J. Neves *et al*., JHEP **10**, 160 (2022)
[24] A. H. Compton, Phys. Rev. **21**, 483 (1923)
[25] P. A. M. Dirac, Proc. Roy. Soc. Lond. A **117**, 610 (1928)
[26] J. R. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Gen. Rel. Grav. **32**, 127 (2000)
[27] J. R. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, arXiv: gr-qc/9909085 [gr-qc]
[28] J. R. Ellis, K. Farakos, N. E. Mavromatos *et al*., Astrophys. J. **535**, 139 (2000)
[29] J. Alfaro and G. Palma, Phys. Rev. D **67**, 083003 (2003)
[30] R. Eisberg and R. Resnick, *FÍSICA QUÂTICA: Átomos, Sólidos, moléculas, Núcleos e Partículas*, First edition (Elsevier, 1979), p. 34
[31] I. Silva, O. Freire Jr., and A. P. B. da Silva, Rev. Bras. Ensino Fís. **36** 1601 (2014)
[32] R. Ruffini, G. V. Vereshchagin, and S. S. Xue, Astrophys. Space Sci. **361**, 82 (2016)
[33] T. Jacobson, S. Liberati, and D. Mattingly, Annals Phys. **321**, 150 (2006)
[34] S. Liberati and L. Maccione, Ann. Rev. Nucl. Part. Sci. **59**, 245 (2009)