

Searching for vectorlike T quarks in the $T \rightarrow tZ$ channel at a future muon-proton collider*

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Abstract: We investigate the discovery potential for a singly produced vector-like top quark T in the $T \rightarrow tZ$ decay channel at future muon-proton (μp) colliders, considering three benchmark center-of-mass energies: $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV. The baseline signal final state consists of a semileptonic top decay $t \rightarrow bW \rightarrow b\ell\nu$ and a hadronic $Z \rightarrow q\bar{q}$ decay, in which the collimated quark pair from the Z boson is reconstructed as a single large-radius fat jet J . Full detector-level Monte Carlo simulations are performed for both the signal process and the dominant Standard Model backgrounds. We derive the 5σ discovery contours and the 95% confidence-level exclusion limits in the two-dimensional parameter space spanned by the VLQ mass m_T and the effective coupling g^* . A comparison with the $T \rightarrow Wb$ channel and with searches at other collider facilities shows that the $T \rightarrow tZ$ decay mode provides complementary sensitivity and offers unique advantages for probing high-mass vector-like T quarks in the boosted-jet topology.

Keywords: vectorlike quarks, muon-proton colliders, fat jet

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I. INTRODUCTION

Vector-like quarks (VLQs) with masses at the TeV scale are widely predicted in various extensions of the Standard Model (SM), such as little Higgs models [1, 2], composite Higgs models [3, 4], two-Higgs-doublet models [5–10], and other extended models [11–14]. A common feature of these new particles is that their left- and right-handed chiral components transform in the same way under the electroweak (EW) symmetry group of the SM [15]. Unlike chiral quarks, VLQ bare mass terms are gauge invariant and therefore can avoid the stringent constraints from Higgs boson data [16, 17]. Moreover, VLQs have the potential to stabilize the EW vacuum [18, 19], address the so-called Cabibbo-Kobayashi-Maskawa (CKM) unitarity problem [20–24], and may also provide explanations for various experimental anomalies, such as the W -mass anomaly [25–29]. VLQs can give rise to a rich variety of phenomena at the Large Hadron Collider (LHC) and future high-energy colliders (see, e.g., [30–65]).

The VLQ T state (denoted T hereafter) carries an electric charge of $+2e/3$ and can appear in various weak-isospin multiplets. For a singlet VLQ- T , three decay

modes are possible: $T \rightarrow bW$, tZ , and th (unless an extended Higgs sector is present [45, 48]). In the high-mass limit, the branching ratios (BRs) satisfy $\text{BR}(T \rightarrow th) \approx \text{BR}(T \rightarrow tZ) \approx \frac{1}{2} \text{BR}(T \rightarrow Wb)$. To date, the ATLAS and CMS Collaborations have conducted extensive searches for pair-produced VLQs, obtaining mass constraints at the 95% confidence level (CL) [66–77]. For instance, the ATLAS Collaboration has set a lower bound of about 1.36 TeV on the mass of a singlet VLQ- T using 140 fb^{-1} of data [66]. The CMS Collaboration has excluded a singlet VLQ- T mass below 1.46 TeV at 95% CL using 138 fb^{-1} of pp collision data in leptonic final states [68]. In addition, VLQ- T can be singly produced at the LHC via EW interactions, and these processes are highly sensitive to the couplings between VLQs and SM quarks [78, 79]. Recent searches by the ATLAS and CMS Collaborations have set limits on VLQ masses and couplings using Run 2 data [80–87].

In this work, we consider a muon-proton collider with multi-TeV beam energies [88–98]. Compared with the LHC, BSM studies at such a machine typically benefit from significantly reduced QCD backgrounds. Recent phenomenological studies of a future μp collider can be

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found in Refs. [99–110]. Very recently, the authors of Ref. [111] systematically investigated the single production of vector-like quarks T and Y decaying into Wb at a future μp collider, considering both the leptonic and hadronic decay channels of the W boson. Their results show that the hadronic channel, in which the W decay products merge into a fat jet, enables a 5σ discovery up to $m_T = 3750$ GeV and $m_Y = 4100$ GeV at $\sqrt{s} = 9.16$ TeV with an integrated luminosity of 100 fb^{-1} , thereby significantly extending the reach beyond LHC capabilities. Nevertheless, the complementary decay mode $T \rightarrow tZ$ has not yet been explored in the context of a μp collider. In the high-mass limit, the branching ratio of $T \rightarrow tZ$ is approximately half that of $T \rightarrow Wb$, i.e., $\text{BR}(T \rightarrow tZ) \approx \frac{1}{2} \text{BR}(T \rightarrow Wb)$, making this channel a promising and complementary avenue for VLQ searches. Moreover, the distinctive final-state topology, featuring a leptonically decaying top quark and a hadronically decaying Z boson (reconstructed as a fat jet), provides unique kinematic handles for suppressing SM backgrounds.

Motivated by the above considerations, we focus on the observability of singly produced vector-like T quarks at a future μp collider via the $T \rightarrow tZ$ decay channel. The top quark decays semileptonically as $t \rightarrow bW \rightarrow b\ell\nu$, and the Z boson decays hadronically into $q\bar{q}$, with the collimated quark pair reconstructed as a single fat jet J . We perform full detector-level Monte Carlo simulations for three benchmark center-of-mass (c.m.) energies $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, optimize the kinematic event selection cuts, and extract the 5σ discovery contours and 95% CL exclusion limits in the two-dimensional (g^*, m_T) parameter plane. We further provide a dedicated comparison with the $T \rightarrow Wb$ channel to demonstrate the complementary discovery potential of these two decay signatures.

The remainder of this paper is organized as follows. In Sec. II, we introduce the simplified effective model of the vector-like quark T and calculate its single-production cross sections at the μp collider for three different c.m. energies. In Sec. III, we investigate the observability of the signal through the full decay chain $T \rightarrow tZ \rightarrow bWZ \rightarrow b\ell\nu J$, which gives rise to a final state with missing transverse momentum from neutrinos. We perform full detector-level Monte Carlo simulations, construct optimized kinematic selection cuts, and present the resulting 5σ discovery and 95% CL exclusion reaches. Section IV presents the main conclusions and provides additional discussions of alternative decay topologies.

II. SIMPLIFIED MODEL AND SINGLE T PRODUCTION AT A μp COLLIDER

In this work, we adopt an $SU(2)_L$ singlet vector-like quark T as our benchmark particle. This minimal particle content is well motivated by numerous ultraviolet (UV)-

complete BSM frameworks and has been widely used as a benchmark scenario for collider searches targeting vector-like quarks. This simplified setup allows us to perform a largely model-independent leading-order analysis, without introducing additional free parameters associated with extra exotic particles.

Following the conventions of Ref. [31], the general effective Lagrangian describing the singlet VLQ- T is written as

$$\mathcal{L}_T = \frac{gg^*}{2} \left\{ \frac{1}{\sqrt{2}} [\bar{T}_L W_\mu^+ \gamma^\mu b_L] + \frac{1}{2c_W} [\bar{T}_L Z_\mu \gamma^\mu t_L] - \frac{m_T}{2m_W} [\bar{T}_R h t_L] - \frac{m_t}{2m_W} [\bar{T}_L h t_R] \right\} + \text{H.c.}, \quad (1)$$

where g is the $SU(2)_L$ gauge coupling constant, $c_W = \cos\theta_W$, and θ_W is the Weinberg angle. Thus, only two model parameters remain: the VLQ- T mass m_T and the effective coupling g^* , defined in units of the SM gauge coupling. As mentioned, the singlet VLQ- T has three possible decay modes: $T \rightarrow bW$, tZ , and th . For $m_T \geq 1$ TeV, the branching ratios satisfy $\text{BR}(T \rightarrow th) \approx \text{BR}(T \rightarrow tZ) \approx \frac{1}{2} \text{BR}(T \rightarrow Wb)$, as expected from the Goldstone boson equivalence theorem [112–116]. For $m_T = 1.5$ (2.0) TeV, g^* is constrained to be smaller than about 0.37 (0.59) [117, 118]. In this work, we adopt a phenomenological upper limit of $g^* \leq 0.5$ for $m_T \geq 1.5$ TeV.

The VLQ- T can be singly produced in μp collisions via Wb fusion and subsequently decays through $T \rightarrow tZ$. Leading-order (LO) cross sections are computed with MadGraph5-aMC@NLO [119], using the default NNPDF23L01 parton distribution functions (PDFs) [120] together with the standard renormalization and factorization scales. Three representative collision setups are considered, corresponding to c.m. energies $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, with a fixed proton beam energy $E_p = 7$ TeV and muon beam energies $E_\mu = 1.0, 1.5$, and 3.0 TeV, respectively. In our leading-order simulations, the muon beam is assumed to be unpolarized.

In Fig. 1, we show the effective cross sections $\sigma \times \text{BR}(T \rightarrow tZ)$ as a function of m_T for the benchmark coupling $g^* = 0.1$, where all conjugate production processes are summed. The cross section decreases gradually with increasing m_T , driven by the reduced available phase space. For $g^* = 0.1$ and $m_T = 2$ TeV, the corresponding cross sections are 0.19, 0.61, and 2.74 fb at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, respectively. As expected, the single-production cross section scales quadratically with the effective coupling, i.e., $\sigma \propto (g^*)^2$.

III. COLLIDER SIMULATION AND ANALYSIS

We perform Monte Carlo (MC) simulations for both the signal and SM background events corresponding to

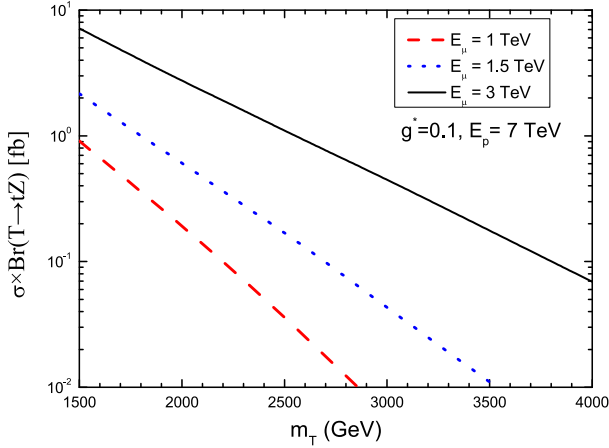


Fig. 1. (color online) Effective cross section $\sigma \times \text{BR}(T \rightarrow tZ)$ as a function of the VLQ- T mass for $g^* = 0.1$ at a μp collider with three different center-of-mass energies.

the process

$$\mu^+ p \rightarrow \bar{\nu}_\mu T(\rightarrow tZ) \bar{b} \rightarrow \bar{\nu}_\mu (b\ell^+ \nu_\ell) (q\bar{q}) \bar{b},$$

where the intermediate W^+ boson decays leptonically as $W^+ \rightarrow \ell^+ \nu_\ell$ with $\ell = e, \mu$, and the Z boson decays hadronically via $Z \rightarrow q\bar{q}$. The resulting final state comprises an isolated charged lepton, one b -tagged jet originating from the top-quark decay, a single fat jet J reconstructed from the collimated products of the $Z \rightarrow q\bar{q}$ decay, and missing transverse momentum arising from undetected neutrinos. The kinematic features of all charged-conjugate production processes are analogous, and their contributions are included in our event yields. A representative tree-level Feynman diagram for this signal process is shown in Fig. 2.

The dominant irreducible SM backgrounds are categorized as follows:

- $\mu p \rightarrow \nu_\mu t j$, where the top quark decays semileptonically via $t \rightarrow Wb \rightarrow b\ell\nu$;
- $\mu p \rightarrow \nu_\mu t Z j$, with semileptonic top quark decay,

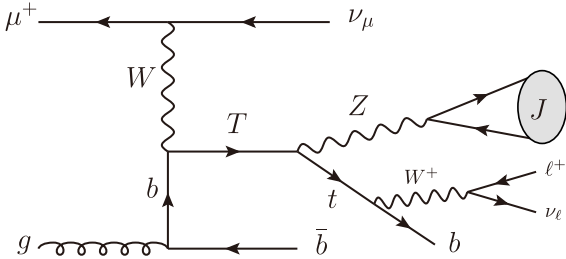


Fig. 2. Representative tree-level Feynman diagram for single VLQ- T production at a μp collider with the decay cascade $T \rightarrow tZ$. The top decays semileptonically via $t \rightarrow W^+ b \rightarrow \ell^+ \nu_\ell b$, and $Z \rightarrow q\bar{q}$ is reconstructed as a merged fat jet J .

$t \rightarrow Wb \rightarrow b\ell\nu$, and hadronic $Z \rightarrow q\bar{q}$ decay; $\mu p \rightarrow \nu_\mu WZj$, with leptonic $W \rightarrow \ell\nu$ and hadronic $Z \rightarrow q\bar{q}$ decays.

In addition, we study reducible backgrounds dominated by QCD multi-jet processes, including $\mu p \rightarrow \nu_\mu jjj$ and $\mu p \rightarrow \nu_\mu b\bar{b}j$. These backgrounds enter the signal region through two main fake sources: charged hadrons that fail the isolation criteria and are misreconstructed as prompt light leptons, and fake missing transverse momentum $\cancel{E}_T$ arising from fluctuations in the jet energy scale. To suppress these QCD contributions, we apply a set of tight kinematic vetoes throughout the analysis. A stringent lepton isolation working point largely eliminates fake leptons from hadronic activity, while complementary requirements on the number of b -tagged jets, a high minimum $\cancel{E}_T$ threshold, and a dedicated boosted Z -jet veto collectively further reduce the QCD background yields.

Taking the $\mu p \rightarrow \nu_\mu b\bar{b}j$ process as a concrete example, its inclusive cross section at a c.m. energy of $\sqrt{s} = 5.29$ TeV is approximately 440 fb. After all analysis selections are imposed, the overall event acceptance efficiency is below the 10^{-7} level, as estimated from full MC simulation. Although these reducible QCD backgrounds do not vanish entirely, their total contribution to the background budget is far below the statistical uncertainty of the expected signal yield. For this reason, they are safely neglected in the subsequent signal extraction procedure.

Table 1 lists the cross sections times branching ratios for all considered irreducible backgrounds at the three c.m. energies.

Signal and background events are generated at LO with MadGraph5-aMC@NLO using the same PDFs. Parton showering and hadronization are performed with Pythia 8.3 [121]. Detector simulation is carried out with Delphes 3.4.2 [122] using the standard LHeC detector card. Small-radius jets are reconstructed using the anti- k_t algorithm [123] with $R = 0.4$. Fat jets are reconstructed using the Cambridge-Aachen algorithm [124, 125] with $R = 0.8$. The b -tagging efficiency is 80%, with misidentification rates of 0.1% for light-flavor jets and 5% for charm jets. The analysis is performed with MadAnalysis 5 [126, 127].

Basic object selection cuts are:

$$p_T^{\ell/b/j} > 15 \text{ GeV}, \quad |\eta_\ell| < 2.5, \quad |\eta_{j/b}| < 5, \\ \Delta R_{xy} > 0.4, \quad \cancel{E}_T > 30 \text{ GeV},$$

where $p_T^{\ell/b/j}$ and $\eta_{\ell/b/j}$ denote the transverse momentum and pseudorapidity of leptons, b -jets, and light jets, respectively. Here, $\Delta R(x, y) = \sqrt{\Delta\phi^2 + \Delta\eta^2}$ is the separation in the pseudorapidity-azimuth plane between the objects x and y , where $x, y = \ell, b, j$.

Table 1. Cross sections multiplied by branching ratios $\sigma \times \mathcal{B}$ (in fb) for three SM backgrounds at three center-of-mass energies $\sqrt{s} = 5.29, 6.48, 9.16$ TeV.

Process	Decay Channel	$\sqrt{s} = 5.29$	6.48	9.16
$\mu p \rightarrow \nu_\mu t j$	$t \rightarrow b \ell \nu$	12840	17800	29930
$\mu p \rightarrow \nu_\mu t Z j$	$Z \rightarrow q \bar{q}$	31.6	49.3	99.4
$\mu p \rightarrow \nu_\mu W Z j$	$W/Z \rightarrow \ell \nu / q \bar{q}$	18.3	26.6	48.2

We consider three signal benchmark points with $g^* = 0.1$: T_{1500} ($m_T = 1500$ GeV), T_{2000} ($m_T = 2000$ GeV), and T_{2500} ($m_T = 2500$ GeV).

Figure 3 shows the normalized kinematic distributions of the signal and SM backgrounds samples at $\sqrt{s} = 5.29$ TeV, including the transverse momentum of the leading b -jet (p_T^b), the angular separation between the b -jet and the lepton ($\Delta R_{b,\ell}$), the fat-jet transverse mo-

mentum (p_T^J), the fat-jet invariant mass (M_J), and the invariant mass of the lepton- b -jet-fat-jet system ($M_{b\ell J}$). The signal events exhibit harder p_T spectra and a pronounced peak in M_J near the Z -boson mass.

Based on these distributions, we impose the following selection criteria:

- Cut 1: At least one fat jet satisfying $p_T^J > 400$ GeV and $|M_J - m_Z| < 10$ GeV.

- Cut 2: Require at least one isolated lepton and at least one b -tagged jet from the top decay ($N_b \geq 1$), with the leading b -jet satisfying $p_T^b > 150$ GeV and $\Delta R_{b,\ell} < 1.0$.

- Cut 3: $M_{b\ell J} > 800$ GeV.

The kinematic distributions at $\sqrt{s} = 6.48$ and 9.16 TeV are similar to those at 5.29 TeV, with differences in

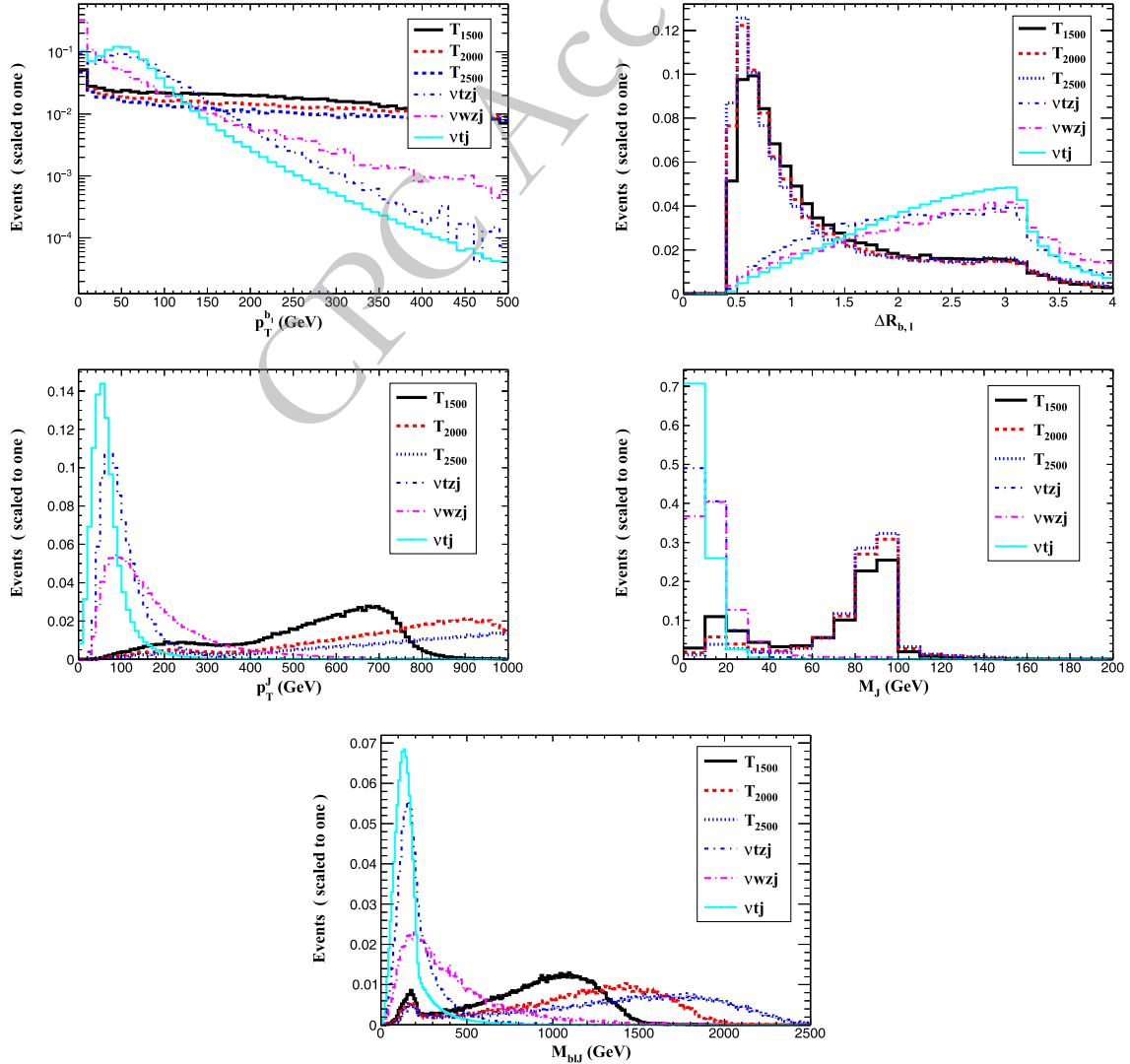


Fig. 3. (color online) Normalized distributions of p_T^b , $\Delta R_{b,\ell}$, p_T^J , M_J , and $M_{b\ell J}$ for the three signal benchmarks with $m_T = 1500$ GeV (solid), 2000 GeV (dashed), and 2500 GeV (dotted), together with the SM backgrounds at $\sqrt{s} = 5.29$ TeV.

the key shape variables below the per-mille level; they are therefore omitted for brevity. The same set of selection cuts is applied uniformly across all three c.m. energies, as the optimal cut values were found to depend only weakly on $\sqrt{s}$. The corresponding cut-flow tables are presented in Table 2. As shown, all SM backgrounds are suppressed very efficiently, while the signal efficiencies remain at the percent level after the full selection, sub-

stantially higher than those of the backgrounds. The total SM background cross sections after all cuts amount to 0.0078 fb at 5.29 TeV, 0.014 fb at 6.48 TeV, and 0.029 fb at 9.16 TeV.

We estimate the expected discovery ($\mathcal{Z}_{\text{dis}}$) and exclusion ($\mathcal{Z}_{\text{exc}}$) significances using the formulas from Ref. [128]:

$$\mathcal{Z}_{\text{dis}} = \sqrt{2 \left[(s+b) \ln \left(\frac{(s+b)(1+\delta_{\text{sys}}^2 b)}{b + \delta_{\text{sys}}^2 b(s+b)} \right) - \frac{1}{\delta_{\text{sys}}^2} \ln \left(1 + \frac{\delta_{\text{sys}}^2 s}{1 + \delta_{\text{sys}}^2 b} \right) \right]}, \quad (2)$$

$$\mathcal{Z}_{\text{exc}} = \sqrt{2 \left[s - b \ln \left(\frac{b+s+x}{2b} \right) - \frac{1}{\delta_{\text{sys}}^2} \ln \left(\frac{b-s+x}{2b} \right) \right] - (b+s-x) \left(1 + \frac{1}{\delta_{\text{sys}}^2 b} \right)}, \quad (3)$$

with $x = \sqrt{(s+b)^2 - 4\delta_{\text{sys}}^2 s b^2 / (1 + \delta_{\text{sys}}^2 b)}$, where s and b denote the numbers of signal and background events, respectively, and δ_{sys} represents the systematic uncertainty. The integrated luminosity is assumed to be 1000 fb^{-1} . We take a systematic uncertainty of $\delta_{\text{sys}} = 20\%$ as the default benchmark. This choice is conservative and representative of the combined systematic effects expected at a future multi-TeV collider, including uncertainties in the jet

energy scale, b -tagging efficiency, lepton identification, missing transverse momentum resolution, and luminosity determination. To illustrate the impact of this assumption, we also present results for the idealized scenario with no systematic uncertainty ($\delta_{\text{sys}} = 0$).

Figure 4 shows the 95% CL exclusion limits and 5σ discovery reaches in the g^*-m_T plane for the three c.m. energies. To assess the impact of systematic uncertainties

Table 2. Cut flow of cross sections (in fb) for the signal and SM backgrounds in the tZ channel with $g^* = 0.1$ at different c.m. energies.

	Cuts	T_{1500}	T_{2000}	T_{2500}	$\nu t j$	$\nu t Z j$	$\nu W Z j$
(a) $\sqrt{s} = 5.29 \text{ TeV}$	Basic	0.0544	0.0091	0.0013	7961	21	13.4
	Cut 1	0.026	0.005	0.00074	0.28	0.023	0.155
	Cut 2	0.0091	0.002	0.00031	0.022	0.0052	0.00053
	Cut 3	0.009	0.0198	0.0003	0.00038	0.00367	0.00024
	Total eff.	8.9%	9%	7.5%	3×10^{-7}	1.16×10^{-4}	1.32×10^{-5}
	Cuts	T_{1500}	T_{2000}	T_{2500}	$\nu t j$	$\nu t Z j$	$\nu W Z j$
(b) $\sqrt{s} = 6.48 \text{ TeV}$	Basic	0.132	0.029	0.0061	11036	31	19.4
	Cut 1	0.062	0.016	0.0036	0.43	0.049	0.266
	Cut 2	0.021	0.006	0.0014	0.03	0.0093	0.00079
	Cut 3	0.021	0.006	0.0014	0.0053	0.0083	0.00032
	Total eff.	8.94%	8.94%	7.4%	3×10^{-7}	1.69×10^{-4}	1.19×10^{-5}
	Cuts	T_{1500}	T_{2000}	T_{2500}	$\nu t j$	$\nu t Z j$	$\nu W Z j$
(c) $\sqrt{s} = 9.16 \text{ TeV}$	Basic	0.436	0.129	0.039	18557	53.7	34.2
	Cut 1	0.21	0.07	0.022	0.89	0.18	0.67
	Cut 2	0.069	0.027	0.0089	0.051	0.026	0.0014
	Cut 3	0.068	0.027	0.0089	0.009	0.019	0.00057
	Total eff.	8.55%	8.8%	7.3%	3×10^{-7}	1.92×10^{-4}	1.18×10^{-5}

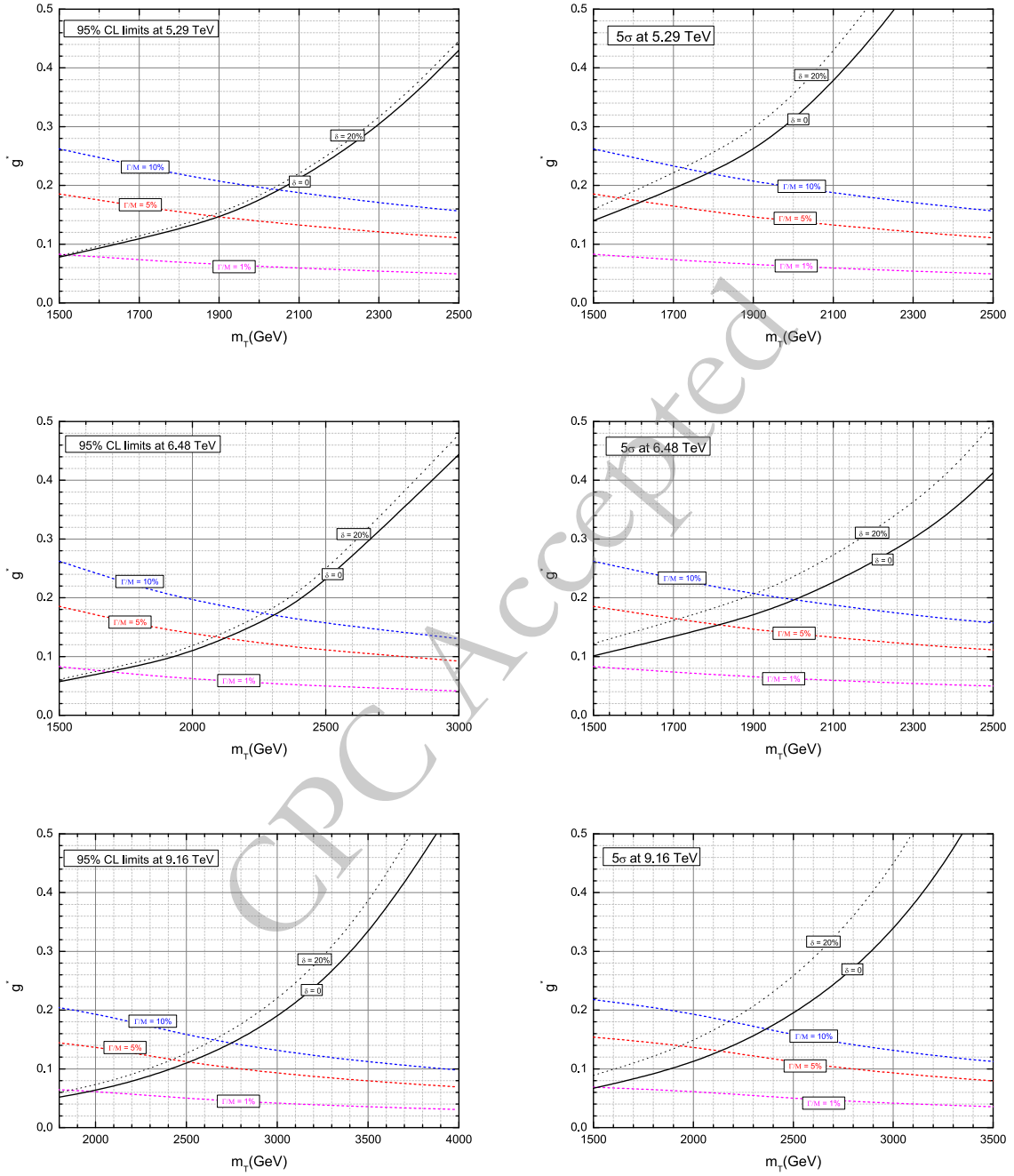


Fig. 4. (color online) 95% CL exclusion limit (left panel) and 5σ discovery reach (right panel) contour plots for the signal in the $g^* - m_T$ plane at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV with 1000 fb^{-1} and $\delta_{\text{sys}} = 0, 20\%$. Dashed lines indicate $\Gamma_T/m_T = 1\%, 5\%, 10\%$.

on the significances, we consider two scenarios: no systematic uncertainty ($\delta_{\text{sys}} = 0$) and a typical systematic uncertainty of $\delta_{\text{sys}} = 20\%$. As expected, larger systematic uncertainty degrades both the discovery potential and the exclusion sensitivity. For $\delta_{\text{sys}} = 20\%$, the 5σ discovery regions are as follows: at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.16, 0.5]$ for $m_T \in [1500, 2200]$ GeV; at $\sqrt{s} = 6.48$ TeV, $g^* \in [0.12, 0.5]$ for $m_T \in [1500, 2500]$ GeV; and at $\sqrt{s} = 9.16$ TeV, $g^* \in [0.09, 0.5]$ for $m_T \in [1500, 3100]$ GeV. The corresponding 95% CL exclusion regions are $g^* \in [0.08, 0.44]$

for $m_T \in [1500, 2500]$ GeV at 5.29 TeV, $g^* \in [0.06, 0.48]$ for $m_T \in [1500, 3000]$ GeV at 6.48 TeV, and $g^* \in [0.06, 0.5]$ for $m_T \in [1500, 3700]$ GeV at 9.16 TeV.

So far, we have treated the VLQ- T signals within the narrow-width approximation (NWA). In this approximation, the total width Γ_T is computed at leading order as the sum of the partial widths for the $T \rightarrow bW, tZ$, and th decays, derived from the Lagrangian in Eq. (1). This treatment is valid when the width-to-mass ratio Γ_T/m_T is at the per-mille level. However, as demonstrated in Refs.

[129, 130], when Γ_T/m_T reaches the percent level or higher, a full Breit-Wigner prescription should be adopted. To quantify the impact of finite-width effects on our sensitivity projections, we overlay in Fig. 4 contours of constant Γ_T/m_T for three representative values: 1%, 5%, and 10%. For a fixed ratio $\Gamma_T/m_T = 10\%$ and a systematic uncertainty of 20%, the 5σ discovery reach for the VLQ- T mass is approximately 1700, 1900, and 2200 GeV at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, respectively, with corresponding 95% CL exclusion lower limits of about 2050, 2280, and 2650 GeV at the three c.m. energies.

We also note that theoretical cross-section calculations including finite-width effects and non-resonant contributions are reliable only for relative T -quark widths up to $\Gamma_T/m_T \sim 50\%$ (see, e.g., Ref. [131]). Our results are therefore presented only within this restricted regime. A complete off-shell calculation, including interference effects over the full parameter space, will be pursued in future work.

To facilitate a direct comparison of the VLQ- T search capabilities across different collider scenarios, we summarize in Table 3 the exclusion and discovery reaches in the (g^*, m_T) parameter space, and present in Fig. 5 a visual comparison of the 95% CL exclusion limits from current LHC bounds (based on the combined ATLAS and CMS exclusion limits for the singlet VLT scenario from Ref. [117]), projected HL-LHC sensitivities [134, 135], and our results for two representative μp collider energies, $\sqrt{s} = 5.29$ TeV and 6.48 TeV. For the singlet VLQ- T scenario, the mixing parameter κ used in the LHC analyses of Ref. [117] is related to the effective coupling g^* adopted in this work by $g^* = \sqrt{2}\kappa$. After applying this

conversion, the current LHC bounds exclude $g^* \gtrsim 0.37$ at $m_T \sim 1.5$ TeV, whereas our μp collider analysis can probe g^* values as low as 0.06 – 0.08 at the same mass, demonstrating a significant improvement in sensitivity. Our analysis assumes a conservative systematic uncertainty of 20%, whereas the other studies adopt different assumptions (see the corresponding references for details). As demonstrated clearly in both the table and the figure, a future μp collider would provide significantly enhanced and complementary sensitivity for singlet VLQ- T searches, particularly in the high-mass region, owing to a cleaner collision environment and substantially reduced QCD backgrounds compared with the LHC and HL-LHC.

It is instructive to compare the sensitivities obtained in this work with those reported in the recent study of the $T \rightarrow Wb$ channel at the same μp collider [111]. That analysis, performed with 100 fb^{-1} and a 10% systematic uncertainty, reported 5σ discovery reaches of $m_T = 2520, 2900$, and 3750 GeV at $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV, respectively, assuming $g^* = 0.5$. In the present work, with 1000 fb^{-1} but a more conservative 20% systematic uncertainty, we obtain 5σ discovery limits of $m_T = 2200, 2500$, and 3100 GeV at the three center-of-mass energies for the same coupling, with corresponding 95% CL exclusion limits of $m_T = 2500, 3000$, and 3700 GeV. Although the branching ratio of the $T \rightarrow tZ$ channel is roughly half that of $T \rightarrow Wb$, the two channels are complementary beyond their branching fractions. The $T \rightarrow tZ$ signature features a boosted fat jet from the Z boson, whereas the $T \rightarrow Wb$ channel relies on a fat jet from the W boson. These distinct final-state topologies imply different sensitivities to

Table 3. Selected results from searches for the singlet VLQ- T at various high-energy colliders are presented. Here, the symbol “\” indicates that no relevant results are available in the reference. The results in this work correspond to a moderate systematic uncertainty of 20% at a μp collider with an integrated luminosity of 1000 fb^{-1} .

Channel	Data Set	Excluding capability		Discovery capability		Reference
		g^*	m_T/TeV	g^*	m_T/TeV	
$T \rightarrow tZ (Z \rightarrow \ell^+ \ell^-)$	LHC @14 TeV, 3 ab^{-1}	[0.06, 0.25]	[0.9, 1.5]	[0.10, 0.42]	[0.9, 1.5]	[132]
$T \rightarrow th$	LHC @14 TeV, 3 ab^{-1}	[0.16, 0.5]	[1.0, 1.6]	[0.24, 0.72]	[1.0, 1.6]	[133]
$T \rightarrow bW^+$	LHC @14 TeV, 3 ab^{-1}	[0.19, 0.5]	[1.3, 2.4]	[0.31, 0.5]	[1.3, 1.9]	[134]
$T \rightarrow tZ (Z \rightarrow \nu \bar{\nu})$	LHC @14 TeV, 3 ab^{-1}	[0.09, 0.5]	[1.0, 1.84]	[0.14, 0.5]	[1.0, 1.6]	[135]
$T \rightarrow bW^+$	$e\gamma$ collider @2 TeV, 1 ab^{-1}	[0.13, 0.5]	[0.8, 1.6]	\	\	[136]
$T \rightarrow tZ$	$e\gamma$ collider @3 TeV, 3 ab^{-1}	[0.15, 0.23]	[1.3, 2.0]	[0.23, 0.5]	[1.3, 2.0]	[137]
$T \rightarrow th$	$e\gamma$ collider @3 TeV, 3 ab^{-1}	[0.14, 0.50]	[1.3, 2.0]	[0.27, 0.5]	[1.3, 2.0]	[138]
$T \rightarrow bW^+$	e^+e^- collider @3 TeV, 5 ab^{-1}	[0.15, 0.40]	[1.5, 2.6]	[0.24, 0.44]	[1.5, 2.4]	[139]
$T \rightarrow tZ$	e^+e^- collider @3 TeV, 5 ab^{-1}	[0.19, 0.40]	[1.3, 2.5]	[0.31, 0.5]	[1.3, 2.3]	[140]
$T \rightarrow tZ \rightarrow b\ell\nu J$	μp collider @5.29 TeV	[0.08, 0.44]	[1.5, 2.5]	[0.16, 0.5]	[1.5, 2.2]	This work
	μp collider @6.48 TeV	[0.06, 0.48]	[1.5, 3.0]	[0.12, 0.5]	[1.5, 2.5]	
	μp collider @9.16 TeV	[0.09, 0.4]	[1.5, 3.7]	[0.09, 0.5]	[1.5, 3.1]	

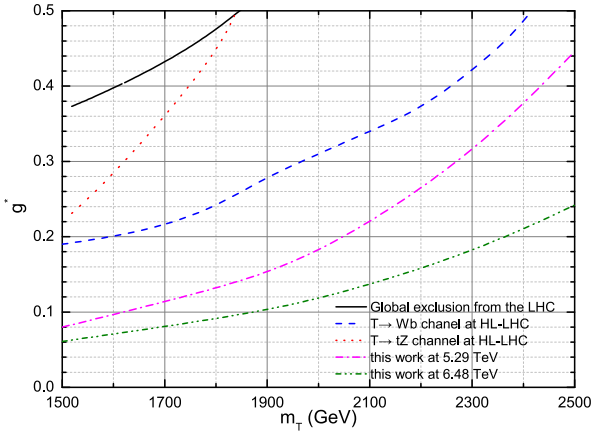


Fig. 5. (color online) Comparison of the 95% CL exclusion limits on the singlet VLQ- T in the (g^*, m_T) plane among current LHC bounds [117], projected HL-LHC sensitivities at $\sqrt{s} = 14$ TeV with 3 ab^{-1} [134, 135], and our results for two representative μp collider energies, $\sqrt{s} = 5.29$ TeV and 6.48 TeV, assuming an integrated luminosity of 1000 fb^{-1} and a systematic uncertainty of 20%.

jet energy scale uncertainties and QCD background modeling. A combined analysis of both decay modes would therefore not only increase the overall signal yield but also provide a robust cross-check against systematic biases, thereby strengthening the case for a potential discovery at a future μp collider.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we investigate the discovery potential of a future μp collider for a heavy singlet VLQ- T produced via the single-production mode $\mu g \rightarrow \nu_\mu b T$, followed by the decay chain $T \rightarrow tZ$, with $t \rightarrow bW$ (leptonic W decay) and $Z \rightarrow q\bar{q}$ (reconstructed as a single fat jet). Within a simplified two-parameter framework defined by m_T and g^* , we perform a detailed detector-level simulation for three center-of-mass energies: $\sqrt{s} = 5.29, 6.48$, and 9.16 TeV. After optimizing the event selection criteria, we derive the 5σ discovery and 95% CL exclusion limits in the (g^*, m_T) parameter space.

Assuming an integrated luminosity of 1000 fb^{-1} and a systematic uncertainty of 20%, the 5σ discovery regions are as follows: at $\sqrt{s} = 5.29$ TeV, $g^* \in [0.16, 0.5]$ for $m_T \in [1500, 2200]$ GeV; at $\sqrt{s} = 6.48$ TeV, $g^* \in [0.12, 0.5]$ for $m_T \in [1500, 2500]$ GeV; and at $\sqrt{s} = 9.16$ TeV, $g^* \in [0.09, 0.5]$ for $m_T \in [1500, 3100]$ GeV. The corresponding 95% CL exclusion regions are $g^* \in [0.08, 0.44]$ for $m_T \in [1500, 2500]$ GeV at 5.29 TeV, $g^* \in [0.06, 0.48]$ for $m_T \in [1500, 3000]$ GeV at 6.48 TeV, and $g^* \in [0.06, 0.5]$ for $m_T \in [1500, 3700]$ GeV at 9.16 TeV.

These sensitivities are competitive with, and complementary to those obtained from the $T \rightarrow Wb$ channel, as well as from other collider platforms.

For completeness, we briefly comment on three alternative final-state topologies of the $T \rightarrow tZ$ cascade that are not analyzed in this work, in contrast to our baseline channel with a semileptonic top decay and a hadronic Z decay. The omitted signal modes are: · invisible $Z \rightarrow \nu\bar{\nu}$ decay with a fully hadronic top decay, $t \rightarrow bW \rightarrow jjb$; · invisible $Z \rightarrow \nu\bar{\nu}$ decay with semileptonic top decay $t \rightarrow bW \rightarrow b\ell\nu$; · Leptonic $Z \rightarrow \ell^+\ell^-$ decay with fully hadronic top decay $t \rightarrow bW \rightarrow jjb$.

Preliminary MC estimates indicate that all three alternative channels suffer from two critical drawbacks relative to our baseline. First, their effective signal cross sections are significantly smaller, resulting in far fewer signal events for the same integrated luminosity. Second, they are subject to substantially larger SM backgrounds. In particular, the two invisible Z channels are overwhelmed by irreducible backgrounds from single-top and multijet QCD production, which generate large missing transverse momentum and cannot be efficiently suppressed by simple kinematic cuts. Meanwhile, the channel with fully hadronic top decay and leptonic Z decay loses the strong background rejection provided by an isolated high- p_T charged lepton from the semileptonic W decay and therefore suffers from severe multijet contamination. Given the limited signal statistics and intractable background levels, we do not perform full detector simulations or significance calculations for these channels. More sophisticated multivariate discriminants or advanced jet-substructure tagging algorithms may partially improve their discovery potential, and could be investigated in dedicated follow-up studies.

Finally, we note that our cross-section calculations are performed at leading order using a single PDF set (NNPDF23LO1). A complete next-to-leading-order (NLO) analysis, including the corresponding K -factors and PDF uncertainties, is beyond the scope of this work. However, such higher-order corrections are not expected to qualitatively alter the main conclusions of our study. The dominant effect of NLO corrections is typically a moderate shift in the overall normalization, whereas the shapes of the kinematic distributions and the relative signal-to-background ratios are generally stable under such corrections. Moreover, PDF uncertainties are expected to be subdominant compared with the 20% systematic uncertainty adopted in our significance estimates. Therefore, our sensitivity projections remain robust as a conservative benchmark for the discovery potential at a future μp collider.

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